

AN
ELEMENTARY; PRACTICAL AND
THEORETICAL
TREATISE ON NAVIGATION:

WITH A
NEW AND EASY PLAN

FOR FINDING
DIFF. LAT., DEP., COURSE, AND DISTANCE BY PROJECTION.

BY M^R F. MAURY,
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SECOND EDITION, REVISED AND CORRECTED.



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EDWARD C. BIDDLE, NO. 6 SOUTH FIFTH STREET.
1843.

NOTICES
OF FIRST EDITION OF
MAURY'S NAVIGATION.

“U. S. N. S., New York, January 19, 1836.

“Dear Sir,—I have had much pleasure in the perusal of your “New Theoretical and Practical Treatise on Navigation;” the plan and arrangements of which are original; it contains little or nothing superfluous, and every part of it appears to be as clear and intelligible as the nature of the subject will admit. Such a work has long been wanted in our Naval Schools, and on board our vessels of war. I intend to make use of it in the Naval School on this station; and I recommend it to be used by all the professors of Mathematics, and Nautical Science in the Navy of the United States.

Yours respectfully,

EDW. C. WARD.

“Passed Midshipman M. F. Maury.
U. S. Navy.”

Prof. Math. U. S. Navy.”

“U. S. Navy Yard, Gosport, March 7, 1836.

“I have examined a Treatise on Navigation written by M. F. Maury of the U. S. Navy; and have no hesitation in recommending it to the students of that science. The explanations are clear, the rules are illustrated by many examples, and the new arrangement of some of the tables exemplify the calculations of the navigator. Mr. Maury is deserving of great credit for the work, and I wish him every success.

P. J. RODRIGUEZ.

“Navy Department, April 9, 1836.

“Sir,—I have to request that you will add the “New Theoretical and Practical Treatise on Navigation,” by M. F. Maury, Passed Midshipman, to the list of books furnished vessels of the navy going to sea.

I am respectfully yours.

(Signed,) M. DICKERSON.”

“COM. JOHN RODGERS,
President of the Board of Navy Commissioners.”

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PREFACE.

THE object of the present volume, is to place in the hands of students, and especially of the Midshipmen of the United States Navy, a Text Book, in which the theory as well as the practice of Navigation, is explained and taught.

It is not pretended that new theories are set forth, or that new principles are established in this work; but it is believed that those which have already been established, are here embodied in such a form, that the means of becoming a theoretical as well as a practical navigator, are placed within the reach of every student.

For this purpose the works of Bonnycastle, Colburn, Hutton, Legendre, Davies, Bowditch, Lardner, Hassler, Kelly, Keith, and La Place, have been consulted.

Care has been taken to introduce only those theorems upon which the problems in Navigation immediately depend, and which it is necessary to understand, in order, satisfactorily to comprehend the principles of Mathematics and Astronomy, involved in the solution of these problems.

The fear of introducing more than is essentially requisite for this purpose, may have led to an error on the other extreme, by causing something to be omitted, which should have been inserted; but if such a fault be detected in the work, while it will be readily admitted on one hand to be a fault, it can scarcely be unjust on the other, to say that the error is on the safe side; especially when they who judge, are reminded, that there is not throughout the whole work, a single principle laid down which does not serve at once as a rule, or as the basis of a rule, or as reference in some succeeding demonstration, position, or explanation either to prove, establish, or elucidate; and moreover, that there are many, (perhaps the greater number,) of those for whose benefit the work is chiefly designed, who, during the whole period of their service at sea, never have the advantage of instruction from a teacher of Navigation; consequently they have to depend upon their own exertions, and the books before them, for their proficiency as Navigators. How necessary is it then, that the work on Navigation for them, should be an elementary one, adapted to the capacity of all, and that it should not embrace the widest range; more particularly so, as there is not yet any regular system of education provided for the Navy.

The idea of such a work as the present, grew out of the author's own experience, and was suggested to him by his own wants while

a student of Navigation ; if it be not sufficient for the supply of similar wants on the part of others, it is hoped that it will, at least, serve to provoke some more capable pen to undertake and complete what is here attempted.

A more elementary work than any hitherto published on Navigation is much required as a school book in the United States. The attention of teachers of Navigation throughout the country is respectfully invited to it.

These pages were written chiefly on board of a man-of-war, in the midst of the various calls of duty, and the thousand interruptions incident to such a place ; the author trusts that this circumstance will ensure him on the part of his brother officers, and those into whose hands his work may fall, the indulgence usually claimed for inexperienced authors.

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ALGEBRA.

B

ALGEBRA.

§ I. **ALGEBRA** is a method of computation, in which magnitudes, or quantities, are represented by means of the letters of the alphabet. These letters have no positive or fixed value; they only stand for the quantities to be computed.

§ II. In algebra, known quantities are expressed either according to their numerical value, or by the first letters *a, b, c, etc.*, of the alphabet.

§ III. And quantities of unknown value are usually represented by the last letters, *x, y, z, etc.*

§ IV. In algebraical computations, certain characters, called *signs*, have been introduced, and are used in the place of written words: thus,

+ (*plus*) is the sign of addition.

− (*minus*) is the sign for subtraction.

× or . is the sign for multiplication.

= is the sign of equality.

÷ is the sign for division.

$a + b - c \times d = x + y$, or $\frac{x}{y}$, is read *a plus b minus c multiplied by d equals x divided by y.*

§ V. $a > b$ signifies that the former quantity (*a*) is greater than the latter (*b*).

$a < b$ signifies that the latter quantity (*b*) is greater than the former (*a*).

§ VI. (*:*) is to (*:*) as, represents equality of ratio, and denotes proportion: thus $a : b :: c : d$, (read *a is to b as c is to d*), signifies that the ratio of *a* to *b* is equal to the ratio of *c* to *d*, and that these four quantities are proportional.

§ VII. ∽ represents the difference between any two unknown quantities between which it is placed; thus, $x \smile y$ denotes the difference between *x* and *y*.

§ VIII. ∞ signifies that the quantity standing before it, thus, $x \infty$, is infinite in value.

§ IX. The numbers 2, 3, 4, etc., placed after a letter, thus, a^2 , a^3 , a^4 , denotes the 2d, 3d, 4th, etc., power of the quantity which that letter represents. The second power is the *square*, the 3d the *cube*, the 4th the *biquadrate*:

$$a \times a = a^2,$$

$$a^2 \times a = a^3,$$

$$a^3 \times a = a^4.$$

§ X. The numbers 2, 3, 4, etc., placed as above, are called the *indices* of the quantities to which they are affixed.

§ XI. The quantity which is a constant multiplier in the involu

tion of a power, is called the root of that power: thus, (§ IX.,) a is the cube root of a^3 , and the square root of a^2 .

§ XII. $\sqrt{\quad}$ is called the *radical sign*; and \sqrt{x} , or $\sqrt[2]{x}$, also $x^{\frac{1}{2}}$, denotes that the square root of x is the quantity alluded to; so $\sqrt[3]{y}$, or $y^{\frac{1}{3}}$, denotes the cube root of y : and so on with the other numbers, or with letters thus placed.

The number or letter placed over the radical sign, thus, $\sqrt[n]{\quad}$, or placed thus, $x^{\frac{1}{n}}$, etc., is called the *exponent*.

§ XIII. A *surd* is a quantity to which the radical sign ($\sqrt{\quad}$) is prefixed, and whose root cannot be expressed by numbers; thus, $\sqrt{3}$, $\sqrt[3]{5}$, are surds.

§ XIV. A *term* is any quantity that is separated from another by a sign; thus, a , b and x , are terms in the compound quantity $a + b - x$.

§ XV. The number, (5,) or the letter, (c,) that is prefixed to any quantity, (5 a , or $c a$,) is called the *coefficient* of that quantity.

§ XVI. *Like quantities* consist of the same letters as $a b + 4 a b - 2 a b$.

§ XVII. *Unlike quantities* consist of different letters, as $x + 2 y \times a + b$.

§ XVIII. When a quantity has no sign prefixed to it, +, or plus, is understood.

§ XIX. $\sqrt{x^2 + y}$, or $(x^2 + y)^{\frac{1}{2}}$, denotes that these two quantities are as one, and that the square root of their *sum* is alluded to by the radical sign, or the exponent $\frac{1}{2}$. Thus, the square root of $64 + 36$, or $\sqrt{64 + 36}$, is $8 + 36 = 42$, but the square root of $64 + 36$, or $(64 + 36)^{\frac{1}{2}}$, under a *vinculum*, is 10, for $64 + 36 = 100$, and the square root of 100 is 10.

§ XX. Addition is performed by collecting several quantities in a more simple form: $3 a + 10 a + a = 14 a$, (read *fourteen times a*). 1 is understood to be the coefficient of every quantity that has no coefficient prefixed to it.

§ XXI. When the quantities to be cast up have unlike signs, the problem is solved, partly by addition and partly by subtraction. If the *negative* be greater than the *positive* quantities, the negative sign must be retained in the result. The sum of $6 - 10 + 9$, is 5; for 6 and 9 are here positive quantities, and make 15; and $15 - 10 = 5$: and the sum of $6 - 10 - 9$, is -13 ; for $-10 - 9 = -19$; and $-19 + 6$, or $6 - 19 = -13$. Every quantity, which has not a sign prefixed to it, is understood to be positive.

To add together, $4 x - 9 x + 3 x - 5 x$. Collecting all the positive quantities into one sum, and all the negative quantities into another, then taking the less from the greater, the remainder, (prefixing the sign of the greater,) is the answer. Thus:

$$\begin{array}{r}
 4 x - 9 x \\
 x - 5 x \\
 3 x \\
 \hline
 8 x - 14 x = -6 x
 \end{array}$$

$$\begin{array}{r}
 -6ay^2 \\
 4ay^2 \\
 9ay^2 \\
 \hline
 \text{Ans. } 7ay^2
 \end{array}
 \qquad
 \begin{array}{r}
 10ba \\
 2ba \\
 -19ba \\
 \hline
 -7ba
 \end{array}
 \qquad
 \begin{array}{r}
 -ac \\
 7ac \\
 -9ac \\
 \hline
 -3ac
 \end{array}$$

§ XXII. If all the terms be not like quantities, the positive and the negative of those that are like must each be added up separately; then these two sums must be subtracted, the one from the other, and the remainder thus obtained, connected by its proper sign to the unlike quantities, shows the answer. To add together:

$$\begin{array}{r}
 4x + 6a - c^2 + 9y \\
 -10x - 5a + c^2 - 5y \\
 x - a + 4c^2 + 3y \\
 2x + 2a \\
 \hline
 -3x + 2a + 4c^2 + 7y
 \end{array}$$

The positive x 's are, $4x + x + 2x = 7x$; the negative are, $-10x$. The difference is, $(7x - 10x =)$, $-3x$. The positive a 's are, $6a + 2a = 8a$; the negative are, $-5a - a = -6a$. The difference is, $(8a - 6a =)$, $2a$. The positive c^2 's are, $c^2 + 4c^2 = 5c^2$; the negative is, $-c^2$. The difference is, $(5c^2 - c^2 =)$, $4c^2$. The positive y 's are, $9y + 3y = 12y$; and the negative are, $-5y$. The difference is, $(12y - 5y =)$, $7y$.

To add together:

$$\begin{array}{r}
 \sqrt{ab - a^2 + 2xy - 6x + 10a^2 - 9xy - a^2b + y - a} \\
 -\sqrt{6ab + 3a^2 + \sqrt{5ab - x + xy + a^2b - 4xy - 2x} \\
 a - c^2 + 4y - \sqrt{2ab - 2a^2 - 3xy + 4x + 14xy + a^2b} \\
 \sqrt{ab - a^2 + 2xy + 6x - a^2b + a - c^2 + 4y} \\
 -\sqrt{6ab + 10a^2 - 9xy - x - 2a^2b + a + y} \\
 \sqrt{5ab + 3a^2 + xy - 2x + a^2b} \\
 -\sqrt{2ab - 2a^2 - 4xy + 4x} \\
 \qquad \qquad \qquad -3xy \\
 \qquad \qquad \qquad 14xy \\
 \hline
 -\sqrt{4ab + 20a^2 + 2xy + 2x - a^2b + 2a - 2c^2 + 10y}
 \end{array}$$

To add together:

$$\begin{array}{r}
 y + ab + dx^3 - z^2a \\
 y + cb + bx^3 - z^2x \\
 \hline
 2y + (a+c)b + (d+b)x^3 - z^2(a+x)
 \end{array}$$

To add together:

$$\begin{array}{r}
 a + b + c - x \\
 2a - b - 2c - 2x \\
 -a + ab - c - 4x \\
 \hline
 2a + ab - 2c - 7x
 \end{array}$$

§ XXIII. Subtraction is the reverse of addition. The method of performing it consists in changing, or reversing, all the signs of the subtrahend, (i. e., making the positive negative, and the reverse,) and then proceeding as in addition; viz., by adding together like quantities that have like signs; and subtracting from each other like quantities that have unlike signs. The quantity or the quantities that result from this operation is the remainder.

To subtract $a - b$ from $a + b$.

$$\begin{array}{r} \text{Changing the signs of the subtrahend} \\ a + b \\ a - b \\ \hline \text{The remainder} \\ \hline \hline = 2b \end{array}$$

This will be readily understood by ascribing a numerical value to a and b . Let $a = 6$ and $b = 4$; now subtract $6 - 4$ from $6 + 4$; $6 - 4 = 2$, and $6 + 4 = 10$, and $10 - 2 = 8$ (or twice $4 = 2b$.)

To subtract $a + b$ from $a - b$.

$$\begin{array}{r} a - b \\ a + b \\ \hline - 2b \\ \hline \hline \end{array}$$

To subtract :

$$\begin{array}{r} 2xy + b^2c - 3a \\ xy - b^2c + a \\ \hline xy + 2b^2c - 4a \\ \hline \hline \end{array}$$

To subtract :

$$\begin{array}{r} 10x - y - ac \\ - 3x - 2y - 4ac \\ \hline 13x + y + 3ac \\ \hline \hline \end{array}$$

§ XXIV. In the multiplication of letters, the product of any two factors is obtained and expressed by prefixing, as coefficients, the several letters contained in the multiplier, to those of the multiplicand. Thus:

$$\begin{array}{r} x + y \\ \text{by } a + b \\ \hline ax + ay + bx + by \\ \hline \hline \end{array}$$

§ XXV. Multiplication is denoted thus, $a \times b$; or thus, $a . b$; or thus, ab .

† 1. By recollecting that a times b , and b times a , are expressions for the same product, just as 4 times 9, and 9 times 4, express the same number, it will at once be understood how there is no difference in the value of the quantity, whether it be expressed ab , or ba . But when several letters are contained in the same term, as $a \times c \times b$, it is generally expressed by the letters arranged in alphabetical order; thus, abc .

§ XXVI. To multiply $x + y$ by $a - b$. Ans. $ax + ay - bx - by$.

When terms of *unlike* signs are multiplied together, their product is negative, and (—) must be prefixed to it.

§ XXVII. The product of $x + y$ by $x + y$ is $x^2 + 2xy + y^2$, or $xx + xy + xy + yy$. The former, being the shorter method of writing it, is the better. The small figures that are affixed show how many times the letter with which they are connected enters as a factor in the term.

§ XXVIII. The product of $x + y$ by $-x - y$, is $-x^2 - 2xy - y^2$.

§ XXIX. The product of $x + y$ by $x - y$, is $x^2 - y^2$; for xy occurring twice in the product, and with unlike signs, (+ and —), cancels itself.

$$\begin{array}{r}
 \text{To multiply } x^2 - y^2 + 3xy \\
 \text{by } \quad \quad \quad x + y \\
 \hline
 x^3 - xy^2 + 3x^2y \\
 \quad \quad \quad x^2y - y^3 + 3xy^2 \\
 \hline
 x^3 + 2xy^2 + 4x^2y - y^3
 \end{array}$$

§ XXX. Unless the quantity be under a vinculum, the index applies only to that letter with which it is in juxta-position. The term xy^2 denotes the product of x and y^2 ; and x^2y denotes the product of the two factors x^2 and y . The term $(xy)^2$, or xy^2 , denotes the square of the product of the two factors x and y . Let $x = 4$ and $y = 3$: $x^2 = 16$ and $y^2 = 9$. The value then of the first term (xy^2) is $4 \times 9 (= 36)$; of the second, (x^2y), it is $16 \times 3 (= 48)$; and of the third, ($(xy)^2$), it is the square of the product, (12), of 3 and 4, ($= 144$).

$$\begin{array}{r}
 \text{To multiply } ax + ab - a^2 \\
 \text{by } \quad \quad \quad a + x + b \\
 \hline
 a^2x + a^2b - a^3 \\
 \quad \quad \quad ax^2 + abx - a^2x \\
 \quad \quad \quad \quad \quad \quad abx + ab^2 - a^2b \\
 \hline
 ax^2 - a^3 + 2abx + ab^2
 \end{array}$$

$$\begin{array}{r}
 \text{To multiply } -5a - 4x \\
 \text{by } \quad \quad \quad 2 - a - x \\
 \hline
 -10a - 8x \\
 \quad \quad \quad 5a^2 + 4ax \\
 \quad \quad \quad \quad \quad \quad 5ax + 4x^2 \\
 \hline
 -10a + 5a^2 - 8x + 9ax + 4x^2
 \end{array}$$

§ XXXI. When the same letter enters into both factors, the product is obtained by adding the indices or exponents of the letter, as $a^3 \times a^2 = a^5$; for $3 + 2$ is 5. And $x \times x$ is x^2 ; 1 is the index and coefficient of every letter which has no other index or coefficient expressed.

§ XXXII. Division is the converse of multiplication.

4 times b divided by 4 gives b . So a times b divided by a gives b .

$$\begin{array}{r}
 4ax + 2x = 2a \\
 4ax \div 2a = 2x
 \end{array}$$

§ XXXIII. Division being the converse of multiplication, the product of the divisor by the quotient gives the dividend. Thus, $2x \times 2a = 4ax$.

§ XXXIV. When the dividend and divisor are powers, or are roots, of the same quantity, the index, or exponent, of the divisor, minus that of the dividend, is the quotient. Thus, $x^3 \div x^5 + x^2 = x + x^5$.

§ XXXV. When all the terms of the dividend have letters that are common to the divisor, the operation of dividing is performed

by striking out from the dividend the letters of the divisor, and dividing the coefficients. Thus,

$$\begin{aligned} 8xy + 2y &= 4x \\ 12axz + 3ax &= 4z \\ 6xy^2z + 2xy &= 3yz. \end{aligned}$$

§ XXXVI. Division is sometimes expressed without being performed, as $x + y + a$, or $\frac{x+y}{a}$.

§ XXXVII. This last quantity is a fraction. Fractions in algebra are multiplied, divided, etc., after the same manner by which such operations are performed in common arithmetic.

§ XXXVIII. Two or more quantities, with the sign (=) of equality between them, constitute an equation; as $x + 10 = 14 + 6$.

§ XXXIX. All the quantities or terms that are on either side of the sign (=) constitute a *member* of the equation; $x + 10$ is the *first*, and $14 + 6$ is the *second*, member of the equation $x + 10 = 14 + 6$. The two members, necessarily, are always equal to each other.

§ XL. In the process of solving, or reducing, an equation, the known quantities, or terms, of the equation, are all collected and arranged with their proper signs, in one member of the equation, and the unknown quantities in the other member, as $x = 14 + 6 - 10$. The value of x in this equation, is therefore equal to 10.

§ XLI. In transposing a term from one member to another, the equality of the two members of the equation is preserved, by *changing the sign* of the term transposed. By transposition, *positive* terms become *negative*, and the reverse; also the sign (\times) of multiplication becomes ($+$) the sign of division, and vice versa. Thus, in the equation above $x + 10 = 20$, (or $6 + 14$;) by transposition, $x = 20 - 10$. Also, in the equation $12 \times 4 = 8 \times 6$, by transposing we have $\frac{12}{8} = \frac{6}{4}$; and the equation $a + b = x + y$, by transposition, becomes $a \times y = x \times b$.

To find the value of x in the equation $a + x - c = b + 14$; by transposing a and $-c$, and changing their signs, the value of x is obtained: it stands thus, $x = b + 14 - a + c$.

Ex. $a - x^2 + y = 90 + a - 40$. To find y ; a being in each member, and having the same sign, cancels itself, and may therefore be stricken from the equation; for if transposed, the expression would be $a - a$, which two balance each other. The value of y is obtained then, by transposing $-x^2$, when the equation stands thus, $y = 90 - 40 + x^2$; in its most simple form, thus, $y = 50 + x^2$.

Ex. $8x - 10 = 80 + 2x$. To find x ; transposing and placing the known and unknown quantities on opposite sides, the equation stands, $8x - 2x = 80 + 10$. Subtracting and adding, it becomes $6x = 90$; and $x = 15$, (by division).

Ex. $x + 2 + 10 = 14 + 8$. To find x ; $\frac{x}{2}$ or $\frac{x}{2} = 14 + 8$

—10. The value of the second member of the equation is 12; therefore $\frac{x}{2} = 12$; and by multiplication, $x = 24$.

From the two last examples, this general conclusion may be drawn, viz.:

§ XLII. When a multiplier of either member is transposed, it becomes divisor to the other member; and vice versa.

Ex. $8x^2 + ax - 10x = 14x + 4x^2 + 16x$. To find x . x is common in every term of each member of the equation. Then, dividing by x , the equation becomes $8x + a - 10 = 14 + 4x + 16$; transposing and placing all the x terms alone in one member, $8x - 4x = 14 + 16 + 10 - a$; subtracting and adding, $4x = 40 - a$; dividing, $x = 10 - \frac{a}{4}$.

Ex. $3bx - 9ab = 3y$. To find x . Transposing, $3bx = 3y + 9ab$; dividing, $x = \frac{3y + 9ab}{3b} = \frac{y}{b} + 3a$. Therefore, $x = \frac{y}{b} + 3a$.

Ex. $\frac{x}{3} + \frac{x}{2} = 10$. To find x . Clearing the equation of fractions; 1st, $x + \frac{3x}{2} = 30$; 2d, $2x + 3x = 60$; adding, $5x = 60$; dividing, $x = 12$.

§ XLIII. Thus an equation is cleared of fractions *by multiplying every term (except the fraction itself) by the denominator*.

Ex. 1. The commander of a man-of-war is desirous of having his ship calked, that he may proceed on his voyage. His own calkers can finish the job of calking in 10 days. But he employs a gang from the shore, that could finish the whole work in 6 days, to assist his. How many days' job are there for both gangs together?

Let x denote the job of work. Then $\frac{x}{10}$ is one day's work for the ship's calkers; $\frac{x}{6}$ is one day's work for the shore gang; and $\frac{x}{10} + \frac{x}{6} = 1$ day's work for both gangs together. Clearing this equation of fractions, $6x + 10x = 60$; and $16x = 60$, or $x = 3\frac{3}{4}$, the number of days.

Ex. 2. A vessel, after an engagement, could muster only 238 able bodied men; on examining her list of sick and wounded, she found her loss in killed to be $\frac{1}{3}$ of her whole crew, and in wounded $\frac{1}{2}$ of the whole crew. What crew had she when she went into action?

Let x denote her crew when the action commenced. Thus, $\frac{x}{3} + \frac{x}{2} + 238 = x$. Clearing the equation of fractions, $2x + 3x + 3570 = 6x$; transposing, $3570 = x$, or $510 = x$, the whole crew when the action commenced.

§ XLIV. When the value of more than one unknown quantity is required, the problem, if definite, comprises conditions for as many equations as there are quantities required. In such cases the unknown quantities (x, y, z , etc.) have the same value in all the equations; i. e. x in one equation is equal to x in another of the same set. When the value of x, y , or z , is found, its value is substituted in its stead.

Ex. $x + y = 44$
 $x - 3y = 36$ } To find x and y ; $x = 44 - y$, and $x = 36 + 3y$; thus $44 - y = 36 + 3y$; transposing, $4y = 8$, or $y = 2$; and substituting, $x = 44 - y$, or $x = 42$.

Ex. $x + y + z = 60$
 $x + 4y + 3z = 144$
 $2x + y + 8z = 132$ } To find x, y , and z ; $x = 60 - y - z$; $x = 144 - 4y - 3z$; and $x = 66 - \frac{y}{2} - 4z$. Eliminating x ; $60 - y - z = 144 - 4y - 3z$; and $144 - 4y - 3z = 66 - \frac{y}{2} - 4z$. Transposing, to find the value of y ; $3y = 144 - 60 - 2z$, or $y = 28 - \frac{2z}{3}$; again, $7y = 288 - 132 - 8z + 6z$, or $y = \frac{156 - 2z}{7}$; eliminating y , $28 - \frac{2z}{3} = \frac{156 - 2z}{7}$; clearing the equation, $14z - 588 = 468 - 6z$; transposing, $20z = 120$ or $z = 6$.

For z , in the equation $y = 28 - \frac{2z}{3}$, substituting its value (6), $y = 28 - \frac{12}{3}$, or $y = 24$. And for z and y , in the equation $x = 60 - y - z$, substituting their values (24 and 6), $x = 60 - 24 - 6$, or $x = 30$.

§ XLV. There is another manner of expressing certain equations which do not involve more than 4 terms, and when thus expressed the terms are said to be *proportional*. Thus the proportion $3 : 4 :: 6 : 8$, is but another method of expressing the equation $3 + 4 = 6 + 8$, or $\frac{3}{4} = \frac{6}{8}$. The dots (:) being an abbreviation of the sign (+) of division; and the dots (::) being another form for expressing the sign (=) of equality, to show that the *ratio* between the quantities on each side of it is the same.

§ XLVI. Whence it may be inferred as a general rule, that, *if the quotient of two quantities be equal to the quotient of two others*, these four quantities are proportional. And,

† 1. That the ratio between either divisor and its dividend, is equal to the ratio between the other divisor and its dividend.

§ XLVII. By this rule we have $4 : 3 :: 8 : 6$, for $\frac{4}{3} = \frac{8}{6}$. This form of expression for the proportion ($3 : 4 :: 6 : 8$) first quoted, is called "*invertendo*," (from inverting the divisor and dividends,) thus $3 + 4 = 6 + 8$, and *inversely* $4 + 3 = 8 + 6$.

§ XLVIII. By the same rule we also have $3 : 6 :: 4 : 8$. This form of expression for the proportion ($3 : 4 :: 6 : 8$) is called

“*alternando*,” for by taking the terms *alternately* we have $3 \div 6 = 4 \div 8$.

§ XLIX. Whence also another general rule in proportions: that, if four quantities be proportional, they are also proportional when taken *inversely*, or when taken *alternately*.

§ L. $3 \div 4 = 6 \div 8$, by transposition $3 \times 8 = 6 \times 4$; wherefore, also, if the product of two factors be equal to the product of two other factors, those four factors are proportional, as $3 : 6 :: 4 : 8$.

GEOMETRY.

PART I.

GEOMETRY.

DEFINITIONS.

- § 1. *A point* is an atom of space.*
§ 2. *A line* (——) is length without breadth.
§ a. The track of a moving point would be a line.
§ b. The extremities of every line are points.
§ c. A line may be of any length.
§ d. Lines are either straight or curved.
§ 3. *A straight or right line* (——) is the shortest line that can be drawn from one point to another.

§ a. If two lines meet each other they form an angle.



§ 4. *A plane* is an extent, (as the surface of this page), on which the straight line will lie that joins any two points which are in that extent.

§ a. A plane is without thickness, and it may be of any length and breadth.

§ b. The limits of every plane are lines.

§ c. If two planes cross each other, the line of their intersection is a straight line.

§ 5. Two lines diverging from, or meeting in, the same point, form an *angle*.

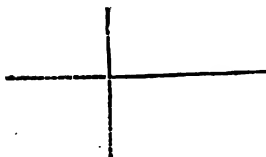


§ a. The point in which the lines join or cross each other, is called the *angular point*.

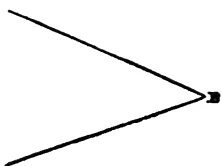
§ b. Angles are either right, acute, or obtuse.

§ 6. When one straight line stands upon another, without inclining to either side, the angle, or angles, which these two lines form, are called *right angles*.

§ a. All right angles are equal, and every one contains 90° (*degrees*). (Vide § 49, § d.)

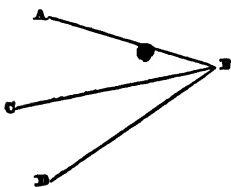


* According to the *atomic* theory of matter, all bodies are composed of indivisible and impenetrable particles, called atoms; an atom then is to matter what a point is to space; hence the idea of an atom of space.



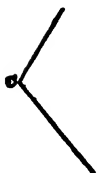
§ *b*. Straight lines that form right angles with each other, are said to be *perpendicular* the one to the other; and those that are perpendicular are at right angles with each other.

§ 7. Every angle which is *less* than a right angle, is an *acute* angle.



§ *a*. Angles are generally particularized by means of letters; as the angle B. But when there are more angles than one at the same angular point, the angle to be particularized is made known by placing the letter at the angular point *between* the letters which stand for the lines that form the angle. As the angle A B C, or C B D.

§ 8. Every angle which is *greater* than a right angle is an *obtuse* angle; as the angle C.



§ *a*. An *oblique* angle may be either acute or obtuse.

§ *b*. Two angles are equal, when they contain the same number of degrees ($^{\circ}$), minutes ($'$), and seconds ($''$), or when the lines which form an angle have the same divergence from each other, which the lines have that form the other angle.

§ *c*. The difference between an oblique angle and a right angle is the *complement* of the oblique angle.



§ 9. *Parallel lines* are lines that have always the same distance between them. They lie in the same direction, and if lengthened, *ad infinitum*, would neither approach, or recede from, each other.

§ *a*. The distance of two parallel lines from each other is measured by any straight line (*p*) that may be drawn between them, perpendicularly from one to the other.

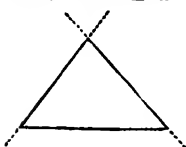
§ 10. A *figure* is any extent bounded by one or more lines, or surfaces.

§ *a*. The space included by a figure is called its *area*.

§ 11. A *superficies* is the surface of a figure.

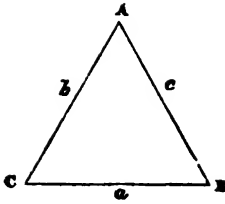
§ *a*. A superficies and a plane coincide, when a straight line, that joins any two points in the superficies, lies on that surface. The superficies of a figure is limited to the extent of the surface of that figure; but its plane is infinite.

§ *b*. When the superficies and the plane of a figure coincide, the former is called a *plane* superficies.



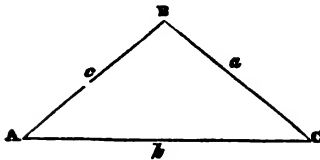
§ 12. A *plane triangle* is a figure that is formed by the intersection of three straight lines, any two of which intersect the other in different points.

§ *a*. The intercepted parts of these lines are called the *sides* of the triangle.



§ b. Every triangle has six parts; viz., three sides, and three angles.

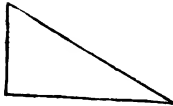
§ c. Triangles, with regard to their sides, are either equilateral, isosceles, or scaline; with regard to their angles, they are either acute, right, or obtuse angled.



§ 13. An equilateral triangle has its sides all equal to each other; viz., b equal to a or c .

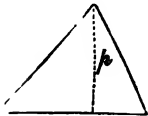
§ 14. An isosceles triangle has two equal sides; as, c and a .

§ a. An isosceles triangle may be either acute, right, or obtuse angled. Its third side (b) is its base.



§ 15. A scaline triangle has none of its sides equal.

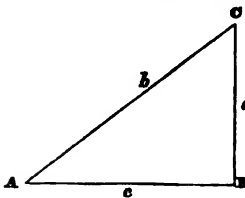
§ a. A scaline triangle may also be right or oblique angled.



§ 16. An acute angled triangle has each of its angles less than a right angle.

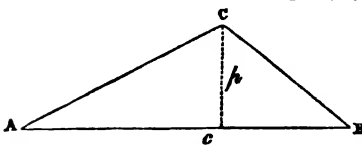
§ a. The vertex of a triangle is the angle that is opposite to the base of the triangle.

§ b. Any angle may be called the vertical angle, and consequently any side may be made the base.



§ 17. A right angled triangle has an angle (B), that is, a right angle.

§ a. The side (b) which subtends the right angle, is called the *hypotenuse*; the two other sides (c and a) are called *legs*.

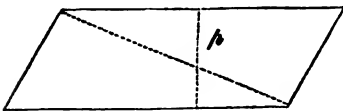


§ 18. An obtuse angled triangle has an angle (C) that is obtuse.

§ a. The altitude of a triangle is the perpendicular distance (p) of the vertical angle (C) (§ 16, § b.) from the base (c). The base must be produced to meet the perpendicular,

if the perpendicular fall without the triangle.

§ b. As any side (§ 16, § b.) may be made the base of a triangle, the perpendicular distance of any angle from its opposite side may be called the altitude, or height of the triangle.

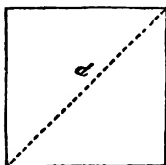


§ 19. A parallelogram is a right lined (§ 3.) quadrilateral figure, the opposite sides of which are equal and parallel.

§ a. The altitude of a parallelogram is the distance (p) (§ 9. § a.) between either pair of its opposite sides. To show the altitude of a parallelogram, either of two

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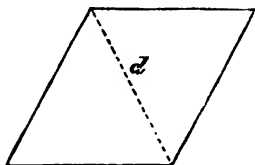
opposite sides may be produced until it meets at right angles, a perpendicular from the other side.



§ b. The measure or *area* of a parallelogram, is the product of its length and breadth, or of its base and altitude.

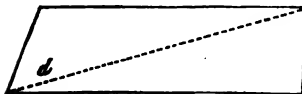
§ c. Any side of a parallelogram may be made its base.

§ 20. A *square* is a parallelogram of which all the sides and angles are respectively equal.



§ a. Every angle of a square is a right angle.

§ 21. A *rhombus* is a parallelogram that has all of its sides equal to each other; but its angles are not right angles.



§ 22. A *trapezoid* is also a four-sided figure, but only two sides of it are parallel, though they are not equal.

§ a. A diagonal is a straight line (d) that joins two opposite angles in

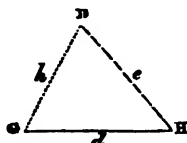
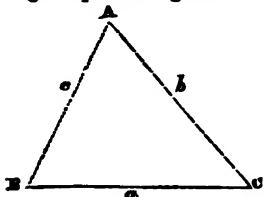
a four-sided figure.

§ b. The space included by a parallelogram is called (§ 10. § a.) its *area*, or measure, and is expressed (§ 19. § b.) by the product of its base and altitude.

§ 23. Two figures are *equal* when every *part* in one is equal to the part in the other, which corresponds to it; and two equal figures are always of the same magnitude.

§ a. And two figures are of the same *magnitude* when their areas are equal: a triangle and a parallelogram may be of the same magnitude, but they cannot be equal. A triangle only can be equal to a triangle; a parallelogram to a parallelogram, etc.

§ b. Two figures are *similar* when every *angle* in one is equal to the angle which corresponds to it in another figure. Figures only of the same class are similar, viz.: triangles are similar to triangles, parallelograms to parallelograms, etc.



§ c. *Homologous* sides, or angles, are the sides, or angles, which, in two equal or similar triangles, correspond by their relative positions to each other; thus c and h

are homologous sides, also b and e, and a and d.

§ d. Homologous angles are equal to each other. B and G are homologous angles; so also are A and D, and C and H.

AXIOMS.

§ 24. *Axioms* are self-evident truths, such as:

§ a. Things that are equal to the same, or to equal things, are themselves equal.

§ b. If equals be added to, or subtracted from, or substituted for, multiplied or divided by, the same or equal quantities, the sums or remainders, quotients or products, will be equal.

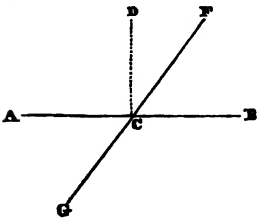
§ c. A part is less than the whole.

§ d. All of the parts are equal to the whole, and the whole to all of its parts.

PROPOSITIONS.

PROPOSITION I.

§ 25. Two straight lines which cross each other, make the two angles that are on the same side of either line, either two right angles, or equal to two right angles.

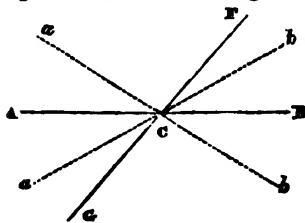


§ a. Let AB and FG be two straight lines that cross each other in C ; the two angles (FCA, FCB) on the same side of either line (AB) are either two right angles, or are equal to two right angles.

§ b. If AB and FG be perpendicular to each other, they cut each other at right angles (§ 6. § b.); and consequently each of the angles FCA and FCB is a right angle. But if AB and FG be not perpendicular to each other, from C , the point of their intersection, draw CD , which shall be perpendicular to (AB) one of them, then will DCA and DCB , the whole angular space from C , on one side of AB , be two right angles (§ 6. § b.); FCA and FCB , together, also comprehend the same angular space; therefore they are equal to $DCA + DCB$ (§ 24. § c.), which are two right angles. In a similar manner, it may be proven, that $BCF + BCG$, or $GCB + GCA$, or $ACG + ACF$, are equal to two right angles.

PROPOSITION II.

§ 26. All the angles, that any number of straight lines, which cross each other in the same point, make with each other, are together equal to four right angles.

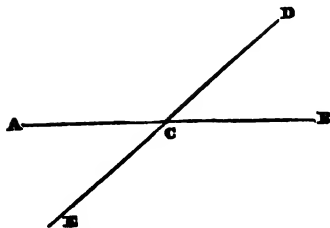


§ a. Since the two angles (§ 25.) FCA, FCB , are equal to two right angles, and also GCB, GCA , equal to two right angles; the four angles FCA, FCB, GCA, GCB , which two straight lines make by crossing each other, are equal to four right angles. If these four angles be divided into any number of other

angles, by straight lines ab, ab , crossing in the point C , the

sum of the divisions thus made will be equal (§ 24. § d.) to four right angles.

PROPOSITION III.

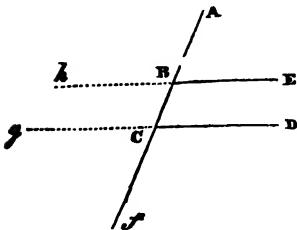


§ 27. When two straight lines $A B$, $D E$, cross each other, they make the angles $D C B$ and $A C E$, or $A C D$ and $B C E$, that are vertically opposite, equal to each other.

§ *a*. The angles $A C D$ and $D C B$, (§ 25.) are together equal to two right angles; by the same proposition $A C D + A C E$, are also equal to two right angles; wherefore (§ 24. § c.) $A C D + D C B = A C D + A C E$; and $A C D$ is a term in each member of the equation, and being taken away or cancelled, we have (§ 24. § b.) $D C B = A C E$.

§ *b*. It may be demonstrated in a similar manner, that the vertical and opposite angles $A C D$ and $B C E$, are equal to each other.

PROPOSITION IV.



§ 28. When a straight line ($A C$) crosses two others ($B E$ and $C D$) that are parallel, it makes the angles ($A B E$ and $A C D$), which are on the same side of the two lines, equal to each other.

§ *a*. If the straight line $A C$ cross the two others perpendicularly, the proposition becomes evident, for the angles formed would be (§ 6. § b.) right angles, and consequently equal.

§ *b*. But if they cross obliquely, it is obvious that if two lines be parallel to each other, they must have the same divergence from any straight line which crosses them; wherefore (§ 8. § b.) $A B E = A C D$; for $B E$ and $C D$ have the same divergence from $A C$.

§ *c*. In the same manner it may be shown that $E B f$ is equal to $D C f$.

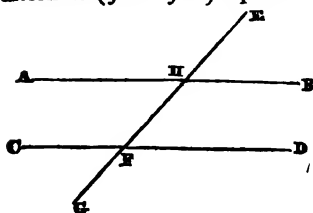
§ *d*. The angles ($A B E$, $h B f$, $A C D$, $g C f$, etc.) taken alternately on each side of $A f$, are called *alternate* angles.

§ *e*. The angles ($A B h$, $A B E$, $f C g$, and $f C D$), on the outside of the two parallel lines are called the *external* or *exterior* angles.

§ *f*. And the others are called *internal* or *interior* angles.

PROPOSITION V.

§ 29. A right line that crosses two others which are parallel, makes the interior angles (§ 28. § f.) (A H G and D F E) that are alternate (§ 28. § d.) equal to each other.

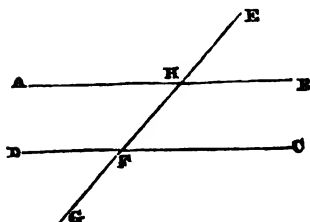


each other.

§ a. A H G and E H B are opposite and vertical angles, wherefore (§ 27.) they are equal to each other, and E H B (§ 28.) is equal to D F E; therefore (§ 24. § a.) A H G is equal to D F E.

§ b. In the same manner it may be proven that the alternate and interior angles B H F and C F H are equal to

PROPOSITION VI.



§ 30. If a straight line cross two others that are parallel, it makes the sum of the two internal angles (B H F and H F C) that are on the same side of it, equal to two right angles; and the alternate (§ 28. § d.) angles (E H B, A H F, H F C, and D F G) equal to each other.

§ a. The angles B H E and B H F (§ 25.) are together equal to two right angles, and (§ 28.) B H E is equal to H F C, therefore (§ 24. § b.) the internal angles (B H F and H F C) that are on the same side of E G, are together equal to two right angles.

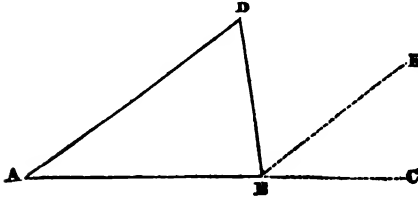
§ b. The alternate angle E H B = A H F, and H F C = D F G, because (§ 27.) they are vertically opposite; and (§ 29.) A H F = H F C; wherefore (§ 24. § a.) the alternate angles E H B, A H F, H F C, and D F G, are equal to each other.

§ c. After the same manner of demonstration, it may be proven, that the alternate angles A H E, B H F, H F D, and C F G, are equal to each other.

§ d. Cor. If a straight line (E G) cross two others (A B and D C) so as to make the sum of the internal angles (§ a.) (B H F and H F C) on the same side of it, equal to two right angles; or the alternate angles (§ b.) (E H B, A H F, H F C, and D F G,) equal to each other; or an exterior angle (§ 28.) (E H B) equal to its alternate and internal angle (H F C), these two straight lines are parallel.

PROPOSITION VII.

§ 31. The exterior angle (D B C) which is formed by producing any side (A B) of a triangle, is equal to the two interior and remote angles (A and D) of the triangle.



§ a. From B let B E be drawn parallel to A D ; also let A B be produced to C .

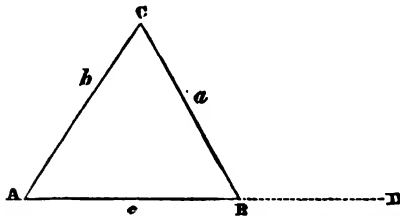
§ b. Because A D and B E are parallel (§ a.) and A C crosses them, the angles A and E B C (§ 28.) are equal to each other ;

moreover D B, by crossing the same two parallels (D A and B E), makes (§ 29.) the alternate and interior angles (D and D B E) equal to each other ; the exterior angle D B C is made up of D B E and E B C, then substituting the whole for its parts (§ 24. § d.) we have (§ 24. § b.) the exterior angle D B C = A + D, the two interior and remote angles in the triangle A D B.

§ c. If either of the two other sides be produced, the proposition is proven in the same way.

PROPOSITION VIII.

§ 32. The sum of the angles of a triangle is 180° , or two right angles.



§ a. Produce a side (c) of any triangle to D. Then C B A and the exterior angle C B D (§ 25.) are equal to two right angles : and the exterior angle C B D, (§ 31.) is equal to the two remote angles C and A : therefore (§ 24. § b.) C, A,

and C B A are equal to two right angles : and C, A, and C B A are the three angles of the triangle (A C B) proposed.

§ b. Cor. If two angles of one triangle be known, the third is also known. It is found by subtracting the sum of the two known angles from 180° .

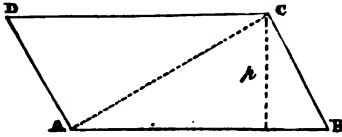
§ c. Cor. And the two acute angles of a right angled triangle (§ 17.) are together equal to one right angle.

§ d. Wherefore the acute angles of any right angled triangle are the complements (§ 8. § c.) of each other.

§ e. It is also evident from the above, that if two angles in one triangle be equal to two angles of another, the remaining angles are also equal to each other.

PROPOSITION IX.

§ 33. A diagonal (A C) of a parallelogram divides the parallelogram into two equal (§ 23.) triangles (A C D and A C B).



§ a. The opposite sides of a parallelogram (§ 19.) are parallel and equal to each other; wherefore $D C = A B$, $A D = C B$, and A C is common to the two triangles (A C D and A C B); therefore their sides are equal,

those of the one to those of the other. The angles D C A and C A B are equal to each other (§ 30.), for they are alternate angles (§ 28. § d.) made by A C crossing the two parallels D C and A B. For the same reason the alternate angles D A C and A C B made by A C with the parallels A D and C B, are equal to each other. Wherefore, the two angles (D C A and D A C) in one triangle being equal to two angles (A C B and C A B) in another, the remaining angles (D and B) (§ 32. § e.) are also equal to each other. Therefore, the triangle A C D having its sides and angles respectively equal to the sides and angles of the triangle A C B, is equal (§ 23.) to the latter, and the diagonal (A C) divides the parallelogram D A B C into these two equal triangles.

§ b. Scholium. The area (§ 10. § a.) of a parallelogram (§ 19. § b.) is the product of its base and altitude. The triangle A B C and the parallelogram D A B C have the same base (A B) and altitude (p), and the triangle is proven to be *half* of the parallelogram; Therefore,

§ c. Cor. The area or magnitude of a triangle is the product of its base by *half* its altitude.

§ d. Cor. The triangles into which a diagonal divides a parallelogram are both equal and (§ 23.) of the same magnitude.

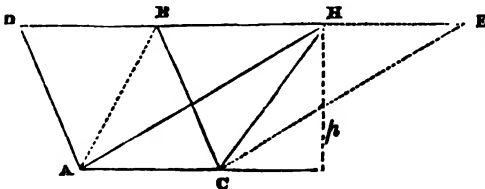
§ e. Scholium. $D A C + B A C$ (§ a.) = $D C A + B C A$, and $D = B$. Therefore,

§ f. Cor. The opposite angles of a parallelogram are equal.

§ g. Cor. If the opposite angles of a quadrilateral figure be equal, figure is a parallelogram.

PROPOSITION X.

§ 34. If a parallelogram (D A C B) and a triangle (A B C) stand upon the same base (A C), and between the same parallels (D B and A C), they have the same altitude (p), and the triangle is equal to *half* the parallelogram.



§ a. If one side (A B) of the triangle be diagonal to the parallelogram, the triangle falls within the parallelogram, and (§ 33.) the proposition is proven.

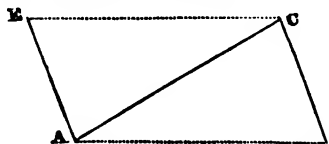
§ b. But if the vertex (H) of the triangle proposed, fall without the parallelogram, as A H C, continue one of the parallels (D B) on, through the vertex H, to E; and upon the base A C construct the parallelogram A H E C, to which the side H C of this triangle is diagonal (§ 22. § a.), therefore A H C (§ 33.) is half of the parallelogram A H E C. The parallelograms A H E C and D A C B (§ 19. § a.) have the same base (A C) and altitude (p), and the area of each (§ 19. § b.) is the product of A C by p ; wherefore (§ 24. § b.) these two parallelograms are of the same magnitude; therefore (§ 24. § a.) the triangle A H C is also equal in *magnitude* to half of either parallelogram, say D B C A; and p (§ 18. § a.) is also the altitude of the triangle A H C.

§ c. *Cor.* If a triangle and a parallelogram have their bases and altitudes equal, the triangle is equal in magnitude to half the parallelogram.

§ d. *Cor.* Triangles which have the same or equal bases and altitudes, are of the same magnitude.

PROPOSITION XI.

§ 35. When two sides (A C and C B) of any triangle (A C B) are equal to two sides (E A and A C) of another (A E C), if the angles (A C B and C A E), which these sides contain, be equal, the two triangles are equal.



§ a. Let one of the equal sides (A C) be made common to the two triangles proposed, by constructing them on opposite sides of it; the figure A E C B, thus formed, will be a quadrilateral.

§ b. By the conditions of the proposition, the angle E A C = A C B, and they are alternate angles, made by A C with the equal lines E A and C B, therefore (§ 30.) E A and C B are parallel; and the opposite straight lines (E C and A B) which join their extremities are also parallel, wherefore (§ 30.) the alternate angles E C A and C A B are equal to each other.

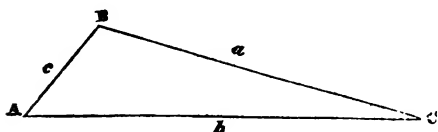
§ c. E and B are the remaining angles of the two triangles, and (§ 32. § e.) they are equal to each other, for (§ b.) E A C + E C A = A C B + C A B, therefore the angles of the two triangles proposed are equal to each other.

§ d. Now, since E A C + B A C = E C A + B C A the whole (§ 24. § d.) E A B is equal to the whole E C B; and (§ c.) E = B; these four are the opposite angles of the quadrilateral A E C B, which therefore (§ 33. § g.) is a parallelogram, and (§ 19.) E C = A B, and (§ 33.) A C divides the parallelogram into the two equal triangles A C B and A E C.

§ e. *Cor.* If the opposite sides of a quadrilateral figure be either parallel or equal, the figure is a parallelogram.

PROPOSITION XII.

§ 36. Either side (b) of any triangle is less than the sum of the two other sides (a & c).

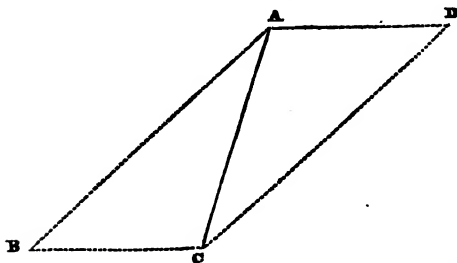


§ a . The straight line b , which joins the two points A and C, is less than the two straight lines c and a which join the same two points,

because the shortest distance between any two points (A and C) (§ 3.) is the straight line (b) which joins them; wherefore b is less than the sum of the two other sides of the triangle, or of any two lines that can join A and C.

PROPOSITION XIII.

§ 37. Any two triangles (C A B and D C A) are equal, if a side (A C), and the two angles (B A C and B C A) adjacent to it, in the one, be respectively equal to a side (C A) and the two angles (A C D and C A D) adjacent to this side in the other.



§ a . Let the equal side (A C) be made common, by constructing the two proposed triangles upon it, so that one of them may be on each side of the common line A C; the figure (B A D C) thus formed is a quadrilateral, and the side A C,

which is common to the two proposed triangles, is a diagonal of it.

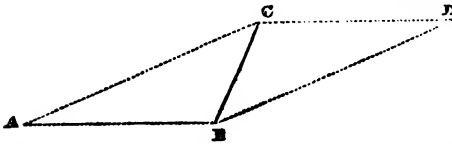
§ b . By the conditions of the proposition, the angles B A C and A C D are equal, and (§ 28. § d .) they are alternate angles, wherefore (§ 30. § d .) the two opposite sides (B A and C D) of the quadrilateral, are parallel. By supposition also, B C A and C A D are equal, and they are likewise alternate angles, wherefore (§ 30. § d .) the other two opposite sides (B C and A D) of the figure, are parallel.

§ c . If the opposite sides of a quadrilateral figure (§ 35. § e .) be parallel, the figure is a parallelogram; therefore B A D C is a parallelogram, and it is divided by the diagonal A C into the two proposed triangles C A B and D C A, which (§ 33.) are therefore equal to each other.

PROPOSITION XIV.

§ 38. Any triangles (B C A and D B C) are equal, if two angles (A & A B C) and an opposite side (C B), in one of the triangles

be respectively equal to two angles (D & $B C D$) and the corresponding side ($B C$), of the other triangle.



§ *a*. Let the equal side ($C B$) be made common to the two triangles, by constructing them so that one will be on each side of $C B$.

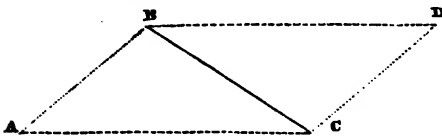
Then $C B$ becomes diagonal to the quadrilateral figure $A C D B$, which is formed by thus constructing the two proposed triangles.

§ *b*. By supposition, $A = D$, and $A B C = B C D$; wherefore (§ 32. § *e*.) the remaining angles $A C B$ and $C B D$ are equal to each other, and they are alternate angles, therefore (§ 30. § *d*.) the two opposite sides, $C A$ and $D B$, of the four sided figure, are parallel; the other two opposite sides, $C D$ and $A B$, are also parallel, the alternate angles $A B C$ and $B C D$ being equal, by the conditions of the proposition. Therefore, the quadrilateral $A C D B$ (§ 35. § *e*.) is a parallelogram, and is divided by the diagonal $B C$; (§ 33.) into the two equal triangles $B C A$ and $D B C$.

§ *c*. *Cor.* Hence it is inferred, that if two angles and a side of one triangle be equal to two angles and a side of another, the two triangles are equal.

PROPOSITION XV.

§ 39. Any two triangles ($B A C$ and $B D C$) are equal, if every side of the former be equal to its corresponding side in the latter: *e. g.* $A B = D C$; $A C = B D$; and $B C = C B$.



§ *a*. Let one of the sides ($B C$) be made common to the two triangles proposed, by constructing one of them on each side of

$B C$; the figure ($A B D C$) thus formed is a quadrilateral, and the common side $B C$ is a diagonal of it.

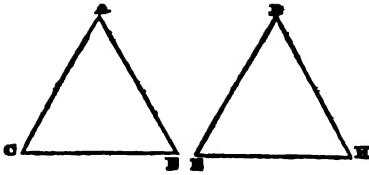
§ *b*. By the conditions of the proposition $A B$ is equal to $D C$, and $A C$ to $B D$, and they are opposite sides of the quadrilateral figure $A B D C$; and if the opposite sides of a quadrilateral figure be equal (§ 35. § *e*.), the figure is a parallelogram; therefore $A B D C$ is a parallelogram, and its diagonal $B C$ divides it into the two proposed triangles $B A C$ and $B D C$, which (§ 32.) are therefore equal.

§ *c*. *Scholium.* Since the corresponding parts in equal figures (§ 23.) are equal, it follows:—

§ *d*. *Cor.* That, if the sides of one triangle be equal to the sides of another triangle, the angles opposite the equal sides are equal.

PROPOSITION XVI.

§ 40. Every equilateral triangle ($A B C$) is also equiangular.



§ a. Let an equilateral triangle (E D H) be drawn, having its sides equal to those of the proposed triangle. The two triangles (§ 39.) are then equal, and the angles A and D which correspond, (§ 39. § d.) are equal to each other.

§ b. By the conditions of the proposition each side of A B C is equal to the same thing, and by construction, equal to either side of E D H; then $AB = EH$; and the two triangles (§ a.) being equal the angles D and C, which are opposite to those equal sides (§ 39. § d.), are equal. Wherefore A and C are each equal to D, and, therefore, (§ 24. § a.) equal to each other. In the same manner B and A may be proven to be equal to each other. Wherefore C and B are each equal to A, and consequently the equilateral triangle A B C is equiangular.

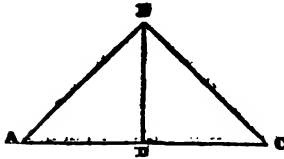
§ c. Cor. Every equiangular triangle is also equilateral.

§ d. Scholium. Since the three angles of any triangle (§ 32.) are together equal to 180° , and the three angles of an equilateral or equiangular triangle, are equal to each other:

§ e. Cor. Every angle in any equilateral or equiangular triangle; contains 60° .

PROPOSITION XVII.

§ 41. The angles (A & C) at the base of an isosceles triangle A B C (§ 14.), are equal to each other.



§ a. Let the base (A C) of the proposed triangle, be divided into two equal parts (A D & D C) by a straight line (B D) drawn from the vertex (B) of the triangle.

§ b. This line (B D) divides the isosceles triangle into the two triangles A B D and D B C; in which every side of the one, is equal to the side which corresponds to it in the other; viz: $AB = CB$ (§ 14.) for they are the legs of the isosceles triangle A B C; $AD = DC$ (§ a.), by construction; and B D is common to both of the triangles; therefore (§ 39.) these two triangles (A B D and D B C) are equal, and the corresponding angles A and C, being opposite to the common side B D, are therefore (§ 39. § d.) equal to each other.

§ c. Scholium. $ABD = CBD$, because (§ 39. § d.) they are opposite to the two equal sides A D and D C; for a similar reason BDA and BDC are also equal to each other, and (§ 25.) these are together equal to two right angles, therefore each of them is a right angle, and the right line B D (§ 6. § b.) is perpendicular to the base (A C) of the isosceles triangle A B C; wherefore:—

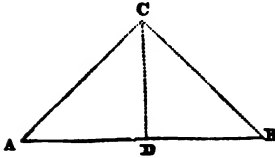
§ d. Cor. A straight line drawn from the vertex, so as to bisect

the base of an isosceles triangle, is perpendicular to the base, and it divides the vertical angle ($A B C$) into two equal parts, and the isosceles into two equal triangles. Also;

§ *e. Cor.* A straight line ($B D$) that is drawn perpendicularly from the vertex, to the base of an isosceles triangle, bisects the vertical angle and the base.

PROPOSITION XVIII.

§ 42. If only two angles (A & B) of a triangle ($A C B$) be equal, the triangle is isosceles.



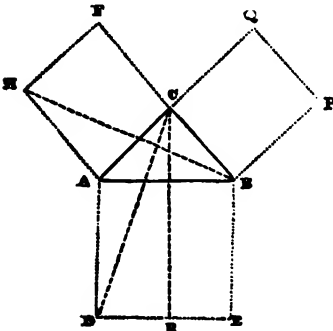
§ *a.* Call the other angle (C) (§ 16. § *b.*) the vertex of the proposed triangle; and from it, let the straight line $C D$ be drawn, so as to bisect the vertical angle (C) and divide the proposed triangle into the two $C A D$ and $C B D$, in which, by construction, the side $C D$ is common, the angles $A C D$ and $B C D$ equal, and (§ 42.) $A = B$; wherefore in these two triangles ($C A D$ & $C B D$), two angles and a side of the one are equal to the homologous angles and side of the other, and therefore (§ 38. § *c.*) these two triangles are equal to each other; consequently (§ 23.) the corresponding sides, $C A$ & $C B$, are equal, and hence, (§ 14.) the triangle $A C B$ is isosceles.

PROPOSITION XIX.

§ 43. In every right angled triangle ($A C B$), the square ($D A B E$) of the hypotenuse (§ 17. § *a.*) is equal in magnitude to the squares ($A C F H$ & $B C Q P$) of the two legs.

§ *a.* Let $D A B E$ represent the square (§ 20.) of the hypotenuse ($A B$), and $A C F H$ & $B C Q P$, the squares of the two legs $A C$ and $B C$; then (§ 43.) $D A B E = A C F H + B C Q P$.

§ *b.* From the right angle ($A C B$) let $C R$ be drawn parallel to the parallels (§ 20. & § 19.) $A D$ & $B E$, join $C D$ and $H B$.



§ *c.* The angles of a square (§ 20. § *a.*) are right angles, and all right angles (§ 6. § *a.*) are equal, therefore $H A C = D A B$; to each of these equals add the angle $B A C$, and the sums $H A B$ and $D A C$ (§ 24. § *b.*) will be equal; also $H A = A C$, because (§ 20.) they are sides of the same square $A F$; and $D A = A B$, because they also are sides of a square $A E$.

§ *d.* Wherefore, in the two triangles $A H B$ and $A D C$, the two sides $H A$ and $A B$ of the one,

are respectively equal to the two sides CA and AD , of the other, and the angles HAB and DAC , contained by these sides, are also equal (§ *c.*), therefore (§ 35.) the two triangles are equal.

§ *e.* The triangle ADC is equal in magnitude (§ 34.) to half of the parallelogram AR , for they stand upon the same base DA , and between the same parallels (§ *b.*) CR and AD . And the triangle AHB is equal in magnitude to half of the parallelogram AF , because they stand upon the same base AH , and between the same parallels HA and FCB .

§ *f.* The parallelograms AF and AR , being each double of either of the equal triangles (§ *d.*) AHB and ADC , are therefore (§ 24. § *b.*) equal to each other in magnitude; but the parallelogram AF (§ *a.*) is the square of the leg AC ; wherefore the square of the leg AC , and the parallelogram AR , are of the same magnitude.

§ *g.* By joining AP and CE , it may be demonstrated in the same manner, that the parallelogram BR , and the square (BPC) of the other leg BC , are of the same magnitude.

§ *h.* The parallelograms AR and BR make up the square ($DAEB$) of the hypotenuse (§ *a.*) AB , therefore (§ 24. § *a.*) the square of the hypotenuse (AB) is equal in magnitude to the sum of the squares of the two legs (AC & CB).

GEOMETRY.

PART II.

§ *c.* When the straight lines which contain an angle, are produced to the circumference of any circle that may be described from the angular point as a centre, that part of the circumference which the two lines intercept, contains the degrees, etc., which express the value of said angle. An angle is said to *stand* upon the part of the circumference that is thus intercepted. Neither the length of the lines which contain the angle, or the distance of it from the circumference by which it is measured, affects its angular value.

§ 50. *The radius* of a circle is a straight line (*e*) that extends from the centre to the circumference of the circle.

§ *a.* All the radii of the same, or equal circles, are themselves equal.

§ 51. *The diameter* of a circle is a straight line (*A C*) that passes through the centre of the circle, and is terminated at each end by the circumference of the circle.

§ *a.* Either of the two parts (*A B C* & *A H C*) into which the diameter divides the circle, is a *semicircle*.

§ *b.* *A segment* of a circle is any part (*g*) cut from the circle by a line (*f*), or a plane, which crosses the circle.

§ 52. *An arc* of a circle is any part of its circumference. That part (*A H*) of the circumference which bounds a segment (*g*) is an arc; and the straight line (*f*) which joins the extremities of an arc, is a chord.

§ *a.* The *complement* of an arc, or angle, is the difference between either and 90° ; and the *supplement* is what either wants of being 180° .

§ *b.* The radius and centre of a segment, or of an arc, are the radius and centre of the circle, of which the segment, or the arc, is a part.

§ *c.* Every arc or angle has its sine and co-sine, tangent and co-tangent, secant and co-secant, besides its versed sine and semi-tangent.

§ *d.* The "*co*" is an abbreviation for *complement*: the co-sine, co-tangent, etc., of an angle or an arc, are the sine, tangent, etc., of the *complement* of that arc or angle.

§ *e.* The sine, tangent, secant, etc., of an arc, are also the sine, tangent, secant, etc., of the supplement of that arc.

§ 53. *A chord* is a straight line (*t*) (§ 52.) that joins the extremities of an arc (*B C*).

§ *a.* Every arc has its chord.

§ *b.* When a chord passes through the centre of a circle, it becomes a diameter, and the arc it subtends is a semicircle.

§ 54. *A sine* is a straight line (*o*) that extends from one extremity (*B*) of an arc (*B C*) perpendicularly to the radius (*a C*) that joins the other extremity.

§ 55. *A versed sine* is that part (*s C*) of the radius which is intercepted between the sine and the extremity of the arc.

§ 56. *A tangent* is a line (*b*) that touches one extremity of an arc (*B C*), is perpendicular to the radius (*a C*) at that extremity,

and extends to another radius (e) produced through (B) the other extremity of the arc.

§ 57. A *secant* ($a B e$) is the produced radius (e) that intersects the tangent.

§ *a*. The co-sine, co-tangent, and co-secant, of the arc BC , are p , $D t$, and $a t$; and p (§ 19.) is equal to $a s$.

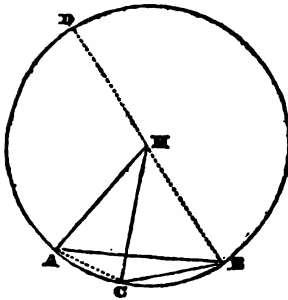
§ *b*. The sine, co-sine, tangent, co-tangent, etc., of an angle, is the sine, co-sine; tangent, co-tangent, etc., of the arc which subtends that angle.

§ *c*. These are sometimes called *trigonometric functions*.

§ 58. A *sector* is a figure (h) contained by two radii ($a B, a C$), and the arc ($B O$) between them.

PROPOSITION I.

§ 59. An angle ($A B C$) at the circumference, is equal to half of an angle ($A H C$) at the centre, if they stand upon the same arc ($C A$), of a circle.



§ *a*. Draw the chord AC , and from B let a diameter ($B H D$) be drawn. The radii HA, HC , and HB (§ 50. § *a*.) are equal, and the triangles $A H B$ and $C H B$ (§ 14.) are isosceles. The exterior angle $C H D$ (§ 31.) is equal to the two interior angles ($H B C$ and $H C B$) of the triangle $C H B$; for the same reason the exterior angle ($A H D$) of the triangle $A H B$, is equal to its two interior angles $H A B$ and $H B A$, which are equal to each other, because (§ 41.) they are at the base of the isosceles

triangle $A H B$; therefore $A H D$ is double of either of them, say of $H B A$. For the same reason $H B C$ and $H C B$ are equal to each other, and $C H D$ is double of either, say of $H B C$.

The difference between $C H D$ and $A H D$, is $C H A$, and the difference between their equals, viz., twice $H B C$ and twice $H B A$, is twice $A B C$. Wherefore $C H A$ (§ 24. § *b*.) is equal to twice $A B C$; or, which is the same thing, the angle ($A B C$) at the circumference, is equal to half of the angle $A H C$ at the centre of a circle, and both of them stand upon the same base.

§ *b*. An angle at the centre of a circle, is measured (§ 49. § *e*.) by the arc it stands upon; Wherefore—

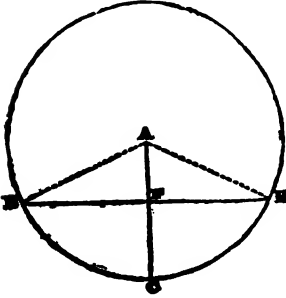
§ *c*. *Cor.* An angle at the circumference of a circle is measured by half the arc which subtends it.

§ *d*. *Cor.* All angles are equal that are at the centre, or all angles are equal that are at the circumference, if they stand upon the same, or equal arcs.

§ *e*. *Cor.* An angle that is at the circumference, and that stands upon a semicircle (§ *c*.), is a right angle.

PROPOSITION II.

§ 60. If the radius (A C) of a circle cut a chord (B H) at right angles, it bisects that chord and its arc (B C H).



§ a. Let the radii, A B and A H, be drawn to join to extremities of the chord (B H). The triangle (B A H) thus formed (§ 50. § a. & § 14.) is isosceles. And the straight line A F C bisects the vertical angle H A B (§ 41. § e.); for by the conditions of the proposition, A C is perpendicular to the chord B H, which is the base of the isosceles triangle B A H. Wherefore (§ 41. § d.) the two triangles B F A and H F A are equal, B H is bisected, and the angles B A C and

H A C are equal to each other, and being equal (§ 59. § d.) they stand upon equal arcs (B C & C H). Therefore the chord (B H), and its arc (B C H), are bisected by the radius A C.

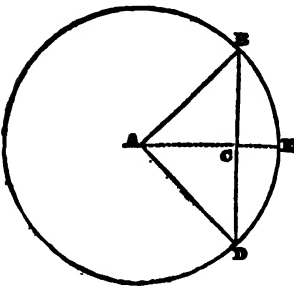
§ b. Cor. If a radius bisect an arc, it also bisects the chord of that arc, and cuts the chord at right angles.

§ c. Cor. If a radius bisect a chord, it also bisects the arc of that chord, and cuts the chord perpendicularly.

§ d. Cor. If a radius bisect a chord, or its arc, it also bisects the angle at the centre, which stands upon that arc.

PROPOSITION III.

§ 61. B C, half the chord (B D) of an arc (B E D), is the sine of C A B, half the angle (D A B) which that arc subtends.

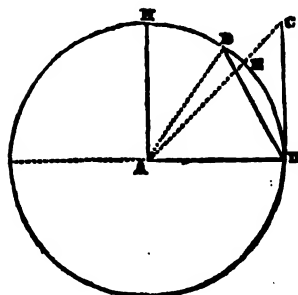


§ a. Let the chord be bisected at C, and through C let the radius A C E be drawn, then this radius (§ 60. § c.) bisects the arc B E D, cuts the chord B D at right angles, and also (§ 60. § d.) bisects the angle (D A B) which this arc subtends. Wherefore, B A C is half of the angle D A B, B C is half the chord of B E D, and it is perpendicular to the radius A C E; therefore (§ 54.) B C, half the chord of B E D, is the sine of C A B, which is half of the angle (D A B) proposed.

PROPOSITION IV.

§ 62. In any circle, radius (A D), the sine (A H) of 90° , the

chord (BD) of 60° , and the tangent (DC) of 45° , are all equal to each other.



§ a. Let A be the centre of the circle, the arc H B E D = 90° , the arc B E D = 60° , and the arc E D = 45° , and join B A and C A. The angles at the centre (A) (§ 49. § e.) are measured by the arcs they stand upon; therefore the angle D A C = 45° , D A B = 60° , and D A H = 90° ; A H then (§ 6. § b.) is perpendicular to A D, and A H (§ 54.) is the sine of the arc H B E D = 90° , it is a radius, and therefore (§ 50. § a.) equal to A D.

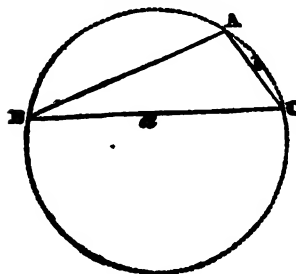
§ b. The radii A B and A D being equal (§ 50. § a.), makes the triangle B A D (§ 14.) isosceles; consequently (§ 41.) the angles A B D and A D B are equal. The angle D A B = 60° (§ 49. § e.) because it stands upon the arc B E D; wherefore (§ 32.) A B D + A D B = 120° , and being equal, the value of each is 60° ; therefore the triangle B A D is equiangular, and (§ 40. § c.) also equilateral; consequently (§ 13.) the chord (B D) of 60° is equal to radius (A D).

§ c. C D is the tangent of E D = 45° , and consequently (§ 56.) is perpendicular to A D; wherefore (§ 6. § b. & § a.) the angle C D A = 90° , and (§ 17.) the triangle A D C is right angled. The angle C A D = 45° (§ 49. § e.), because it stands upon the arc E D, therefore (§ 32. § d.) the angle A C D is also equal to 45° , and is equal to C A D, and (§ 42.) the triangle A D C is isosceles, and (§ 14.) the tangent (D C.) of 45° is equal to radius (A D).

§ d. Therefore the tangent of 45° (§ c.), the chord of 60° (§ b.), and the sine of 90° (§ a) being each equal to radius, are (§ 24. § a.) equal to each other.

PROPOSITION V.

§ 63. In every triangle, the angle (A) that is opposite to the greatest side (a) is the largest, and (B), that which is opposite to the smallest side (b), is the smallest angle of the triangle.



§ a. Describe a circle (A C H B) about the proposed triangles B A C, the circumference of which touches the three angular points (B, A, & C) of the triangle. Each side of the triangle then becomes the chord to the arc, which subtends the angle that is opposite to it.

§ b. The angle A stands upon the arc B H C, which is greater than either of the arcs (B A, A C) upon which the

two other angles (C & B) of the triangle stand. Each of these three angles (§ 59. § c.) is measured by half the arc that subtends it. Therefore A, which stands upon the greatest arc, and subtends the greatest side (a), is the largest angle of the proposed triangle.

§ c. Half the smallest arc (A C) (§ 59. § c.) measures the angle (B) that stands upon it; therefore B (the angle that is opposite to the smallest side) (b) is the smallest angle of the triangle.

DEFINITIONS.

§ 64. The *multiple* of a magnitude is the product of this magnitude and any other factor.

§ a. *Equimultiples* of magnitudes are the product of each of these magnitudes by the same or equal multipliers.

§ b. Thus, 15 and 30 are equimultiples of 3 and 6; for $3 \times 5 = 15$, and $6 \times 5 = 30$. Consequently (§ 24. § b.)—

§ 65. The same or equal multiples of equal magnitudes are equal to each other.

§ 66. *Ratio* is the relation which the value of any magnitude bears to that of another, and it is shown by dividing one magnitude by the other.

§ a. Thus, the ratio of a to b is $a \div b$, and is expressed thus,— $a : b$.

§ b. And if the ratio between two quantities (a & b) be equal to the ratio between two other quantities (c & d), this equality of ratio is expressed (§ XLV. Algebra) by writing the sign ($::$) between the two former and the two latter quantities, thus,— $a : b :: c : d$.

§ 67. *Proportion* consists in the equality of the ratios between magnitudes.

§ a. Thus, the ratio (§ 66.) of 3 to 4 is $\frac{3}{4}$; and the ratio of 6 to 8 is $\frac{6}{8} = \frac{3}{4}$; and these quantities are proportional; *i. e.* $3 : 4 :: 6 : 8$.

§ b. Four quantities are in *direct* proportion, when the 4th is equal to the quotient, which arises from dividing by the 1st quantity, the product of the 2d and 3d. Thus ($3 : 4 :: 6 : 8$), $4 \times 6 = 24$, and $24 \div 3 = 8$. So, also, $a : b :: c : d$, which in Algebra (§ XLV.) is but another form for expressing that $a \div b = c \div d$; and by transposition $a \times d = c \times b$; also, $(c \times b) \div a = d$.

§ c. The first and third (a & c) of four magnitudes that are in *direct* proportion, are called *antecedents*; and the second and fourth (b & d) are called *consequents*.

§ d. Also the first and fourth (a & d) are called *extremes*; and the second and third (b & c) are called *means*.

§ e. It is a rule in proportion, that the quantities for calculation be so arranged that the product of the two extremes be equal to the product of the two means. Thus, $a \times d = b \times c$; and as $a \times d = d \times a$. (§ XXV. †1. Algebra), $d \times a = c \times b$. Wherefore the means may be made extremes, and the extremes means; as, $b : a :: d : c$ (§ XLVIII. Alg.); the antecedents may be made consequents and the consequents antecedents; as, $d : c :: b : a$, without interrupting the harmony of the proportion between the quantities.

§ *f*. Hence if the value of three magnitudes be known, the unknown value of the fourth, which is in the same ratio of proportion, is determinable. It is the quotient that arises (§ *b*.) from dividing the product of the two means by the known extreme. Thus, $3 : 4 :: 6 -$; a 4th quantity, $4 \times 6 = 24$, and $24 \div 3 = 8$, the 4th quantity. Generally speaking, the magnitude whose value is required is expressed last in the order of proportion.

AXIOMS.

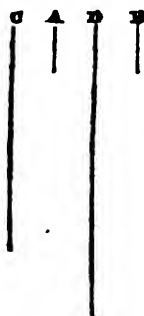
§ 69. Among quantities that are proportional, equals may be substituted for equals.

§ *a*. Magnitudes that are proportional to the same two, or to other magnitudes equal to these, are proportional to each other.

§ *b*. Equal magnitudes have the same ratio to the same magnitudes.

PROPOSITION VI.

§ 70. The same, or equimultiples (C and D), of any two magnitudes (A and B), are to each other as the magnitudes themselves.



§ *a*. Let C be the same multiple, say the 5th, of A, that D is of B; then (§ 64. § *a*.) $A \times 5 = C$, and $B \times 5 = D$; by transposition $C \div A = 5$, and $D \div B = 5$; wherefore (§ 24. § *a*.) $C \div A = D \div B$; then (§ XLVI.) $C : A :: D : B$, and alternately (§ XLVIII.) $C : D :: A : B$. Therefore, the equal multiples are as the magnitudes.

§ *b*. Scholium. The magnitude of a triangle (§ 33. § *c*.) is the product of its base and half of its altitude.

§ *c*. Wherefore the magnitudes of triangles of the same altitude, are equimultiples of their bases. And so of parallelograms of equal altitudes; and therefore,

§ *d*. Cor. Triangles, or parallelograms, that have the same altitude, are, in magnitude to each other as their bases; or those that have equal bases are to each other as their altitudes.

PROPOSITION VII.

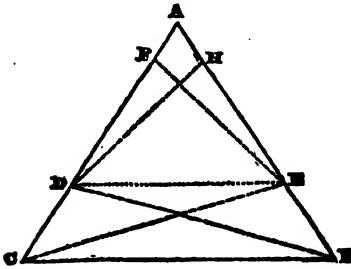
§ 71. When the product of any two quantities (*a* & *b*) equals the product of two others (*c* & *d*), the ratio of the greater multiplicand to the greater multiplier, is equal the ratio of the less multiplicand to the less multiplier; and the four quantities are proportional.

§ *a*. Let $a \times b = c \times d$ be the four quantities proposed, and let *a* be the greater multiplicand, then *d* will be the greater multiplier, *b* the less, and *c* the less multiplicand; and (§ 71.) $a : d :: c : b$.

§ *b*. By the conditions of the proposition $a \times b = c \times d$, and by transposition (§ XLII.) $a \div d = c \div b$; therefore these quantities (§ XLV.) are proportional; *i. e.*, $a : d :: c : b$, and consequently (§ 67. § *a*.) their ratios are the same.

PROPOSITION VIII.

§ 72. If a straight line (DE) be drawn parallel to the base (CB) (§ 16. § b.), and so as to cut the two sides (AC and AB) of any triangle (ABC), the sections (AD, DC, AE, & EB) of these two sides will be proportional; *i. e.*, $AD : DC :: AE : EB$.



§ a. Join DB and CE; and let EF be drawn from the vertex (E) of the two triangles AED and CED, perpendicularly upon AC, then (§ 18. § a.) EF is the altitude of each of these two triangles. In the same manner, by drawing DH perpendicular to AB, it is shown that the two triangles ADE and BDE have the same altitude (DH).

§ b. The triangles AED and CED, having the same altitude (EF), are to each other (§ 70. § d.) as their bases; *i. e.*, $AED : CED :: AD : DC$. And the triangles ADE and BDE are to each other also as their bases; *i. e.*, $ADE : BDE :: AE : EB$.

§ c. The triangle ADE is a magnitude in each set of these proportions; and the triangles CED and BDE are of the same magnitude (§ 34. § d.), for they stand upon the same base (DE), and between the same parallels (§ 72.) DE and CB.

§ d. Wherefore AD and DC, AE and EB, are proportional to the same or equal magnitudes; they are therefore (§ 69. § a.) proportional to each other; *i. e.*, $AD : DC :: AE : EB$.

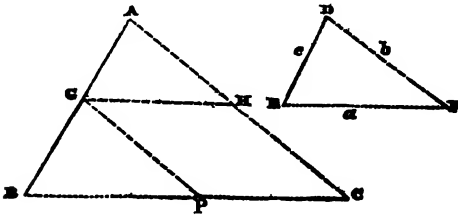
§ e. By drawing a straight line from the vertex B, perpendicularly upon AC, as a base; and another from C as a vertex, perpendicularly upon AB, it may be proven in the same way, that $AC : DC :: AB : EB$. And also that $AC : AD :: AB : AE$. Wherefore,—

§ f. *Cor.* If the two sides of a triangle be crossed by a line that is parallel to the base, these two sides will be proportional to their sections.

PROPOSITION IX.

§ 73. The homologous (§ 23. § c.) sides of similar triangles (ABC & DEF) are proportional to each other, *viz.*, $AB : c :: AC : b :: CB : a$.

§ a. By the conditions of the proposition, the triangles ABC and DEF are similar, wherefore (§ 23. § b.) their corresponding angles are equal, *viz.*, $B = E$, $A = D$, and $C = F$. Upon the two sides AB and AC of the greater triangle, (ABC) let AG be set off, equal to c, and AH = b; let GH be joined, then (§ 35.) the two triangles AGH and DEF are equal to each other; therefore (§ 23.) $GH = a$, the angle $AGH = E$, etc.



§ b. The angle A G H (§ a.) = E, and B = E, therefore (§ 42, § a.) A G H = B, and (§ 30. § d.) G H is parallel to B C. Consequently (§ 72. § f.) $AB : AG :: AC : AH$; by construction $AG = c$,

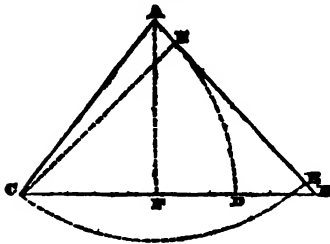
and $AH = b$; then substituting for AG and AH their equals, we have (§ 69.) the proportion $AB : c :: AC : b$.

§ c. Then by drawing from G, GP parallel to AC, the parallelogram GPHC (§ 35. § e.) is formed; and (§ 72. § f.) we have $AB : AG :: CB : CP$. But (§ 19.) $CP = GH$, and (§ a.) $GH = a$; also $AG = c$. Therefore (§ 69.) $AB : c :: CB : a$.

§ d. Cor. Whence it may be inferred, that if a straight line be within a triangle and parallel to its base, the ratio of the base to this line will be equal to the ratio of the sides to their sections.

PROPOSITION X.

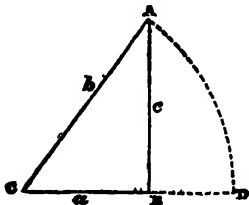
§ 74. In every triangle (ABC) the sides are proportional to the sines of their opposite angles.



§ a. From C and A, as centres, and with AC as the radius, let the arcs AD and CE be described, and draw AF perpendicular upon BC, and CH perpendicular upon AB, then (§ 54.) AF is the sine of the angle ACB, and CH is the sine of the angle CAB.

§ b. Of the two triangles BHC and BAF, the angle at B is common, the right angles AFB and CHB are equal, wherefore (§ 32. § e.) the remaining angles HCB and FAB are equal, and (§ 23. § b.) these two triangles are similar.

§ c. The homologous sides of similar triangles (§ 73.) are proportional; wherefore $AB : CB :: AF : CH$; and AF & CH (§ a.) are the sines of the angles ACB and CAB; therefore $AB : CB :: \text{sine } ACB : \text{sine } CAB$. In the same manner it may be proven that $AC : CB :: \text{sine } ABC : \text{sine } CAB$.



§ d. If the proposed triangle be right angled, and the hypotenuse be made radius, the hypotenuse becomes (§ 62. § a.) the sine of the right angle, and the legs (c and a) the sines of their opposite angles (C & A), whence we have a self-evident truth, viz., $AB : CB :: c : a$; but c and a are the sines of C and A, therefore $AB : CB :: \text{sine } C : \text{sine } A$. Also $AC : AB :: b : c$;

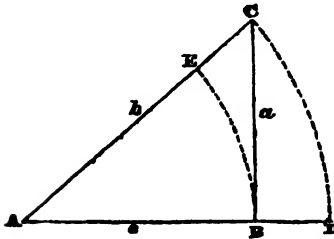
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but b is the sine of B , and c of C , therefore $AC : AB :: \text{sine } B : \text{sine } C$; and (§ 62.) the sine of B is equal to radius. By like reasoning, it is shown that $AC : CB :: \text{sine } B : \text{sine } A$. Wherefore,

§ *e. Cor.* If an hypotenuse be made radius, that hypotenuse is to either leg as radius is to the sine of the angle opposite that leg.

PROPOSITION XI.

§ 75. In every right angled triangle, the sides are proportional to the several *trigonometric functions* (§ 57. § *c.*) of the two acute angles.



§ *a.* Let the hypotenuse (b) of the proposed triangle, be made radius (AC) to an arc $CD = A$. Then (§ 54. & § 57. § *a.*) CB is the sine, and AB the co-sine of A ; likewise AB is the sine, and CB the co-sine of C .

§ *b.* Now (§ XXV. †1.) $b \times a = CB \times AC$; then (§ 71.) $b : AC :: CB : a$; by construction (§ *a.*) AC is radius, and CB is sine A or co-sine C ; therefore $b : \text{radius} :: \text{sine } A \text{ or co-sine } C : a$. Again, $b \times c = AB \times AC$, and AB (§ *a.*) is the co-sine of A , and sine of C ; therefore, $b : \text{radius} :: \text{co-sine } A \text{ or sine } C : c$. And these quantities (§ XLVII.) are also proportional, when taken inversely or alternately.

§ *c.* Let either of the legs (c) be made radius (AB) to another arc $EB = A$. Then CB becomes tangent (§ 56.) to A , and (§ 57. § *b.*) co-tangent to C ; and AC (§ 57.) the sec. of A , and co-sec. of C . And (§ XXV. †1.),

$$1st. AB \times a = CB \times c, \text{ and } (§ 71.) AB : c :: CB : a.$$

$$2d. AB \times b = AC \times c, \quad ,, \quad AB : c :: AC : b.$$

$$3d. AC \times a = CB \times b, \quad ,, \quad AC : b :: CB : a.$$

Now (§ *c.*) AB is radius, CB is tangent A , or co-tangent C , and AC is sec. A , or co-sec. C . Therefore in these three sets of proportions, the first is, radius : $c ::$ tangent A , or co-tangent $C : a$. The second is, radius : $c ::$ sec. A , or co-sec. $C : b$. And the third is, sec. A , or co-sec. $C : b ::$ tangent A , or co-tangent $C : a$. And (§ XLVIII.) these quantities are also proportional, when taken inversely, or alternately.

§ *d.* If BC be made radius, the same method of demonstration will show that radius, the tangent and secant of C , and of its complement, are proportional to the sides (a, b & c) of the proposed triangle.

PROPOSITION XII.

§ 76. The chords (a & b), sines ($d p$ & $e s$), tangents ($e s$ & BC), etc., of equal arcs ($d s$ & $e B$), which have different radii ($AB, A s$), are to each other, as the radius of one of the arcs is to the radius of the other.

LOGARITHMS.

LOGARITHMS.

§ 80. The purpose of logarithms is to facilitate arithmetical calculations. The term is derived from two Greek words (*logos* and *arithmos*), and may rightly be called the language of numbers; for by means of logarithms, not only multiplication and division, but also the tedious operations of involving powers, extracting roots, etc., are simplified. They are performed by the simple process of addition and subtraction, multiplication and division.

§ a. Through the intervention of logarithms, angular and linear magnitudes are compared with each other, as quantities of the same denomination are in common arithmetic; and the unknown value of a line, arc, or an angle, may be deduced from the known ratio between other lines and angles.

§ b. Before the invention of Lord Napier had introduced into mathematical calculations the use of logarithms, the solution of trigonometrical problems was referred to synthesis, and obtained by construction. The process of finding the value of unknown quantities in trigonometry was tedious; and the result, owing to the mechanical manner in which the operation was conducted, was subject to partial inaccuracies.

§ c. If the logarithms that correspond to a series of numbers in geometrical progression, be taken out and noted down, they will be found to constitute another series of numbers in arithmetical progression. And if two series of numbers be arranged, one in the order of geometrical progression (the first term of which shall be unity or 1, and the common ratio 10), and the other in the order of arithmetical progression, (the first term of which shall be zero (0), and the common difference 1), these two series will constitute a basis, or the ground work for forming a table of logarithmic numbers, like those most generally used.

§ d. By arranging the two series of numbers, the one in geometrical, the other in arithmetical order of progression, thus,

Geometrical progression.	-	-	-	-	-	Arithmetical progression.*
1	-	-	-	-	-	0
10	-	-	-	-	-	1
100	-	-	-	-	-	2
1000	-	-	-	-	-	3
10000	-	-	-	-	-	4
100000	-	-	-	-	-	5
1000000	-	-	-	-	-	6
10000000	-	-	-	-	-	7
100000000	-	-	-	-	-	8
1000000000	-	-	-	-	-	9
10000000000	-	-	-	-	-	10,

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the relation and connexion between the orders of the two series will appear. The terms in the order of arithmetical progression, are the logarithms of those in the geometrical order: each the logarithm of the term to which it is opposite; i. e., 0 is the logarithm of 1; the logarithm of 10 is 1, of 100 is 2, of 1000 is 3, of 10,000 is 4, and of 100,000 it is 5, etc.

§ e. By observing the different relations which exist between the terms in the two series above, it appears,—

1st. That the logarithm of any whole number has for its index the figure, which expresses the number of digits, *minus one*, that are in the natural number for which the logarithm is required. Thus 1 is the logarithmic index of 10, which has two digits; 2 is the logarithmic index of 100, which has 3 digits; 3 of 1000; 4 of 10,000, etc.

2d. That the logarithm of any number intervening between 1 and 10, must be a fraction less than unity; for the logarithm of 1 is 0, and of 10 it is 1; then the logarithm of any number between 1 and 10 must be less than 1. The logarithm that corresponds to any number between 10 and 100, must be greater than 1 and less than 2, etc.

3d. That the product of any two numbers in the series of geometrical progression, answers to the sum of the two numbers in the arithmetical order, which correspond to them. Thus; 100,000,000 in the geometrical, answers to 8 in the arithmetical order, and 100,000,000 is the product of $1000 \times 100,000$, which two numbers correspond with 3 and 5 in the arithmetical series; and in this series $3 + 5$, or $8 = 100,000,000$ in the geometrical series. Now 3 is the logarithm of 1000, and 5 of 100,000. Hence, therefore, if the logarithms of any quantities whose product is required, be added together, this sum will be the logarithm of the required product.

4th. That the difference (4) between any two numbers (2—6) in the series of the arithmetical order of progression, is the logarithm of the quotient (10,000) which arises from dividing by each other, the two numbers ($1,000,000 \div 100$) that correspond, in the geometrical order, to the two said arithmetical terms (2 and 6). And,

5th. That logarithms of numbers are a series of terms in the order of arithmetical progression, which series corresponds to another series of numbers in the order of geometrical progression.

§ f. The logarithm of 10 is not necessarily 1. Any other number as well as 1 may be assumed as the logarithm of 10. But if any other number were taken as the logarithm of 10, it would establish another base for calculating a table of logarithms, and this change would effect the logarithm of every other number in the same ratio. From this it appears that the value of a logarithm is entirely conventional. But in order to simplify, and facilitate their application to practice, mathematicians have assumed, in the order of geometrical progression, a series of numbers in the ratio of 1, 10, 100, 1000, 10,000, etc.; and as logarithms to these, they have assigned, in arithmetical progression, a series of numbers in the

order of 0, 1, 2, 3, 4, etc., having 1 for the common difference. The latter numbers, taken in the order in which they stand, are called the logarithmic *indices*, or *characteristics*, of the former, taken in the same order. Whence (§ e. 1st) the logarithmic index of every whole number, is made always to express *one less* than the number of digits contained in the whole number. Thus, the characteristic of 100 is 2, as the number of digits or figures in 100, is 3. The number of digits in 1000 is 4, and the logarithmic characteristic of 1000, is 3.

§ g. The logarithmic signs and tangents, co-sines and co-tangents, etc. of degrees, minutes, and seconds, are also of conventional value. They are expressed in parts of the radius of a circle; the value of which radius is assumed to be equal to 10,000,000, etc., with as many ciphers affixed as the compiler intends the *mantissa* shall consist of digits.

§ h. The *mantissa* is the decimal part of a logarithm; or the part which is not the index. The logarithm of 250 is 2.397940; 2 is the index, and .397940 is the mantissa.

§ 81. The characteristic of a logarithm depends upon the number of digits which are contained in its *geometrical** number; being one less than the number of digits, it is known by counting the figures of the geometrical number, and writing down one less than their number for the index.

§ 82. The mantissæ have been previously calculated and arranged in a tabular form, and in such a manner that the mantissa for any geometric number, (however great or small), may be determined with readiness.

§ 83. The first column of the tables (I.) is marked N. (Numbers.) It contains the geometrical numbers, from 1 to 999. The mantissa of each term in this series, is in a line with its geometrical number, and in the next column which is marked 0. The nine other columns, headed with the cardinal numbers 1, 2, 3, etc., to 9, also contain mantissæ; but these are of geometrical numbers that have *four* digits; the last figure of which, being found at the head of one of these columns, and the three first in the first column (N), show at their angle of meeting, the proper mantissa. Thus, the logarithm of 3681 is 3.565966; 368 being found in the first column, N., the mantissa for 3681 is found in a line with 368, and in the column, at the top and bottom of which, stands 1.

§ 84. To take from the table (I.) the mantissa for any geometrical number less than 1000.

§ a. Find the given number in the first column (N.); its mantissa is opposite to it in the next column (0).

§ b. The mantissa for 9 is .954243; for 14 it is .146128; and for 964 it is .984077.

* The natural numbers, which correspond to, or are represented by, a logarithm, are here called *geometrical* numbers, from their being terms in the geometrical series¹ of progression. Thus 419, of which 2.622214 is the logarithm, is the *geometrical* number of log. 2.622214.

§ c. These mantissæ, with the proper characteristics prefixed, constitute the logarithms of those numbers; thus, logarithm of 9 = 0.954243; of 14 = 1.146128; and of 964 = 2.984077.

§ 85. To take from the tables, the mantissa for any geometrical number that is greater than 1000, but less than 10,000.

§ a. Find, in the first column (N.) of the tables, the three first figures of the proposed number; in a line with them, and in the column over which the fourth figure stands, is the proper mantissa.

§ b. The mantissa (.973590) for 9410 is found in the column marked (0), and opposite to 941 in the first column; and the mantissa (.954918) for 9014, is found in the column marked (4), and opposite to 901 in the first column (N.).

§ c. The geometrical numbers in this case have four digits; 3, then, is the characteristic of their logarithms; which are, 3.973590 = 9410; and 3.954918 = 9014.

§ 86. To take from the tables the mantissa of any geometrical number that has more than *four digits*, or that is greater than 10,000; say of 43568.

§ a. Take out the mantissa for the four first digits (§ 85. § b.), and subtract it from the mantissa next in order. Then say,—If unity (1), prefixed to as many ciphers as there are digits remaining of the proposed geometrical number, give this mantissal difference, what mantissal difference will the remaining digits of the geometrical number, give? This last mantissal difference being added to the mantissa for the four first digits, produces the required mantissa.

§ b. The mantissa for the four first digits (4356) of 43568 is .639088; the mantissal difference between this and the next (.639188) in order, is .000100; one digit (8) of the proposed geometrical number remains; and unity (1) (§ a.) must be prefixed to one cipher (0), which makes (10); then the order and terms of the proportion are, 10 : .000100 :: 8 : .000080. This last term, *plus* the former mantissa, (or .639088 + .000080) = .639168, the mantissa required.

§ c. The geometrical number (43568) in this example has *five* digits; then (4) being prefixed as an index to the mantissa (.639168) makes the logarithm of 43568 = 4.639168.

§ d. The mantissa for the four first digits (7419) of 741946 is .870345; the difference between the mantissa of 7419, and of 7420, is .000059. And the order and terms of the proportion are, 100 : 000059 :: 46 : .000027. This last term being added, and the characteristic (5) being prefixed to the mantissa (.870345), makes the logarithm of 741946 = 5.870372.

§ e. The mantissa for the four first digits (5941) of 594106734 is .773860; the mantissal difference = .000073; the remaining digits (06734) are *five* in number. Prefixing 1 to *five* ciphers, the proportion is 100000 : 000073 :: 06734 : 000004. Prefixing the characteristic (8) to .773860 + 000004, makes the logarithm of 594106734 = 8.773864.

§ f. The mantissa for 124, and the mantissa for 124000000 are

the same (.093422). But the logarithm of 124 = 2.093422; and the logarithm of 124000000 = 9.093422. Hence,

§ g. The effect of final ciphers in a geometrical number, on its logarithm, is confined to the characteristic of the logarithm, for the logs. (§ f.) of 124 and 124000000 have the same mantissa.

§ 87. To find the logarithm of a geometrical number that is mixed with a decimal fraction.

§ a. The mantissa is found as it would be, were the proposed number integral; but the characteristic of the logarithm must express one less than the number of digits in the *integral* part of the number proposed. Thus;

Mixed Geometrical Nos.	Their Logarithms.
9414.5 - - -	= 3.973797
941.45 - - -	= 2.973797
94.145 - - -	= 1.973797
9.4145 - - -	= 0.973797

§ 88. As the logarithm of unity, or (1), is 0, it follows that the logarithm of any quantity less than unity, or the logarithm of a fraction, must be less than 0; that is, it is a *negative quantity*. Thus,—

Logarithm of 1 = 0	.000000
Logarithm of 0.1 = -1	+ .000000
Logarithm of 80 = 1	.903090
Logarithm of 0.08 = -2	+ .903090

§ a. But the use of negative and positive quantities (such as the logarithms of integers and of fractions, as just quoted above), in the same operation, has a tendency sometimes to perplex the calculator. It is better, therefore, that all quantities in the same sum should be affected with like signs. This may be effected with the logarithms of fractions by means of a little artifice, by which negative quantities are made to change their signs, or become as positive quantities.

§ b. This artifice consists in borrowing and applying 10, or 20, or 100, etc., to the logarithmic index of the fraction, and restoring the borrowed quantity again during the progress of the calculation. Thus, in the summing up of the logarithms of 80, and of 0.08, instead of adding the mantissæ and subtracting the characteristics of the two logarithms (1.903090 - 2 + .903090 = 0.806180), the operation may be performed entirely by addition, if the arithmetical complement of - 2, which is 8, be used as the index; thus, 1.903090 + 8 + .903090 = 10.806180; rejecting the 10, which was borrowed in the logarithmic index of 0.08, there remains 0.806180.

§ c. Any quantity with the sign (—) prefixed, may be commuted into a positive quantity by substituting for that quantity its *arithmetical* complement. In this way subtraction may be performed by addition, as we have seen (§ b.) in the case of the negative index - 2.

§ d. The *arithmetical complement* of a number, is the difference

between that number, and the first number that ends with a cipher, and is one scale higher in the decimal order of notation. The arithmetical complement of 6, is 4, ($10 - 6 = 4$); of 77, is 23, ($100 - 77 = 23$); of 115, is 885, ($1000 - 115 = 885$), etc.

§ c. If it be required to subtract 6 from 9, the result is the same, whether we say $9 - 6 = 3$, or take the arithmetical complement (4) of 6, add it to 9, ($4 + 9 = 13$), and reject the borrowed (10) from their sum ($13 - 10 = 3$).

§ 89. To find the arithmetical complement of a logarithm.

§ a. Prefix (1) to as many ciphers as there are digits in the proposed logarithm; then from this number subtract the proposed logarithm, and the remainder will be the required arithmetical complement.

§ b. Thus, the arithmetical complement of 1.903090 is 8.096910; for 1000000.

$$\begin{array}{r} 1.903090 \\ \hline 8.096910 \\ \hline \end{array}$$

§ c. The same result may be obtained by beginning at the left, and subtracting from 9, every figure except the last significant one, which must be subtracted from 10.

$$\begin{array}{r} \text{Thus---} \quad 9.9999(10)0 \\ \quad \quad \quad 1.9030 \ 9 \ 0 \\ \hline \quad \quad \quad 8.0969 \ 1 \ 0 \\ \hline \end{array}$$

§ d. If the index be greater than 9, it must be subtracted from 19.

§ 90. To find the logarithm of a decimal fraction.

§ a. The mantissa for the proposed fraction is taken from the table, as for an integral geometrical number of the same significant figures which the fraction has.

§ b. The logarithm for the same figures expressed both integrally and fractionally, differs only in the characteristic; the mantissa is the same in both cases. Therefore we have only to teach how to determine a *positive* characteristic, or the logarithm of a decimal fraction.

§ c. The characteristic is determined by means of the number of ciphers which precede the first significant figure of the decimal, for the number of these subtracted from (9), gives the required characteristic.

$$\begin{array}{l} \text{Thus, logarithm of the decimal} \quad .301 = 9.478567 \\ \text{Logarithm of the decimal} \quad .0301 = 8.478567 \\ \text{Logarithm of the decimal} \quad .000301 = 6.478567 \end{array}$$

§ 91. To find the logarithm of a vulgar fraction that is less than unity.

§ a. Take from the tables the mantissa for the numerator, and for the denominator, separately, and as if each were a whole number expressed by the same figures. Then, from the logarithm of the numerator, *plus* 10 to the index, subtracting the logarithm of the denominator, leaves the logarithm of the proposed fraction. Thus,

§ b. To find the logarithm of $\frac{25}{100}$.

$$\begin{aligned} \text{Logarithm of } 25 &= 1.397940 \\ \text{Logarithm of } 100 &= 2.000000 \end{aligned}$$

$$\text{Logarithm of } \frac{25}{100} = 9.397940$$

To find the logarithm of $\frac{3}{8}$.

$$\begin{aligned} \text{Logarithm of } 3 &= 0.477121 \\ \text{Logarithm of } 8 &= 0.903090 \end{aligned}$$

$$\text{Logarithm of } \frac{3}{8} = 9.574031$$

§ 92. To find the logarithm of a mixed number.

§ a. If the number proposed (9.05) consist of a whole and a decimal fraction, take from the tables the mantissa for the mixed number as if it were a whole, and then prefix the index, *which expresses one less than the number of digits in the integral part of the mixed number proposed*. Thus, the index of the logarithm for 9.05 is 0, and for 90.5 it is 1; the mantissa (.956649) is the same for each number; because it is taken out for the figures (905) as though they stood for a whole number.

Then, the logarithm of 9.05 = 0.956649

The logarithm of 90.5 = 1.956649

§ b. But if the proposed mixed number consist partly of a vulgar fraction, let it be reduced to an improper fraction; then find the logarithm of the numerator, and of the denominator, as if they were separately whole numbers; and the remainder of logarithm of the numerator, *minus* logarithm of the denominator, is the required logarithm.

Thus, logarithm $12\frac{2}{3} = 1.102663$; for $12\frac{2}{3}$ reduced to an improper fraction is $\frac{38}{3}$; and

$$\text{Logarithm } 38 = 1.579784$$

$$\text{Logarithm } 3 = 0.477121$$

$$\underline{\underline{1.102663 = 12\frac{2}{3}}}$$

§ c. The index to the logarithm of an improper fraction, is always a *proper* index; for the numerator, being greater than the denominator of such a fraction, the logarithm of the latter can be subtracted from that of the former, without borrowing 10 in the index.

§ 93. To find the geometrical number which corresponds to a logarithm.

§ a. The number of digits in the geometrical number for a logarithm, is known by the characteristic of the logarithm; for 1 added to the *characteristic* tells the number of digits.

§ b. Find, in the table, the mantissa of the proposed logarithm. The figures in the first column (N.) being prefixed to the figure that stands at the top of the column in which the mantissa is found, compose the geometrical number that is required.

§ c. The geometrical number of log. 3.759214 = 5744.

The geometrical number of log. 2.759214 = 574.4.

The geometrical number of log. 1.759214 = 57.44.

The geometrical number of log. 0.759214 = 5.744.

This mantissa is found in the column (4) of the table, and opposite to 574 in the first column (N.).

§ d. Had the index of the logarithm been greater than (3), the number of digits for the geometrical number required, would have been made up by affixing ciphers to the numbers (574 and 4) found above, and opposite to, the mantissa. Thus, the geometrical number for 4.976854 is known (§ a.) to consist of five digits, because of the index (4). The mantissa (.976854) is found opposite to 948 (in the column N.), and under 1; making up with ciphers, the proper number of digits.

The geometrical number for 4.976854 = 94810

The geometrical number for 5.976854 = 948100

The geometrical number for 6.976854 = 9481000

§ e. If the proposed mantissa cannot be found in the tables, take that mantissa in the tables, which, being less, comes nearest to it, and affixing any number of ciphers to the difference between these two mantissæ, divide it by the difference between said tabular mantissa and the one next in order after it; the quotient, being affixed to the numbers opposite to, and at the head of the column of, said least mantissa, comprise the figures of the required geometrical number. The digits for the integral part of the geometrical number are determined by the index of the proposed logarithm (§ a.), and the figures which are to the right of these digits, constitute the fractional part of the geometrical number. Thus,

§ f. Geometrical number for 4.979744 = 95443.

Geometrical number for 2.979744 = 954.43.

Geometrical number for 0.979744 = 95.443.

The proposed mantissa (979744) cannot be found in the tables. The difference between the next less (.979730) and the next greater (979776) found in the tables, is 46; and the difference (14) between the less of these two and the proper mantissa being prefixed to any number of ciphers, (14000 - - -), and divided by the tabular difference (46), gives 3 + to be affixed to the four figures 9544, from which the required digits for the integral part of the geometrical number, are to be separated according to the index of the logarithm proposed.

§ g. If the index of the logarithm proposed be an *improper* characteristic, the geometrical number of the logarithm is a decimal fraction. And the difference between this characteristic and (9), tells how many ciphers must intervene between the decimal point (.) and the first significant figure of the fraction. Thus (the indices being *improper*),

The geometrical number for 7.911424 = 0.008155.

The geometrical number for 9.897627 = 0.79.

The geometrical number for 5.698970 = 0.00005.

§ 94. To perform multiplication by logarithms.

§ a. Add the logarithms of the factors together; the sum of these is the logarithm of the product. Thus,

§ b. To multiply 3 by 4; 20 by 2.5; and .25 by .30.

$\begin{array}{r} \text{Logarithm } 3 = 0.477121 \\ \text{“ } 4 = 0.602060 \\ \hline 1.079181 = 12 \end{array}$	$\begin{array}{r} \text{Logarithm } 20 = 1.301030 \\ \text{“ } 2.5 = 0.397940 \\ \hline 1.698970 = 50 \end{array}$
---	--

$$\begin{array}{r} \text{Logarithm } 0.25 = 9.397940 \\ \text{“ } 0.30 = 9.477121 \\ \hline 8.875061 = 0.075 \end{array}$$

§ 95. To perform division by logarithms.

§ a. Subtract the logarithm of the divisor from the logarithm of the dividend, the remainder will be the logarithm of the quotient.

§ b. To divide 36 by 3; 8941 by 19; 50 by .05; and 82.7 by 70.91.

$\begin{array}{r} \text{Log. } 36 = 1.556302 \\ \text{“ } 3 = 0.477121 \\ \hline 1.079181 = 12. \end{array}$	$\begin{array}{r} \text{Log. } 8941 = 3.951386 \\ \text{“ } 19 = 1.278754 \\ \hline 2.672632 = 470.57 + \end{array}$
--	--

$\begin{array}{r} \text{Log. } 50 = 1.698970 \\ \text{“ } .05 = 8.698970 \\ \hline 3.000000 = 1000 \end{array}$	$\begin{array}{r} \text{Log. } 82.7 = 1.917506 \\ \text{“ } 70.91 = 1.850708 \\ \hline 0.066798 = 1.16 + \end{array}$
---	---

§ 96. To perform involution by logarithms.

§ a. Multiply the logarithm of the proposed geometrical number by the index (§ X.) of the power to be involved; this product will be the logarithm of the required power.

§ b. Raise 8² 17³ 112⁴ 516⁹.

$\begin{array}{r} \text{Log. } 8 = 0.903090 \\ \quad \quad \quad 2 \\ \hline 1.806180 = 64 \end{array}$	$\begin{array}{r} \text{Log. } 17 = 1.230449 \\ \quad \quad \quad 3 \\ \hline 3.691347 = 4913 \end{array}$
---	--

$$\begin{array}{r} \text{Log. } 112 = 2.049218 \\ \quad \quad \quad 4 \\ \hline 8.196872 = 157351936 \end{array}$$

$$\text{Log. } 516 = 2.712650$$

9

$$24.413850 = 2593267484132083176308736$$

§ c. If the geometrical number that is proposed to be raised, be a decimal fraction, the difference between the characteristic of the logarithm of the power, and the product of 10 by the index of the power involved, expresses *one* more than the number of ciphers, that must precede the first significant figure of the required power.

§ d. To raise .17³ .05³ .064⁵.

$$\text{Log. } 0.17 = 9.230449$$

3

$$\text{Log. } 0.05 = 8.698970$$

2

$$27.691347 = 0.00491 +$$

$$17.397940 = 0.0025$$

$$\text{Log. } 0.064 = 8.806180$$

5

$$44.030900 = 0.000001073 +$$

§ 97. Evolution is the converse of involution.

§ a. To perform evolution by logarithms.

§ b. The quotient, obtained by dividing the logarithm of the power proposed, by the exponent (§ XII.) of the root to be extracted, is the logarithm of the root required.

§ c. To evolve the square root of 196, the $\sqrt{\quad}$ of 81, and the $\sqrt[3]{\quad}$ of 128.

$$\text{Log. } 196 = 2.292256 \div 2 = 1.146128 = 14.$$

$$\text{Log. } 81 = 1.908485 \div 3 = 0.636161 + = 4.326 +.$$

$$\text{Log. } 128 = 2.107210 \div 7 = 0.301030 = 2.$$

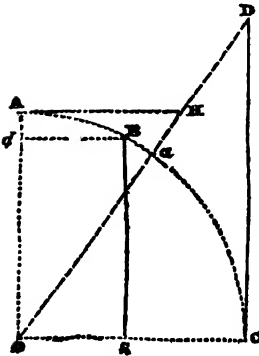
§ d. If the geometrical number whose root is to be evolved, be a decimal fraction, in order to obtain the logarithm of the root required, the exponent, *less one*, of the root to be extracted, must be prefixed to the characteristic of the logarithm of the number proposed; and then the quotient, that arises from dividing this quantity by the exponent of the root to be evolved, will be the logarithm of the power required. Thus, to find the cube root of .27. The logarithmic index of 0.27 is 9; now, to 9, prefixing *one* less (2), than the exponent (3) of the root to be evolved, makes the logarithm of $0.27 = 29.431364 \div 3 = 9.810454 + = 0.646 +$. To find the square root of 0.00776, the logarithmic index of which is (7); prefixing to 7 *one* less (1) than the exponent (2), makes the logarithm of $0.00776 = 17.889862 \div 2 = 8.944931 = 0.088 +$.

§ 98. In the tables (II.) containing logarithms for sines and co-sines,

tangents and co-tangents, the logarithms are calculated from the ratio between the radius and the sine, or the tangent of an arc.

§ a. In these tables, as in all others, the logarithmic value of a sine or a tangent is conventional, as are the logarithms of geometrical numbers.

§ b. In forming these tables, radius is taken as the base, or the ground work for the calculations; and its value is assumed to be 10.000000. The logarithmic value of the sine of 90° , or of the tangent of 45° (§ 62.), shows that this is the value with which radius enters this system. Wherefore, in every calculation, radius = 10.



§ 99. If an arc of 90° , or if a right angle, be divided into any two parts, the sine or the tangent of either part, is the co-sine or co-tangent of the other. Thus, in the quadrant ABC ; $AB = 30^\circ$, $BC = 60^\circ$, $Aa = 35^\circ$, and $aC = 55^\circ$; PB is the sine of $AB = 30^\circ$, and co-sine of $BC = 60^\circ$; BS is the sine of 60° (BC), and co-sine of 30° (AB); therefore the co-sine of AB , is BS , which is sine of BC , and the co-sine of BC is Bp , which is sine of $A B$. In the same manner AH and CD are reciprocally the tangents and co-tangents of Aa and aC . Wherefore the tables, by being calculated as far as 45° , answer for the whole circle, or for two right angles.

§ 100. The logarithmic value of the sec. of an arc, or an angle, is the arithmetical complement of the logarithmic co-sine of the same arc or angle.

§ a. The logarithmic co-sec. is the arithmetical co. of the logarithmic sine, and the logarithmic co-tangent is the arithmetical co. of the logarithmic tangent, and *vice versa* (Vide § 109. § h.).

§ b. Wherefore in logarithmic tables of the trigonometric functions, it is only essential that the logarithms of the sine, co-sine, and tangent, or of the sec., co-sec., and co-tangent, should be given, for the first three are the arithmetical co. of the latter, and *vice versa*; but to facilitate beginners in their calculations, the logarithmic sine, co-sec., co-sine, secant, tangent, and co-tangent, for every degree ($^\circ$) and minute ($'$), are given in the tables (II.), as well as for every hour (h), minute (m), and four seconds (s), of the day.

§ 101. In trigonometrical calculations, when any logarithm is to be subtracted, the proper result may be obtained by adding the arithmetical co. of such logarithm.

§ a. This method of changing subtraction into addition is found very convenient in practice. After a little exercise in it, the arithmetical co. of a logarithm may be read from the logarithm, as readily as the logarithm itself is read from the tables.

§ b. Wherefore in the solution of trigonometrical problems, sub-

traction need never be performed, for (§ 101.) instead of subtracting a logarithm, the same result is obtained by substituting, for this logarithm, its arithmetical co., and adding this arithmetical co. in the calculation; therefore (§ 100. § a.),

When { A logarithmic sine of an arc or an angle is to be subtracted, add its co-sec., and *vice versâ*.
 A logarithmic co-sine of an arc or an angle is to be subtracted, add its sec., and *vice versâ*.
 A logarithmic tangent of an arc or an angle is to be subtracted, add its co-tangent, and *vice versâ*.

§ 102. To find in the tables (II.), the logarithmic value of the sine of an arc or an angle.

§ a. If the proposed arc or angle be an *extreme* one, the number of *degrees* contained in it, is at the top of the page. But if the one proposed be a *mean* arc or angle, the *degrees* contained in it, are to be found at the *bottom* of the page.

§ b. An *extreme* arc or angle is one that contains *less* than 45° , or *more* than 135° . A *mean* arc or angle contains *more* than 45° but *less* than 135° .

§ c. If the proposed arc contain *less* than 45° or 3 H, or having *more* than 90° or 6 H, contain *less* than 135° or 9 H, the odd minutes of it are to be found in the proper minute column at the left of the page. But if it be *greater* than 135° or 9 H, or being *greater* than 45° or 3 H, contain *less* than 90° or 6 H, the odd minutes of it are contained in the minute column, that is at the right hand side of the page.

§ d. The columns marked at the bottom, Cos., Sec., Sin., Co-sec., Co-tang., and Tang., contain the logarithmic co-sine, secant, sine, co-sec., co-tang., and tang., of the degrees, etc., that are at the *bottom* of the page. And those marked Sin., Co-sec., Cos., Sec., Tang., and Co-tang., at the top, contain the logarithmic sine, co-sec., co-sine, secant, tangent, and co-tangent of the degrees, etc., at the *top* of the page.

§ e. To take out of the tables the logarithmic sine of an extreme arc, that is *less* than 45° , say of $34^\circ 40'$. The degrees (34°) of the arc proposed being found at the top of the page, and the minutes ($40'$) in the minute (') column at the left of the page, the logarithm (9.754960) which, in the column of Sin. stands opposite to the minutes ($40'$), is the required logarithm. In the same way the logarithmic co-sine, or tangent, etc., is found in its proper column, and opposite to the given minutes.

Co-sine $21^\circ 14'$ logarithm = 9.969469.

Tangent $19^\circ 56'$ logarithm = 9.559491.

§ f. If the arc ($179^\circ 7'$) proposed be extreme, and *greater* than 135° , the odd minutes ($7'$) must be found in the last minute (') column on the page; the required logarithm (8.187985) is then found opposite, and in the column marked (Sin.) at the top.

§ g. The odd minutes of a mean arc, say $61^\circ 12'$, that is *less* than 90° , are found in the last minute (') column; and those of one, say $114^\circ 59'$, that is *greater* than 90° , are found in the first minute

([']) column of the page; and the character (Sin., Cos., Co-tang.) of the column is taken from the bottom of the page.

Sine $61^{\circ} 12' = 9.942656$.

Co-tangent $114^{\circ} 59' = 9.668343$.

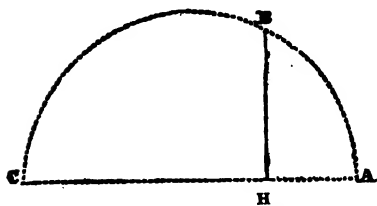
Tangent $114^{\circ} 59' = 10.331657$.

§ *h*. The trigonometric functions of hours, minutes, and seconds, are taken from the tables in the same way.

§ 103. To find the degs. etc. for a log. sine.

§ *a*. The minutes, in the minute ([']) column, which are found opposite to the proposed logarithm, and the degrees, which stand where the column in which the proposed logarithm is found, is marked Sin., are the degrees and minutes required. The same directions answer for taking out the tangent, secant, co-sine, etc., of a logarithm. Thus,

Logarithm 9.627300 sine = $25^{\circ} 5'$ or $154^{\circ} 55'$.
 „ 9.956566 cos. = $25^{\circ} 12'$ or $154^{\circ} 48'$.
 „ 9.831709 tang. = $34^{\circ} 10'$ or $145^{\circ} 50'$.
 „ 9.745494 cos. = $56^{\circ} 11'$ or $123^{\circ} 49'$.
 „ 10.377793 tang. = $67^{\circ} 16'$ or $112^{\circ} 44'$.



§ 104. An arc (A B) and its supplement (B C) have the same sine (B H); therefore the same logarithm answers either to the sine of A B or B C. But, for the most part, in calculations, there is some circumstance connected in the operation, which determines

whether an arc or its supplement be required. If not, the case or the solution is called *doubtful*. The same remarks apply to co-sines, secants, tangents, etc.

PLANE TRIGONOMETRY.

PLANE TRIGONOMETRY.

§ 105. Plane trigonometry is that branch of mathematics by which, with certain data, the unknown parts of triangles are determined.

§ *a.* Plane trigonometry is divided into right, and oblique, angled trigonometry. The solution of right angled triangles pertains to the former; and that of oblique angled triangles to the latter. The methods of solving problems in either, in most cases are similar.

§ 106. Every plane triangle consists of three sides and three angles; which, in a general term, are called *the six parts* of a triangle.

§ *a.* If the value of any three of these six parts (the *three* angles excepted), be known, the value of every one of the remaining parts is determinable by trigonometrical operations.

§ *b.* When the angles constitute the only data, the sides cannot be determined (except in *species*); because there may be any number of triangles which are equiangular to each other, and their homologous sides may all be unequal, as the sides of the two triangles under § 73. are.

§ 107. In order to solve a trigonometric problem, the value of three of five parts, viz., two of the angles and the three sides, must be given, and at least *one* of these three parts must be a side.

§ 108. The several combinations of three, which can be formed of five parts, comprise all the cases that are necessary for solving trigonometric problems.

§ *a.* To one of these every problem in trigonometry resolves itself for solution.

§ *b.* These several combinations are reduced to *five* cases; viz.,

1st, When two sides and the angle which is opposite to either of them, are given:

2d, When two angles and the side which subtends either of them, are given:

3d, When two sides and the angle which they contain, are given:

4th, When two angles and the side that is between them, are given:

5th, When the three sides are given.

CASE I.

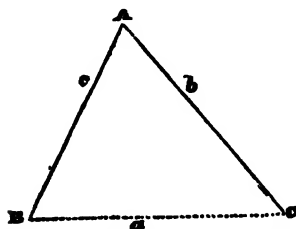
§ 109. Given the sides (c & b) and the angle (B), that is opposite to (b) one of them:

$$c = 450 \text{ yards.}$$

$$b = 600 \text{ yards.}$$

$$B = 74^\circ 49'$$

Required the remaining parts (A , C , & a)?



§ *a*. First, to find the value of C .

By § 74., $b : \text{sine } B :: c : \text{sine } C$;

and substituting for these given parts (b , c , and C), their values, the proportion is, $600 : \text{sine } 74^\circ 49' :: 450 : \text{sine } C$.

$$\text{Log. sine of } 74^\circ 49' = 9.984569$$

$$\text{Logarithm of } 450 = 2.653213$$

$$12.637782$$

$$\text{Logarithm of } 600 = 2.778151$$

$$\text{Log. sine } C = 9.859631 = 46^\circ 22' 15''.$$

§ *b*. The sum of the two angles B and C (§ 32. § *b*.) being subtracted from 180° , gives the remaining angle (A) of the proposed triangle ABC . Thus, $B + C = 121^\circ 11' 15'' - 180^\circ = 58^\circ 48' 45'' = A$.

§ *c*. The logarithmic process (§ *a*.) of finding the value of C , might have been abbreviated, and the operation might have been performed entirely by addition. This abbreviation (§ 101.) consists in using the arithmetical complement of the logarithm of the first term in the order of proportion, in the place of said logarithm. This arithmetical complement, added to the logarithm of the second and third terms, gives the logarithm, (rejecting 10 from the index), of the fourth or required term.

§ *d*. When one or more logarithms are to be subtracted in the process of a calculation, the operation may be simplified (§ 88. § *c*.) by substituting for such logarithms, their arithmetical complements (§ 88. § *e*.), and adding these arithmetical complements in the calculation, and then rejecting the borrowed 10 from the index of the sum thus obtained, the remainder is that which would have resulted by adding together the logarithms which were to be added, and then subtracting from their sum, the logarithms which were to be subtracted.

§ *e*. Hereafter, instead of the logarithm of the first term in the order of proportion being used, its arithmetical complement will be adopted in calculation. This substitution of the arithmetical complement renders the arrangement for calculation uniform; for it changes the process of addition and subtraction, into the simple operation of addition.

§ f. Let it be borne in mind that the *arithmetical complement* (§ 88. § d.) of any numerical expression, is obtained, by subtracting the expression proposed from the number which is made by prefixing unity (1) to as many cyphers, as there are figures in said expression. Thus the arithmetical complement of 89 is 11, for 89—100 = 11. Or, the arithmetical complement is obtained by beginning at the left, and subtracting each figure from 9 (§ 79. § c.), except the last significant figure, which must be subtracted from 10. Thus the arithmetical complement of 9.984569 is 0.015431 (what the former wants of 10.000000), and is obtained thus:

$$\begin{array}{r} 9.99999(10) \\ \text{Logarithm } 9.984569 \\ \hline \text{Ar. co. } 0.015431 \end{array}$$

§ g. And the arithmetical complement of 3.450000 (§ f.) is 6.550000, and is obtained thus:

$$\begin{array}{r} 9.9(10)0000 \\ \text{Logarithm } 3.450000 \\ \hline \text{Ar. co. } 6.550000 \end{array}$$

§ h. When the logarithmic index of any of the *trigonometric functions exceeds 9*, the arithmetical complement of such index is the difference between itself and 19, (§ 89. § d.) and consequently the arithmetical complement is found by subtracting said *index* from 19. Thus the logarithmic tangent of 74° is 10.542504, and its arithmetical complement is 9.457496, which is the co-tang. of 74°.

$$\begin{array}{r} 19.99999(10) \\ 74^\circ \text{ log. tang. } 10.542504 \\ \hline \text{Ar. co. } 9.457496 \end{array}$$

§ i. To find the value of the side *a*; according to § 74, sin. B : *b* :: sin. A : *a*; or, substituting their values, sin. 74° 49' : 600 :: sin. 58° 48' 45'' : *a*.

$$\begin{array}{r} \text{Log. sine } 74^\circ 49' \text{ ar. co.} = 0.015431 = \text{co-sec. B } (\S 100. \S a) \\ \text{Logarithm } 600 \quad \quad \quad = 2.778151 \\ \text{Log. sine } 58^\circ 48' 45'' \quad \quad = 9.932208 \end{array}$$

$$\text{Log. } a = 2.725790 = 531.8 \text{ yards.}$$

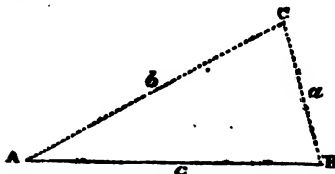
The 10, which was borrowed for the arithmetical complement of the logarithmic sine of 74° 49', being rejected in the sum, gives 2 for the index of the logarithmic value of *a*.

§ j. In the solution of all trigonometric problems by calculation, the *logarithmic* values of the functions employed are always used;

therefore the repetition of the word *log.* before *sine*, *tangent*, etc., will be omitted; and the arithmetical complement of the logarithm of a function or number, is that which is to be understood, and not the arithmetical complement of the number itself.

CASE II.

§ 110. Given two angles (A & C) and the side (c) that subtends one of them.



$$A = 23^{\circ} 56'$$

$$C = 76^{\circ} 4'$$

$$c = 1760 \text{ feet.}$$

To find the other parts (B, a and b),

§ a. The process of solution in this case is a repetition of the methods shown § 109. § b. & § i.; for $A + C$ subtracted from 180° (§ 32. § b.) gives $B = 80^{\circ}$; and (§ 74.) $\sin C : c :: \sin A : a :: \sin B : b$.

§ b. To find a. $\sin 76^{\circ} 4' : 1760 :: \sin 23^{\circ} 56' : a$.

$$\text{Sine } 76^{\circ} 4' \text{ ar. co.} = 0.012970 = \text{co-sec. } C (\S 101. \S b.)$$

$$\text{Log. } 1760 = 3.245513$$

$$\text{Sine } 23^{\circ} 56' = 9.608177$$

$$\text{Log. } a = 2.866660 = 735.6 \text{ feet.}$$

§ c. To find b. $\sin 76^{\circ} 4' : 1760 :: \sin 80^{\circ} : b$.

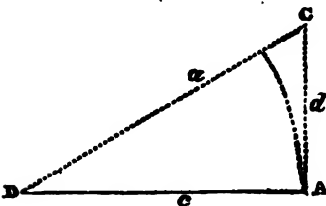
$$\text{Sine } 76^{\circ} 4' \text{ ar. co.} = 0.012970 = \text{co-sec. } C$$

$$\text{Log. } 1760 = 3.245513$$

$$\text{Sine } 80^{\circ} = 9.993352$$

$$\text{Log. } b = 3.251835 = 1785.8 \text{ feet.}$$

§ d. If, with similar data, the proposed, be a right angled triangle, the problem may be solved as the one above is, by means of the proportion between the sides and sines of opposite angles; or it may be solved by means of the principles involved (§ 75.) in the proportion between the sides and trigonometric functions of a right angled triangle. Thus, the data being,



$$C = 67^{\circ} 19'$$

$$c = 1760 \text{ feet.}$$

The value of A is known by the rectangularity of the proposed triangle (D A C). To find the other parts (D, d & a),

§ c. The third angle (D) is *always* known (§ 32. § b.) by subtracting the sum of the two other angles from 180° ; $A + C - 180^\circ = D$, or $22^\circ 41'$.

§ f. To find the two sides a and d ; making D A radius, the following proportions (§ 75. § c.) are deduced, viz.:

Rad. : $c :: \text{tang. D} : d$; and Rad. : $c :: \text{sec. D} : a$.

Radius (*always*, § 98. § b.) = 10.000000

$c = 1760 \log.$ = 3.245513

$D = 22^\circ 41' \text{ tang.}$ = 9.621142

Log. d = 2.866655 = 735.6 \text{ feet.}

Radius = 10.

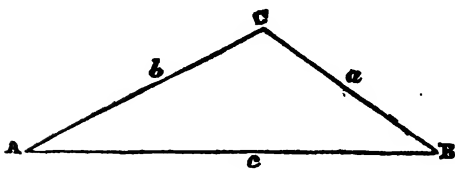
$c = 1760 \log.$ = 3.245513

$D = 22^\circ 41' \text{ sec.}$ = 0.034963

Log. a = 3.280476 = 1907.5 \text{ feet.}

CASE III.

§ 111. Given two sides (b & a) and the angle (C) they contain;



$b = 479$ fathoms.

$a = 419.8$ "

$C = 119^\circ 41'$.

To find the other parts (A, B & c),

§ a. The solution of this problem is founded

upon § 77.

§ b. One angle (C) of the proposed triangle being known, the sum of the two other angles (A & B) (§ 32. § b.) is obtained by subtracting the given angle from 180° . Thus, $180^\circ - 119^\circ 41' = 60^\circ 19'$, the sum of A & B.

§ c. To find the values of A and B; according to § 77., $(b + a)$:

$\text{tang. } \frac{A+B}{2} :: (b \oslash a) : \text{tang. } \frac{A \oslash B}{2}$. Therefore, by substitution,

$898.8 : \text{tang. } 30^\circ 9' 30'' :: 59.2 : \text{tang. } \frac{A \oslash B}{2}$.

$(b + a) = 898.8 \log. \text{ ar. co.} = 7.046337$

$(A + B) + 2 = 30^\circ 9' 30'' \text{ tang.} = 9.764207$

$(b \oslash a) = 59.2 \log. = 1.772322$.

Tang. $\frac{A \oslash B}{2} = 8.582866 = 2^\circ 11' 30''$.

Now half the difference ($2^{\circ} 11' 30''$) between A and B, being added to half their sum ($30^{\circ} 9' 30''$) (§ 78. § h.), gives B the greater angle; and being subtracted from said half sum, gives A the less angle. Thus $30^{\circ} 9' 30'' + 2^{\circ} 11' 30'' = 32^{\circ} 21'$, or B; and $30^{\circ} 9' 30'' - 2^{\circ} 11' 30'' = 27^{\circ} 58'$, or A. A is known to be the less angle (§ 63.), because its subtending side (a) is the less of the sides b & a .

§ d. The angles and two sides being known, the value of the third side (c) is deducible by means of calculations conducted upon the proportion (§ 74.) between the sides and sines of the angles of plane triangles.

§ e. To find c . $\text{Sin. B} : b :: \text{sin. C} : c$.

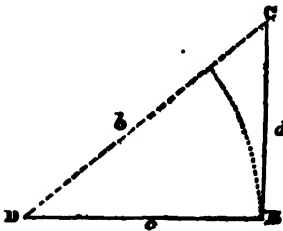
$$B = 32^{\circ} 21' \text{ sin. ar. co.} = 0.271573 = \text{co-sec. B} \text{ (§ 100. § a.)}$$

$$b = 479 \text{ faths. log.} = 2.680336$$

$$C = 119^{\circ} 41' \text{ sin.} = \underline{\underline{-9.988908}} \text{ (§ 102. § g.)}$$

$$\text{Log. } c = \underline{\underline{2.890817}} = 777.7 \text{ faths.}$$

§ f. If the problem comprise a right angle triangle, with similar data, the process of solving it is much more simple. For the two legs and the right angle being the given parts, the calculations for determining the unknown parts, are founded upon the relations of the sides and trigonometric functions of a right angled triangle, as they are shown § 75.



§ g. In the proposed triangle (D B C), right angled at B, the values of the legs, are;

$c = 479$ faths.; and $d = 419.8$ faths.; to find the other parts.

§ h. Making either leg (c) radius, (§ 75. § c.) the terms and order of the proportion for finding the value of D, are, $c : \text{rad.} :: d : \text{tang. D}$. And for finding the hypotenuse (b) they are, $\text{rad.} : c :: \text{sec. D} : b$.

§ i. To find D.

$$c = 479. \text{ Log. ar. co.} = 7.319664$$

$$\text{Radius} \quad \quad \quad = 10.$$

$$d = 419.8 \text{ log.} = \underline{\underline{2.623042}}$$

$$\text{Tang. D} = \underline{\underline{9.942706}} = 41^{\circ} 13' 53''$$

§ j. To find b .

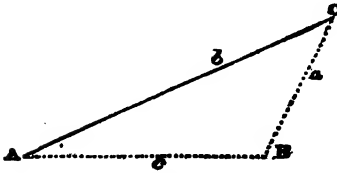
$$\text{Radius} \quad \quad \quad = 10.$$

$$c = 479 \text{ log.} = 2.680336$$

$$D = 41^{\circ} 13' 53'' \text{ sec.} = \underline{\underline{0.123777}}$$

$$\text{Log. } b = \underline{\underline{2.804113}} = 636.9 \text{ faths.}$$

CASE IV.



§ 112. Given two angles (A & C) and the side (b) between them;

A=26° 1'

C=38° 49'

b=179 chains,

To find the values of the other parts (B, c & a) of the proposed triangle. (A B C).

§ a. Two angles being known, the third (B) (§ 32. § b.) is also known; it is B=115° 10'.

§ b. The remaining unknown parts (a & c) of this triangle, are determined by means of the principles involved in the solution of the problem under Case 2. The process of operation in the solution of this, is precisely similar to that of § 109. § i. under Case 1. In the method of their solution, Cases 1, 2, and 4, are only repetitions of each other. For whenever two angles of a triangle are given, the third is also known (§ 32. § b.); and whenever an angle and its subtending side are two of the known parts, the required parts are determinable through calculations conducted upon the proportion (§ 74.) which exists between the sides and sines of angles in all triangles.

§ c. To find the value of a. Sin B : b :: sin. A : a.

B=115° 10', sin. ar. co.=0.043316=co-sec. B (§ 101. § b.)

b=179 chains log. =2.252853

A=26° 1', sin. =9.642101

Log. a = 1.938270 = 86.7 chains.

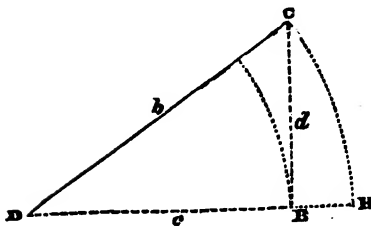
§ d. To find the value of c. Sin. B : b :: sin. C : c.

B = 115° 10' co-sec. = 0.043316

b = 179 chains log. = 2.252853

C = 38° 49' sin. = 9.797150

Log. c = 2.093319 = 123.9 chains.



§ e. Transferring the conditions of this problem to one in right angled trigonometry, the use of sines may be retained in the calculation for determining the unknown parts (c & d); for by making the given side (b), radius to an arc (C H), the two legs of the proposed triangle (D B C)

become (§ 75. § a.) sine and co-sine to the acute angles (D & C);

and (§ 75. § b.) rad. : b :: sin. D : d :: co-sine D : c . The parts given are $D = 26^\circ 1'$, and $b = 179$ chains.

§ f. To find the values of c and d .

$$\begin{array}{rcl} \text{Radius} & - & - = 10. \\ b = 179. & \log. & = 2.252853 \\ D = 26^\circ 1' & \sin. & = 9.642101 \end{array}$$

$$\text{Log. } d = \underline{\underline{1.894954}} = 78.5 \text{ chs.}$$

$$\begin{array}{rcl} & \text{Radius} & = 10. \\ b = 179 & \log. & = 2.252853 \\ D = 26^\circ 1' & \cos. & = 9.953599 \end{array}$$

$$\text{Log. } c = \underline{\underline{2.206452}} = 160.8 \text{ chains.}$$

§ g. The same result may be obtained by making a radius of either leg (c), and introducing different trigonometric functions into calculation, upon the principles shown under § 75. § c. Thus; sec. D : b :: radius : c :: tang. D : d .

§ h. To find c and d by this method of calculation :

$$\begin{array}{rcl} D = 26^\circ 1' \text{ sec. (ar. co.)} & = & 9.953599 = \cos. D \text{ (§ 101. § b.)} \\ b = 179 \text{ chains} & \log. & = 2.252853 \\ \text{Radius} & - & - = 10. \end{array}$$

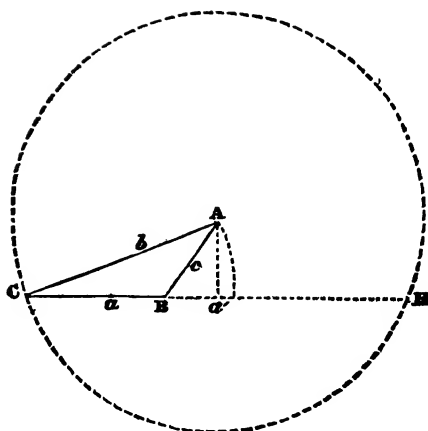
$$\text{Log. } c = \underline{\underline{2.206452}} = 160.8 \text{ chains.}$$

$$\begin{array}{rcl} D = 26^\circ 1' \cos. (\S 101. \S b.) & = & 9.953599 \\ b = 179 \text{ chains} & \log. & = 2.252853 \\ D = 26^\circ 1' & \text{tang.} & = 9.688502 \end{array}$$

$$\text{Log. } d = \underline{\underline{1.894954}} = 78.5 \text{ chains.}$$

§ i. The use of sine and co-sine as functions in plane trigonometrical calculations, being more convenient than that of other functions, the former (§ f.) is generally preferable to the latter method (§ h.) of solving problems.

CASE V.



§ 113. Given the three sides (BC, b, c)
 BC = 144 chains.
 b = 220 chains.
 c = 103 chains.

To find the angles (C, A & B)

§ a. Here is the application to practice of the principles involved in § 79. And in order to determine the value of any one (C) of these angles by calculation, the distance (a a') from the middle (a) of the base (CB), to the perpendicular (A a') upon

it, must be found by means of the proportion, (§ 79.) the base BC : b ∞ c :: b + c : 2 a a'. Then either of the segments (C a') is found (§ 79. § d.) by applying (C a) half the base (CB) to (a a') the distance of the perpendicular at a' from the middle of the base. This segment (C a'), the perpendicular (A a'), and the side (b), adjacent to the required angle (C), form a right angled triangle (C A a'); of which are known the hypotenuse (b) and a leg (C a').

§ b. The value of the required angle (C) is then found according to the principles established under § 75. For by making the hypotenuse (CA) radius, the segment (C a') becomes co-sine to the required angle (C); the value of which may then be determined (§ 75. § b) by means of the proportion, b : radius :: C a' : cos. C.

§ c. Knowing the value of any one (C) of the angles, either of the others (A) may then be determined by means of calculations conducted upon the proportion (§ 74.) of the sides of triangles to the sines of their opposite angles. And the remaining angle (B) (§ 32. § b.) is determinable by subtraction.

§ d. To find the value of the angle C.

1st. Call CB the base, in order to find twice the distance (a a') from the middle of the base, to a perpendicular drawn upon the base, from its opposite angle (A) (§ 79.). Twice the length of this distance (a a') is evolved by the proportion; CB : b ∞ c :: b + c : 2 a a'.

CB = 144.	log. ar. co. =	7.841637
b ∞ c = 117.	log. =	2.068186
b + c = 323.	log. =	2.509203

$$\text{Log. } 2 a a' = 2.419026 = 282.4$$

L

2d. To find the greater segment $C a'$. One half of 262.4 ($2 a'$) is equal to the distance (a') of the perpendicular from the middle of the base; and (§ 79. § d.) $a a' + \frac{B C}{2} = C a'$. Therefore, $131.2 + 72 = C a' = 203.2$.

§ e. If twice the distance (a') of the perpendicular from the middle of the base be *greater* than the base, the perpendicular falls *without* the triangle, as in the case before us. But if it be *less* than the base, the perpendicular falls *within* the triangle.

To find the value of C . According to § b. we have, $b : \text{rad.} :: C a' : \cos. C$.

$$b = 220. \text{ log. ar. co.} = 7.657577$$

$$\text{Radius} = 10.$$

$$C a' = 203.2 \text{ log.} = 2.307924$$

$$\text{Cos. } C = 9.965501 = 22^\circ 32' 11''$$

§ f. To find the value of A (§ c.) we have, $c : \sin. C :: B C : \sin. A$.

$$c = 103. \text{ log. ar. co.} = 7.987163$$

$$C = 22^\circ 32' 11'' \text{ sine} = 9.583504$$

$$B C = 144. \text{ log.} = 2.158363$$

$$\text{Sine } A = 9.729030 = 32^\circ 24' 2''$$

§ g. To find the value of B . According to § 32. § b. $C + A - 180^\circ = B$; therefore $B = 125^\circ 3' 47''$.

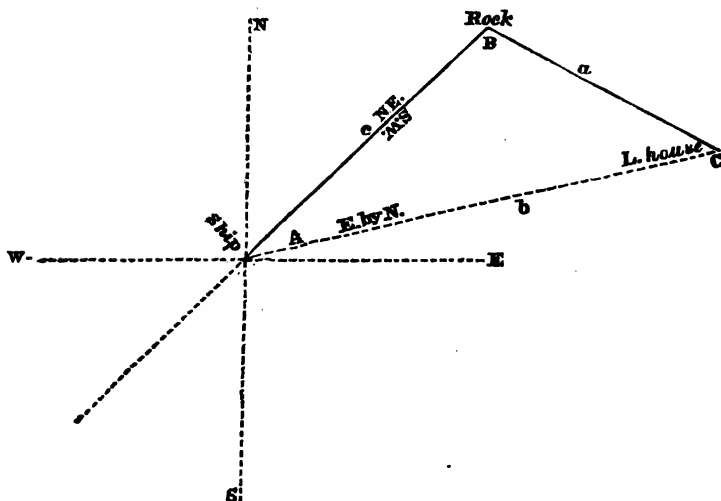
§ h. If, when the perpendicular falls *without* the triangle, the obtuse angle (B) be the first part required, the side (c) adjacent to it becomes hypotenuse to the right angled triangle, of which, the perpendicular to the base and the smaller segment ($B a'$) of the base, are legs, and the supplement ($A B a'$) of the required angle (B) is that which is evolved from the proportion (§ b.), $c : \text{rad.} :: B a' : \cos. A B a'$.

EXAMPLE I.

§ 114. A rock is seven miles from a light-house, and a ship, having the light-house to bear E. by N., is nine miles S. W. of the rock; What is the distance between the ship and the light-house?

Ans. 12.4 miles.

§ a. The solution of this problem is conducted after the methods shown under § a. and § i. (§ 109.) The parts given are the sides a and c , and the angle (A) contained at the ship, between the bearing of the rock and light-house. The value of A is known by converting into degrees, etc., the points contained between the bear-



ing of the rock and light-house, from the ship. The rock bears N. E., and the light-house E. by N. from the ship. Three points intervene between E. by N. and N. E., and each point contains $11^{\circ} 15'$; therefore the angle (A) contained between the lines of these bearings is $11^{\circ} 15' \times 3$, or $33^{\circ} 45'$.

§ b. Unless the problem were embraced in a right angled triangle, the value of C must be found, and B known (§ 32. § b.) before that of b is determined. C is found (§ 109. § b.) from the proportion (§ 74.) which exists between certain parts of every triangle; viz., $a : \sin. A :: c : \sin. C$. Then $C + A - 180^{\circ} = B$; and then, $\sin. C : c :: \sin. B : b$, affords the solution required.

EXAMPLE II.

§ 115. A ship was steering N. and sailing 10 knots per hour; she made a point of land bearing N. E. by N.; two hours afterwards the same point bears E. by S. What is the distance of it from the ship?

Ans. 12. + miles.

§ a. The solution of this problem depends upon the principles which Example 1 involves. The angle (A) between the point of land and the ship's course is three points, or $33^{\circ} 45'$. The angle (C) between the bearing of the ship from the point of land at each station, is six points, or $67^{\circ} 30'$. And the angle (B) between the second bearing of the land and the line upon which the ship sailed,

towr contains $11\frac{1}{2}$ points, or $129^\circ 22' 30''$. The sum of the angles B and C is found by subtracting A (§ 32.) from 180° . And half their difference is determined by the proportion (§ 77.), $b+c :$

$$\text{tangent } \frac{B+C}{2} :: b \propto c : \text{tang. } \frac{B \propto C}{2}.$$

§ c. The value of B and of C is then known by § 77. § h.; and the solution completed by evolving the value of a from the proportion,— $\sin. C : c :: \sin. A : a$.

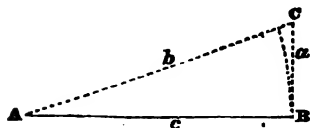
EXAMPLE IV.

§ 117. Wishing to know the height of a Chinese pagoda that stood in a plane, I took the altitude of the sun from an artificial horizon, and at the same time the length of the shadow cast by the pagoda was marked and measured.

The sun's corrected altitude was $21^\circ 10'$.

The length of the shadow 347 feet.

Ans. Height of pagoda 134.3 feet.



§ a. The pagoda is supposed to stand upright. The angle (A) which is formed at the end of the shadow by an imaginary line drawn thence to the top of the pagoda, is $21^\circ 10'$, or whatever be the altitude of the body which

causes the shadow. The solution of this problem may be conducted according to the principles shown § 75. § c.

§ b. The parts given are the length (c) of the shadow, the angles A and B. The latter is known from the primitive condition of the triangle, which is right angled.

§ c. Making the given leg radius, the other leg (a) or height of the pagoda, becomes tangent to the angle (A) of altitude. The value of the required part (a) is evolved by means of the proportion, $\text{rad.} : c :: \text{tang. } A : a$.

EXAMPLE V.

§ 118. A road crosses a mountain, the base of which is nine miles across. The distance from the foot to the top of the mountain, is 4.5 miles on one side, and 5.3 on the other.

What is the angular ascent of the road on each side of the mountain, and how high is the top above the base ?

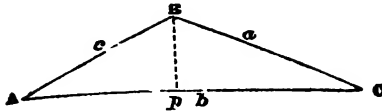
Ans. Angle of ascent on one side = $31^\circ 32' 21'' 25'' 15''$

Angle of ascent on the other = $26^\circ 32' 25'' 45'' 30''$

Height of the mountain 1.94 miles. 1.94

§ a. This problem involves in the solution of it, the principles

demonstrated § 79. and § 75. The parts given are the three sides (a, b, c).



§ b . The operation for finding the ascent of the road, or the angles A and C , is a repetition of the method shown in Case 5, § 113, for finding C and B . Bp , the perpendicular upon the base (b) of the mountain, shows its height; the value of it is determined (§ 75.) after the angles A & C are determined. $AC : c \propto a :: c + a : 2pb$; and (§ 79. § d .) $pb + \frac{AC}{2} = pC$; and (§ 75. § b .) $a : \text{rad.} :: pC : \cos. C$; and (§ 74.) $c : \sin. C :: a : \sin. A$; and $\text{rad.} : a :: \sin. C : Bp$.

EXAMPLE VI.

Wishing to ascertain the breadth of a river, I observed that a cocoa-nut tree, which stood at the water's edge on the opposite bank, bore west of me; then after having walked due south 194 yards, the same tree bore N. W.

How wide was the river?

Ans. 194 yards.

EXAMPLE VII.

Wishing to know the height of a mountain in South America, on the morning of the 22d December, 1832, between eight and nine o'clock, I measured with a sextant the altitude of the sun above the top of the mountain; it was $19^\circ 40' 15''$. The apparent altitude of the sun, measured at the same time from an artificial horizon, was $41^\circ 47' 45''$. Then going from the mountain, I measured 4420 fathoms, and the next day, when the top of the mountain was again in the plane of the sun's azimuth circle, the sun's altitude above the top of the mountain, and from an artificial horizon taken as before, was, from the horizon = $40^\circ 21' 30''$, above the mountain = $25^\circ 14'$

What is its height?

Ans. 3564.1 fathoms, or 21384 feet 7 inches.

EXAMPLE VIII.

Supposing that a ray of light is not refrangible, that the earth is a perfect sphere, that its diameter is 7924 miles, and that the height of Chimborazo is 21384 feet, How far can the top of this mountain be seen from the surface of the earth?

* The mean diameter of the earth is about 7923.57 miles.

SPHERICS.

SPHERICS.

§ 119. Spherical Trigonometry is one of the principal branches upon which the science of navigation is founded. The solution of all problems for finding latitude, longitude, variation, etc., from data obtained by means of observations made upon celestial objects, depends upon the principles of spherical trigonometry.

§ *a*. As the figure of the earth is an approximation to that of a sphere, the whole science of navigation, properly speaking, is based upon the doctrines of the sphere.

§ 120. The figure of a *sphere* may be generated by the revolution of a semicircle about its diameter, as an axis. A perfectly round globe or ball is a sphere.

§ 121. Trigonometrically speaking, all lines upon a sphere are curved lines. These lines are either circles, or arcs of circles. Circles are either *great* or *small*.

§ 122. A *great circle* is concentric with the sphere.

§ *a*. The shortest distance between two points on a plane (§ 3.) is the straight line that joins them; and the shortest distance on a sphere, between two points, is the arc of a great circle, that joins them.

§ *b*. Every great circle has two *poles*.

§ 123. The *poles* of a great circle are two points on the surface of a sphere, that are diametrically opposite to each other, and are equidistant from all points at the circumference of their circle. Consequently each pole is 90° from the circumference of its circle.

§ *a*. The north and south poles are the poles of the equator. The equator is 90° from either pole.

§ 124. The straight line, which, passing through the centre of a great circle, joins its poles, is the *axis* of that circle.

§ 125. All great circles cross each other in points diametrically opposite. Therefore,

§ *a*. Two great circles cannot be parallel; but they divide each other into arcs of 180° each.

§ *b*. The space contained between the intersecting halves of two great circles, is called a *lune*.

§ 126. One great circle is perpendicular to another, when the two cross each other at right angles.

§ *a*. When two great circles cut each other at right angles, the axis of either lies along the plane of the other; and they pass through the poles of each other.

§ 127. A *secondary* is a great circle that crosses another, perpendicularly.

§ *a.* A meridian of longitude is secondary to the equator.

§ 128. Every great circle divides its sphere into two equal parts, as the Equator does the earth.

§ 129. A *small circle* divides the sphere into two unequal parts; as the Tropic of Cancer, or of Capricorn, or as a parallel of latitude, does the earth.

§ *a.* If every point at the circumference of a small circle be equidistant from either pole of a great circle, the small is parallel to the great circle.

§ *b.* The centre of a small circle is in the axis of the great circle to which it is parallel.

§ 130. The radius of a small circle, is the sine of the arc intercepted between its circumference and the pole of the great circle, to which it is parallel.

§ *a.* The radius of a parallel of latitude is the co-sine of its own latitude.

§ 131. All spheric, like all rectilineal, angles, are either acute right, or obtuse; and their values are expressed under the same denominations of degrees ($^{\circ}$), minutes ($'$), and seconds ($''$).

§ 132. Two arcs of great circles, like two straight lines, (§ 25 & § 27.) that cross each other, make the vertically opposite angles equal to each other, the two angles that are on the same side of either, equal to two right angles, and the four angles together equal to four right angles.

§ 133. A spherical angle is at the pole of the circle, upon which it is measured.

§ *a.* And the arc of this circle, which is intercepted between the two arcs forming the angle, is its measure.

§ *b.* The two circles which form an angle are secondaries (§ 127.) to that upon which the angle is measured.

§ *c.* This circle (§ 126. § *a.*) passes through the poles of its secondaries.

§ *d.* The distance between the poles of two great circles that form an angle with each other, measures that angle.

§ 134. Every spherical, like every plane triangle, has three sides and three angles, and any two of the former are greater than the third.

§ 135. The sides of a spherical triangle are arcs of circles. Their values are *always* expressed in degrees ($^{\circ}$), minutes ($'$), and seconds ($''$).

§ 136. Plane triangles are unlimited as to the value of their sides; the sum of the three sides of a spherical triangle is less than 360° . For if two arcs that form an angle be each 180° , they will intersect each other (§ 125.) in another point, that is diametrically opposite to their angular point; then, if a third arc cross these two, it will divide the lune (§ 125. § *b.*) into two triangles, and be a side to each. This third side is less (§ 134.) than the sum of the two other sides of either triangle, and the sum of these four sides, by supposition,

is two semicircles, or equal to 360° ; therefore the sum of the two sides of either triangle plus the third side is *less* than 360° ; and so with any spherical triangle.

§ a. The minimum of the sides of a spherical triangle is without limits.

§ 137. The sum of the angles of a plane triangle is equal to two right angles; the sum of the angles of a spherical triangle, in all cases, is greater than two, but less than six, right angles.

§ a. The exterior angle of a spherical triangle (§ 132.), therefore is not, as in a plane triangle, necessarily equal to the sum of the two remote interior angles of the triangle.

§ b. Most of the other propositions of elementary geometry which show the ratio, relations, or equality between the parts of plane triangles, are likewise applicable to spherical triangles.

§ 138. Two angles, or two legs, of a spherical triangle, are *alike*, or of the *same affection*, when both of them are either acute or obtuse.

§ 139. In spherical, as in plane triangles (§ 63.), the greatest side and angle subtend each other; so also do the mean and least.

§ a. An angle and its opposite side are not always of the same affection.

§ 140. Spherical, besides being, as plane triangles, divided into equilateral, isosceles, and scalene, are either right angled, quadrantal, or oblique.

§ 141. A *right angled spherical triangle* has one right angle.

§ a. The side subtending the right angle is the hypotenuse; but it is not, as in a plane triangle, necessarily the greatest side.

§ b. If the hypotenuse be less than 90° , the legs are of the same affection; and the oblique angles also.

§ c. If the hypotenuse be greater than 90° the legs are unlike, and the oblique angles also.

§ d. A leg of a right angled spherical triangle, and its opposite angle, are always of the same affection.

§ 142. A *quadrantal triangle* has a side that is a quadrant, or 90° .

§ 143. An *oblique spherical triangle* has neither a side, nor an angle of 90° .

§ 144. In plane triangles, the sides, and the sines of their opposite angles, have the same ratio to each other; in spherical triangles, the sines of the opposite sides and angles are proportional.

§ 145. In plane triangles (§ 107.), one side, at least, must be among the three parts that constitute the data necessary for finding the other parts; in spherical triangles any three parts are data sufficient for determining the other parts. Therefore—

§ 146. In spherical trigonometry, the three sides of a triangle can be determined by having the three angles as data; and the reverse.

§ 147. For the purpose of facilitating, by logarithmic calculation, the solution of problems in spherical trigonometry that are included in oblique triangles, spherical trigonometry is here divided into right and oblique angled trigonometry.

§ *a*. All the problems that come under the several cases, two excepted, of oblique spherical trigonometry, may be solved by reducing them to analogous cases in right angled trigonometry; this is done by means of drawing a perpendicular from an angle upon its subtending side, of the proposed triangle, and from such an angle, that the proposed oblique triangle will be divided into two right angled triangles, one of which shall contain two of the parts given in the problem.

§ 148. Every problem in oblique trigonometry may be solved without the direct intervention of a perpendicular, and of right angled triangles.

§ *a*. The process of calculation is shortened by solving the problems without conducting the operation upon the principles of right angled triangles. For this reason spherical trigonometry is here divided into right and oblique trigonometry.

§ 149. Problems comprising as data the three angles, or the three sides of an oblique triangle, do not admit of the intervention of right angles for determining the unknown parts; because, in neither case can a perpendicular be so drawn in the proposed triangle, that two of the given parts will fall in either of the right angled triangles, or on either side of the perpendicular.

§ 150. By some, problems that depend upon quadrantal triangles, are included in a separate division of spherical trigonometry, called "quadrantal trigonometry." But knowing three parts, the other three of a quadrantal triangle may be determined by the rules for finding with similar data, like parts that are unknown in a right angled triangle.

§ *a*. In arranging for calculation the trigonometric functions of the parts in a problem included in a quadrantal triangle, the legs must be called *angles*, and their adjacent angles, *legs*; the *supplement* of the angle that subtends the quadrantal side, must be called the *hypotenuse*, and the quadrantal side, *radius*.

§ 151. The side and angle that are opposite in a quadrantal triangle, are of the same affection.

§ 152. The relations between the parts of a triangle in plane, and the parts of a triangle in spherical, trigonometry, are similar; and the formulæ for the solution of problems in the one, are analogous to the formulæ for the solution of similar problems in the other.

§ *a*. As much, with regard to the sides, can be determined from the three angles of a plane triangle, as can be determined from the three angles of a spherical triangle. In the former case, the triangle can be determined in *species*; and in the latter, the sides are determined in degrees and minutes of arcs, which arcs, in *absolute* length, may be infinitely great or small. And the number of miles, or of any units of a positive quantity, contained by any one of these arcs, depends upon the relations of other quantities to it, and not at all upon the relations between it and the other parts of its triangle.

§ *b*. In order then to arrive at absolute values for the sides and angles of a triangle which are submitted for calculation, a third

quantity, that will answer the purpose of a common measure for lines and angles, or arcs, must be brought into consideration.

§ c. And this common measure is the *radius* of a circle. When we say that a side of a spherical triangle is an arc containing a certain number of degrees and minutes, we have no reference to the *absolute* length of this arc, or to the number of inches, or fathoms, or miles, contained in it. It may be an arc whose radius is infinitely great or small. But if the length of its radius in inches, or miles, be known, the *absolute* length of the arc also in the denominations of the same dimensions is determinable. But this belongs to another branch of mathematics, which is not relevant to the purposes for which this treatise is designed.

§ 153. The analogy between the formulæ for the solution of problems of like data and quæsitæ in plane and spherical trigonometry, grows out of the resemblance between the parts of a plane and a spherical triangle.

§ a. Suppose that the sides of a spherical triangle should remain of the same *absolute* length, and that the radius of their sphere be increased *ad infinitum*. The angles at the centre of the sphere, which these sides subtend, and the number of degrees contained in these sides, or arcs, will be decreased until they reach the *ultimatum*, when the surface included by these three sides becomes a plane, the triangle a plane triangle, and a *finite* portion of infinity.

RIGHT ANGLED SPHERICAL TRIGONOMETRY.

§ 154. In the trigonometrical solution of right angled spherical triangles, there are *five parts*, (three sides and two angles,) any one of which may be the unknown and required part of the problem.

§ a. The right angle is ever known from the condition of the triangle. In calculation the right angle is not called a *part*.

§ 155. Any two of the five parts of a right angled spherical triangle being known, the rest are determinable by means of trigonometrical calculations.

§ 156. In right angled spherical trigonometry, there can be six cases of two different parts as data, viz.:

1. The hypotenuse and a leg.
2. The hypotenuse and an angle.
3. The two legs.
4. The two angles.
5. An angle and its opposite leg.
6. A leg and its adjacent angle.

§ 157. Lord Napier has given *two rules*, by which every problem that can occur in any of these several cases, may be solved. But he has not made known the process of reasoning, by which he arrived at the conclusions, whence he deduced these rules; nor have mathematicians been able to follow the steps of this bold reasoner.

§ a The truth of his, called the Catholic Proposition, is sufficiently

established by practice, and by its utility, and may therefore be admitted as an axiom or received truth.

§ 158. In his Analytical Treatise on Plane and Spherical Trigonometry, Dr. Lardner observes, "We have thus established *Napier's rules*, by proving separately all the several cases which they include. There is no independent or general demonstration of these remarkable theorems, nor is it easy to conceive the process of mind by which their illustrious inventor arrived at them. Professor Woodhouse justly observes, that there are not, perhaps in the whole compass of mathematical science, rules which more completely attain that which is the proper object of rules, namely, brevity and facility of computation. He might have added, that few, or perhaps no theorems equally general, make such an immediate and permanent impression on the memory."*

§ 159. The *five* parts of a right angled triangle, are called, in Napier's rules, the *circular* parts.

§ a. The circular parts are the two legs, and the *complements* of the hypotenuse and its adjacent angles, instead of these three parts themselves.

§ 160. The right angle (§ 154. § a.) is thrown out of consideration; and the five circular parts join each other.

§ a. The complement of the hypotenuse joins the complement of each of the two angles; each of which angular complements joins its adjacent leg, and the two legs join each other.

§ 161. In every problem two of these parts are given and a third is sought. They are named from their relative position with regard to each other.

§ a. One of them is the *middle* part; the two others are *extremes*, either *conjunct* or *disjunct*.

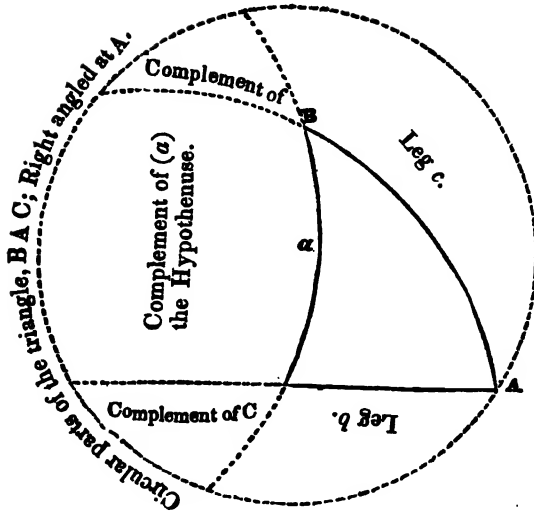
§ 162. If the parts given and the part sought join each other in circular order, the first and last in this order are called *extremes conjunct*; and the part between them connects them together, and is called the *middle* part.

§ a. The triangle A B C, represents a right angled spherical triangle, right angled at A; its hypotenuse is *a*, and *b* & *c* are its two legs. The circular parts are the leg *b*, the complement of C, do. of *a*, do. of B, and the other leg *c*.

§ b. The three parts, *a*, B, *c*, join in circular order. The complement of B is the connecting part between *c* and the complement of *a*. The complement of B is the middle part; *c*, and the complement of *a*, are extremes conjunct.

§ c. Of the parts	Middle part is,	Extremes conjunct are,
B, c, b.	c.	b, and complement of B.
c, b, C.	b.	c, and complement of C.
b, C, a.	Complement of C.	b, and complement of a.
C, a, B.	Complement of a.	Complements of C and B.
a, B, c.	Complement of B.	c, and complement of a.

* See Analytical Treatise on Plain and Spherical Trigonometry, by the Rev. Dionysius Lardner.



§ 163. If the parts given, and the parts sought, be not adjacent to each other in the circular order, that part which stands alone, or is not adjacent to either of the two other parts, is the middle part, and the two others are *extremes disjunct*.

§ a. The three parts, a, c, b , do not join each other in the circular order; the complement of a is not adjacent to either of the two other parts, being separated from them by the complement of the two acute angles, C & B ; it stands alone, and is therefore (§ 163.) the middle part, and the two other parts that do join each other, are the *extremes disjunct*.

§ b. Of the parts	Middle part is,	Extremes disjunct are,
$B, b, C.$	Complement of $B.$	b , and complement of $C.$
$c, C, a.$	$c.$	Complements of C and $a.$
$c, C, B.$	Complement of $C.$	c , and complement of $B.$
$b, a, B.$	$b.$	Complements of B and $a.$
$a, c, b.$	Complement of $a.$	b and $c.$

§ 164. Napier's rules establish an equality of ratio between radius, the *sine* of the middle part, and certain trigonometric functions of the *extremes*. They are,

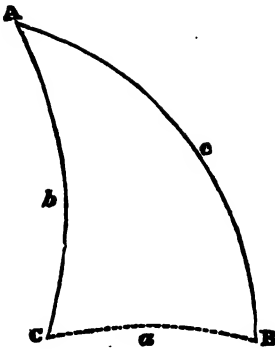
§ 165. The product of radius and sine of the middle part, is equal to the product of the tangents of extremes conjunct. And,

§ 166. The product of the co-sines of extremes disjunct, is equal to the product of radius and sine of the middle part.

§ 167. Every problem in right angled spherical trigonometry can be solved either by the one or the other of these two rules.

§ a. They have been put in the following mnemonic form :

“The *product* of radius and middle part's sine
 Equals *that* of the tangents of parts that combine ;
 Or, *that* of the co-sines of those that disjoin.”



§ 168. If the data and quæsitæ be opposite sides and angles, the problem proposed may be solved by the proportion (§ 144.) between the sines of opposite sides and angles.

§ 169. A B C represents a right angled spherical triangle upon a plane; B is the right angle, and b the hypotenuse.

§ a . The circular parts, commencing with c , and naming them in the order around to the right, in which they join; are c , a , $\cos. C$, $\cos. b$, and $\cos. A$.

CASE I.

§ 170. Given the hypotenuse (b) and a leg (c).

The hypotenuse $b = 74^\circ 40'$.

The leg $c = 60^\circ 14'$.

To find the other parts.

§ a . 1st. To find the other leg (a).

The three parts that, in this case, comprise the data and quæsitum of the problem, are the two legs (c , a), and the hypotenuse (b). The complement of b , (§ 163. § a .) is not adjacent to either of the other parts, (c , a .); therefore (§ 163.) it is the middle part, and c and a are extremes disjunct; and (§ 166.) $\cos. c \times \cos. a = \text{Sin. } b^\circ \times \text{Rad.}$;* then (§ 71.) $\cos. c : \text{Rad.} :: \sin. b^\circ : \cos. a$.

$$c = 60^\circ 14'. \cos. \text{ar. co.} = 0.804108 (= \sec. c.)$$

$$\text{Radius} \qquad \qquad \qquad 10.$$

$$b = 74^\circ 40'. \cos. (= \sin. b^\circ) = 9.423318$$

$$\text{Cos. } a = 9.726426 = 57^\circ 49'.$$

§ c . In looking in the tables for the degrees, etc., which correspond to the logarithmic co-sine, 9.726426, it will be seen that $122^\circ 11'$, the supplement of $57^\circ 49'$, also corresponds to it. Which of the two to take, is known (§ 141. § b .) by the affection of the two legs.

§ d . The sine, tangent, secant, etc., of an arc or angle, (§ 52. § d .) are the co-sine, co-tangent, co-secant, etc., of the complement of that arc, and (§ 104.) the sine, tangent, secant, etc., of the supplement of the same arc. Therefore,

§ e . Cor. The logarithmic value of any trigonometric function

* To obviate the necessity of writing *complement*, the circular parts of the hypotenuse and two angles will be denoted in the rest of the work by writing an ($^\circ$) after the letter which stands for any one of those parts. Thus, a° , C° , B° , stands for the complements of a , C , B .

corresponds to two arcs or angles; viz., an angle and its supplement. Thus, log. sine $9.611576 = 24^\circ 8'$, or $155^\circ 52'$.

§ f. There is generally some circumstance connected with the conditions of the problem, or triangle, under solution, which determines the affection of the required part, and thence the arc required; for every one of the sides and of the angles of a spherical triangle (§ 136. & § 137.), may be either greater or less than a right angle, or 90° .

§ g. To find the value of a . In this case A° is the middle part. It is between, and joins, the two other parts b° and c . Therefore (§ 162.) b° and c are extremes conjunct. And (§ 165.) $\text{rad.} \times \sin. A^\circ = \text{tang.} c \times \text{tang.} b^\circ$. Then (§ 71.), $\text{rad.} : \text{tang.} b^\circ :: \text{tang.} c : \sin. A^\circ$.

Radius	= 10.
$b = 74^\circ 40'$ co-tang.	= 9.438059 (= tang. b° , § 52. § d.)
$c = 60^\circ 14'$ tang.	<u>10.242655</u>

Sin. A° (= co-sin. A . (§ 52. § d.)) = 9.680714 = $61^\circ 21' 9''$.

§ h. $61^\circ 21' 9''$, and *not* its supplement, is known (§ 141. § b. & § d.) to be the required value of the angle A .

§ i. If the angle (C) opposite to (c) one of the given sides be required from the data of this case, the ratio (§ 144.) between the sines of opposite sides and angles, will determine the value of it, viz. : $\text{Sin.} b : \text{sin.} B (= (\text{§ 62.}) \text{rad.}) :: \text{sin.} c : \text{sin.} C$.

§ j. Or the value of C may be determined by means (§ 166.) of the Catholic Proposition; c (§ 163.) is the middle part, and b° and C° are extremes disjunct; and (§ 166.) $\text{cos.} b^\circ \times \text{cos.} C^\circ = \text{sin.} c \times \text{rad.}$; wherefore (§ 71.) $\text{cos.} b^\circ : \text{rad.} :: \text{sin.} c : \text{cos.} C^\circ$; which is the proportion under § i., stated under different denominations, but the same ultimately; for the co-sine b° and $\text{cos.} C^\circ$ (§ 52. § d.) are the same as $\text{sin.} b$ and $\text{sin.} C$.

$b = 74^\circ 40'$ sin. ar. co.	= 0.015741 = co-sec. (§ 101. § b.)
Radius	= 10.
$c = 60^\circ 14'$ sine	<u>9.938547</u>

Sine $C = \underline{9.954288} = 64^\circ 10' 14''$ (§ 141. § d.)

CASE II.

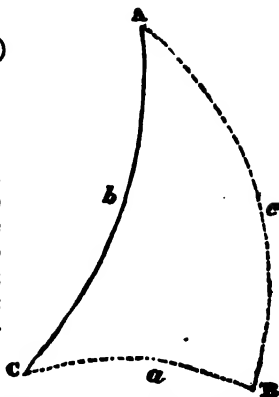
§ 171. Given the hypotenuse (b)
and an angle (C),

$$b = 119^\circ 39'$$

$$C = 104^\circ 01'$$

To find the other parts.

§ a . 1st. To find the other angle (A).
The two angles (A, C) are adjacent to
the hypotenuse (b), which is therefore
(§ 162.) the middle part, and the two
angles (A, C) are extremes conjunct;
then (§ 170. § g .) $\text{tang. } C^\circ : \text{rad.} ::$
 $\text{sin. } b^\circ : \text{tang. } A^\circ$; and (§ 52. § d .) co-
 $\text{tang. } C : \text{rad.} :: \text{cos. } b : \text{cotang. } A$.



$$C = 104^\circ 01' \text{ co-tang. ar. co.} = 0.602691 = \text{tang. } (\S 100. \S a.)$$

$$\text{Radius} = 10.$$

$$b = 119^\circ 39' \text{ cos.} = 9.694342$$

$$\text{Co-tang. } A = 10.297033 = 26^\circ 46' 36''.$$

§ b . The value of A is known to be less than 90° (§ 141. § c),
because C is greater than a right angle.

§ c . 2d. To find a , which (§ 141. § d .) is also less than a right
angle. The three parts, a, C° , and b° , join in circular order; and
(§ 162. § c .) C° is the middle part, and b° and a are conjunct ex-
tremes. Then (§ 170. § g .) $\text{tang. } b^\circ : \text{rad.} :: \text{sin. } C^\circ : \text{tang. } a$; and
(§ 52. § d .) $\text{co-tang. } b : \text{rad.} :: \text{cos. } C : \text{tang. } a$.

$$b = 119^\circ 39' \text{ co-tang. ar. co.} = 0.244709 = \text{tang. } (\S 101. \S b.)$$

$$\text{Radius} = 10.$$

$$C = 104^\circ 01' \text{ cos.} = 9.364182$$

$$\text{Tang. } a = 9.628891 = 23^\circ 2' 58''.$$

§ d . 3d. To find c ; c (§ 141. § d .) is greater than a quadrant. The
method for finding the value of c in this case, is similar to that
(§ 170. § j .) of finding C , Case 1.

$$\text{Rad.} : \text{sin. } b :: \text{sin. } C : \text{sin. } c.$$

$$\text{Radius} = 10.$$

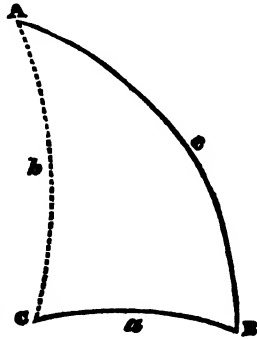
$$b = 119^\circ 39' \text{ sin.} = 9.939052$$

$$C = 104^\circ 01' \text{ sin.} = 9.986873$$

$$\text{Sin. } c = 9.925925 = 122^\circ 31' 18''.$$

CASE III.

§ 172. Given the two legs (a, c),
 $a=31^{\circ} 16'$
 $c=74^{\circ} 14'$



To find the other parts (A, C, b .)

§ *a*. 1st. To find the hypotenuse (b); it is less than a quadrant, because (§ 141. § *b*.) the legs are of the same affection. The three parts, a, b° , and c , do not join each other in the circular order; b° stands alone: then (§ 163. § *a*.) a and c are disjunct extremes, and b° is the middle part; and (§ 170. § *a*. & § XLIX. Alg.) radius :

$$\begin{aligned} \cos. c &:: \cos. a : \sin. b^{\circ} = (\S 52. \S d.) \cos. b. \\ \text{Radius} &= 10. \\ c=74^{\circ} 14' \cos. &= 9.434122 \\ a=31^{\circ} 16' \cos. &= 9.931845 \end{aligned}$$

$$\text{Cos. } b = 9.365967 = 76^{\circ} 34' 12''.$$

§ *b*. 2d. To find an angle (C); it is less (§ 141. § *d*.) than 90° . The three parts C°, a, c , join in circular order; and (§ 162.) C° & c , are conjunct extremes, and a is the middle part: then (§ 165. & § 71.) tang. c : rad. :: sin. a : tang. C° = co-tang. C .

$$\begin{aligned} c=74^{\circ} 14' \text{ tang. ar. co.} &= 9.450777 = \text{co-tang.} \\ \text{Radius} &= 10. \\ a=31^{\circ} 16' \text{ sine} &= 9.715186 \end{aligned}$$

$$\text{Co-tang. } C = 9.165963 = 81^{\circ} 39' 47''.$$

§ *c*. 3d. To find the other angle (A); it is also less than 90° , and it is an extreme conjoined to a by c , which (c) is the middle part; then (§ *b*.) tang. a : rad. :: sin. c : tang. A° = co-tang. A .

$$\begin{aligned} a=31^{\circ} 16' \text{ tang. ar. co.} &= 0.216659 = \text{co-tang.} (\S 101. \S b.) \\ \text{Radius} &= 10. \\ c=74^{\circ} 14' \text{ sin.} &= 9.983345 \end{aligned}$$

$$\text{Co-tang. } A = 10.200004 = 32^{\circ} 15'.$$

CASE IV.

§ 173. Given the two angles (A, C),
 $A=31^{\circ} 56'$
 $C=111^{\circ} 11'$

To find the sides a , b , and c .

§ a. 1st. To find the hypotenuse (b); it is (§ 141. § c.) greater than a quadrant; it connects A° and C° in the circular order, which are conjunct extremes; and (§ 171. § a. § XLIX. Alg.)
 $\text{rad.} : \text{tang. } A^{\circ} :: \text{tang. } C^{\circ} : \text{sin. } b^{\circ}$; or,
 $\text{rad.} : \text{co-tang. } A :: \text{co-tang. } C : \text{cos. } b$.

$$\begin{aligned} \text{Radius} &= 10 \\ A=31^{\circ} 56' \text{ co-tang.} &= 10.205336 \\ C=111^{\circ} 11' \text{ co-tang.} &= 9.588316 \end{aligned}$$

$$\text{Cos. } b = 9.793652 = 128^{\circ} 26' 53''.$$

§ b. 2d. To find the leg c ; it and A° are extremes disjunct; and (§ 166. & § 71.) $\text{cos. } A^{\circ} : \text{rad.} :: \text{sin. } C^{\circ} : \text{cos. } c$; or,
 $\text{Sin. } A : \text{rad.} :: \text{cos. } C : \text{cos. } c$.

$$\begin{aligned} A=31^{\circ} 56' \text{ sin. ar. co.} &= 0.276600 = \text{co-sec.} (\S 101. \S b.) \\ \text{Radius} &= 10. \\ C=111^{\circ} 11' \text{ cos.} &= 9.557932 \end{aligned}$$

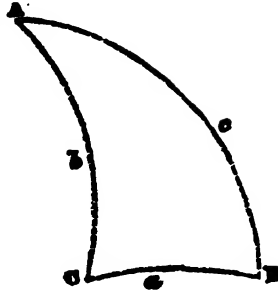
$$\text{Cos. } c = 9.834532 = 133^{\circ} 5' 32'' (\S 139.)$$

C and c (§ 141. § d.) are of the same affection.

§ c. 3d. To find the other leg a ; it is less than a quadrant, and a disjunct extreme; A° is the middle part, and (§ b.) $\text{cos. } C^{\circ} : \text{rad.} :: \text{sin. } A^{\circ} : \text{cos. } a$; or,

$$\begin{aligned} \text{Sin. } C : \text{rad.} &:: \text{cos. } A : \text{cos. } a \\ C=111^{\circ} 11' \text{ sin. ar. co.} &= 0.030384 = \text{co-sec.} (\S 101. \S b.) \\ \text{Radius} &= 10. \\ A=31^{\circ} 56' \text{ cos.} &= 9.928736 \end{aligned}$$

$$\text{Cos. } a = 9.950120 = 24^{\circ} 28' 18''.$$



CASE V.

§ 174. Given an angle (C) and its opposite leg (c),

$$c = 45^\circ 19'$$

$$C = 54^\circ 49'$$

To find the other parts (*a*, *b*, *A*).

§ *a*. This is called the "doubtful case," because there is nothing in the condition of any problem that falls under this case, which will determine the affection of the part whose value is sought.

§ *b*. 1st. To find the other leg (*a*); it is the middle part, connecting the conjunct extremes, C° and *c*; and (§ 172. § *b*.) $\text{rad.} : \text{tang. } C^\circ :: \text{tang. } c : \sin. a$, or,

$$\text{Rad.} : \text{co-tang. } C :: \text{tang. } c : \sin. a.$$

$$\text{Radius} \quad \quad \quad 10.$$

$$C = 54^\circ 49' \quad \text{co-tang.} = 9.848181$$

$$c = 45^\circ 19' \quad \text{tang.} = 10.004801$$

$$\text{Sin. } a = \frac{9.852982}{10} = 45^\circ 27' 54''.$$

§ *c*. 2d. To find the hypotenuse (*b*); it is a disjoined extreme, and is found (§ 170. § *j*.) $\cos. C^\circ : \text{rad.} :: \sin. c : \cos. b^\circ$; or,

$$\text{Sin. } C : \text{rad.} :: \sin. c : \sin. b.$$

$$C = 54^\circ 49' \quad \text{sin. ar. co.} = 0.087612 = \text{co-sec.} (\S 101. \S b.)$$

$$c = 45^\circ 19' \quad \text{sin.} = 9.851872$$

$$\text{Radius} \quad \quad \quad = 10.$$

$$\text{Sin. } b = \frac{9.939484}{10} = 60^\circ 27' 2''.$$

§ *d*. 3d. To find the other angle (*A*); it is a disjunct extreme; and (§ 173. § *b*.) $\cos. c : \text{rad.} :: \sin. C^\circ : \cos. A^\circ$; or,

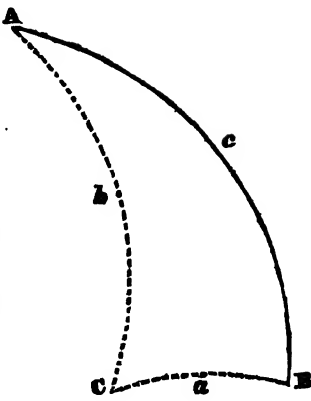
$$\text{Cos. } c : \text{rad.} :: \cos. C : \sin. A.$$

$$c = 45^\circ 19' \quad \text{cos. ar. co.} = 0.152929 = \text{sec.} (\S 101. \S b.)$$

$$\text{Radius} \quad \quad \quad = 10.$$

$$C = 54^\circ 49' \quad \text{cos.} = 9.760569$$

$$\text{Sin. } A = \frac{9.913498}{10} = 55^\circ 1' 30''.$$



CASE VI.

§ 175. Given a leg (c) and its adjacent angle (A),

$$c = 35^\circ 15' 7''$$

$$A = 74^\circ 47'$$

To find the values of the other parts (b, C, a).

§ *a.* 1st. To find the value of the hypotenuse (b); it is (§ 141. § *b.* & § *d.*) less than a quadrant; and the problem is the converse of § *g.*, Case I, (§ 170.). Tang. $c : \text{rad.} :: \sin. A^\circ : \text{tang. } b^\circ$; or,

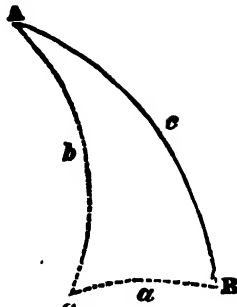
$$\text{Tang. } c : \text{rad.} :: \text{co-tang. } b.$$

$$c = 35^\circ 15' 7'' \text{ tang. ar. co.} = 0.150715 = \text{cot.} \quad (\S 101. \S b.)$$

$$\text{Radius} = 10.$$

$$A = 74^\circ 47' \cos. = 9.419080$$

$$\text{Co-tang. } b = 9.569795 = 69^\circ 37' 37''.$$



§ *b.* 2d. To find the value of the angle C . This is the converse of § *b.*, Case IV. (§ 173.), $\text{Radius} : \cos. A^\circ :: \cos. c : \sin. C^\circ$; or,

$$\text{Rad.} : \sin. A :: \cos. c : \cos. C.$$

$$\text{Radius} = 10.$$

$$A = 74^\circ 47' \sin. = 9.984500$$

$$c = 35^\circ 15' 7'' \cos. = 9.912022$$

$$\text{Cos. } C = 9.896522 = 38^\circ 00' 7''.$$

§ *c.* 3d. To find the other leg a . The calculation for this is the converse of § *c.*, Case III., (§ 172.)

$$\text{Tang. } A^\circ : \text{rad.} :: \sin. c : \text{tang. } a; \text{ and } \text{tang. } A^\circ =$$

$$\text{Co-tang. } A = 74^\circ 47' \text{ ar. co.} = 0.565421 = \text{tang.} \quad (\S 101. \S b.)$$

$$\text{Radius} = 10.$$

$$c = 35^\circ 15' 7'' \sin. = 9.761305$$

$$\text{Tang. } a = 10.326726 = 64^\circ 46' \quad (\S 141. \S d.)$$

§ 176. The rules which have been given for the solution of any problem that can occur in right angled spherical trigonometry, are good in theory; but instances may occur in practice, wherein they will not yield the most accurate result. This apparent defect arises from the trigonometric functions by which certain arcs, or angles, that enter into calculation, are denominated. For instance: If the required part be a very small arc or angle, and the result give its log.

co-sine, the solution is not a good one; because a small error in the log. cos. will produce a much larger one in the arc or angle. This arises from the different manner in which an arc and its sine increase.

§ a. By referring to a logarithmic table, even of six digits in the mantissæ, it will be observed that an arc of $89^{\circ} 51'$ may increase $18'$, or to $90^{\circ} 9'$, and that the value of its sine will only vary .000001.

§ b. The same thing is true of the co-sine of the complement of such an arc.

§ c. For these reasons, *the sine of an arc near 90° , or the cos. of one near 1', should not be used in trigonometrical calculation.*

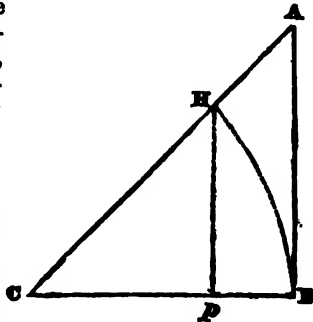
§ 177. The value of sine and co-sine of such arcs may be substituted by their equivalents in other trigonometric functions.

§ 178. The principles from which such equivalents are obtained, are involved in § 75.

§ a. The elements of trigonometry are chiefly contained in the principles of § 43. and § 75.

§ 179. An equivalent for the sine, co-sine, etc., of any angle, may be found by means of the principles developed under § 75.

§ a. Let the proposed angle (C) be made an oblique angle in a right angled triangle (A B C); and from it, with its adjacent side (C B) as radius, describe the arc H B, and draw the sine (H p) of the angle (C) proposed; C p is its co-sine, A B its tangent, and C A its secant.



§ b. Now, instead of naming these sides by letters, denominate them by the trigonometric functions whose offices they fill; thus, calling H p, sin. C; C p, cos. C; A B, tang. C; and so on; we have (§ 75. and § 73.),

$$\text{Sin. C} : \text{cos. C} :: \text{tang. C} : \text{radius}; \text{ or, } \text{sin. C} = \frac{\text{Cos. C} \times \text{tang. C}}{\text{radius.}}$$

$$\text{§ c. } \text{Cos. C} : \text{sin. C} :: \text{rad.} : \text{tang. C}; \text{ or, } \text{cos. C} = \frac{\text{Sin. C} \times \text{rad.}}{\text{tang. C}}$$

§ d. Equivalents for other trigonometric functions may be found after a similar manner; and the same relations exist between the trigonometric functions of plane and spherical triangles. Therefore—

§ 180. The logarithmic sine of any angle is equal to its log. co-sine and tangent, minus radius, or 10. And—

§ 181. The log. cos. of any angle is equal to its log. sine and radius, minus the log. tang. of the same angle.

OBLIQUE SPHERICS.

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OBLIQUE SPHERICS.

§ 182. In every oblique spherical triangle, there are *six parts*, any three of which constitute sufficient data (§ 145.) for determining the value of any of the remaining parts by means of trigonometrical calculation.

§ 183. Every problem in oblique spherical trigonometry is comprised in one of the six following cases; in which the data are,

1st, Two sides and angle opposite to one of them.

2d, Two angles and the side subtending either.

3d, Two angles and the side between them.

4th, Two sides and the angle they include.

5th, The three sides.

6th, The three angles.

§ 184. When any one of the four first cases, comprises the data of the problem, it may be solved by right angled trigonometrical operations; for this purpose the proposed triangle (§ 147. § a.) must be divided into two right angled triangles.

§ a. The perpendicular arc must be so drawn, that *two* of the given parts must be on the *same side* of it, and consequently, in one of the right angled triangles, into which this arc divides the triangle of the problem.

§ 185. The rules in right angled trigonometry, for determining *in species* an unknown side or angle, (i. e. whether it be greater or less than 90°), do not hold good in all cases in oblique trigonometry.

§ 186. The opposite sides and angles of oblique spherical triangles, are not necessarily of the same affection.

§ a. But if the *greatest side* be greater than a quadrant, it and its opposite angle are of the same affection.

§ 187. The converse of § 186. § a. does not hold good; for the sides (§ 136. § a.) of a spherical triangle may be indefinitely small, and the sum of the angles (§ 137.) being greater than two right angles, an obtuse angled spherical triangle may be formed of these indefinitely small sides; in which case an obtuse angle and its subtending side will be unlike.

§ 188. In most cases though, there are circumstances connected either with the condition of the problem, or of the triangle it involves, by which the species of the part required may be determined. Some of these circumstances are mentioned below.

§ 189. If an angle of a triangle be greater than 90° , an arc drawn from it, perpendicular to the subtending side, will fall *within* such triangle.

§ a. If the angles adjacent to the base, be of the same affection, the perpendicular falls upon it, also within the triangle.

§ 190. If all the angles of a triangle be not of the same affection, and a perpendicular arc be drawn from one of the angles which is less than 90° upon the subtending side, it will fall *without* the triangle.

§ 191. An opposite side and angle of a spherical triangle cannot be unlike, unless the sine of this side be greater than the sines of the other sides.

§ *a*. The sine of the angle opposite to such a side is also greater than the sines of the two other angles.

§ *b*. In an isosceles triangle, each side and its opposite angle are alike.

§ 192. If an obtuse side be opposite to an acute angle in any triangle, the two other sides are unlike; and (§ 191.) they and their opposite angles are of the same affection; consequently, the two angles are unlike.

§ *a*. If an obtuse angle subtend an acute side, the other sides and angles are all alike.

§ 193. Whether the perpendicular fall within or without the triangle, the less segment of the base is next to that angle which, of the two adjacent to the base, is the nearer 90° .

§ *a*. When the perpendicular falls within the triangle, the sum of the segments equals the base.

§ *b*. And when it falls without the triangle, the base equals the difference of the segments.

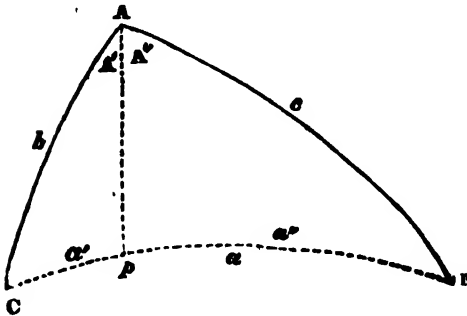
§ 194. If a side and its subtending angle be each less than 90° , a smaller side and its opposite angle, of the same triangle, must be (§ 139.) of the same affection.

§ 195. If the sum of two angles of a triangle be less than 90° , the third angle (§ 137.) is greater than a right angle.

§ 196. By writing co-tang. for the arithmetical complement (§ 101. § *b*.) of the tangent; sec. for the arithmetical complement of cos.; cosec. for the arithmetical complement of sine, etc., greater simplicity and uniformity of calculation will be effected.

§ *a*. The learner will have observed by the reference (§ 101. § *b*.) appended to the arithmetical complement of sines, etc., in some of the calculations in right angled spherical trigonometry, that by taking from the tables the cosec. of an arc, he obtains at once the arithmetical complement of the logarithmic sine of the same arc. So, co-tang. for arithmetical complement of the tangent, etc.

CASE I.



§ 197. Given two sides (*b* & *c*) and an angle (*C*) opposite to one of them,

$$b = 21^\circ 17'$$

$$c = 74^\circ 14'$$

$$C = 69^\circ 10'$$

To find the other parts (*A*, *B* and *a*) by right angular operations.

§ *a*. If the third side (*a*) or its oppo-

site angle (A) be computed, the two other remaining parts are determinable by means of the ratio (§ 144.) between the sines of opposite sides and angles.

§ b. In order that two (*b* & *C*) of the given parts, may be on the same side (§ 184. § a.) of the perpendicular *A p*, it is drawn from *A*, upon its opposite side *a*, so as to divide the triangle of the problem into two right angled triangles *A p C* and *A p B*; the former of which contains two of the given parts (*C* & *b*).

§ c. *C* & *B* are less than 90° (§ 184.); therefore (§ 141. § b.) *C p* & *A p* are each less than 90°; the perpendicular upon the side (*a*) adjacent to *C* & *B* (§ 189. § a.) falls within the triangle; and makes the sum of the segments of the angle *A*, equal to *A*.

§ d. The distance (*C p*) of the given angle (*C*) from the intersecting point (*p*) of the perpendicular and the base, is called auxiliary *a'*.

§ e. To compute the value of the auxiliary arc *a'*.

The method of doing it, is shown under Case II. (§ 171. § c.).

$$\text{Tang. } a' = \frac{\cos. C}{\cot. b} = (\S 196.) \text{ tang. } b \cos. C.$$

$$\begin{aligned} \S f. \quad b &= 21^\circ 17'. & \text{Tang.} &= 9.590562 \\ C &= 69^\circ 10'. & \text{Cos.} &= 9.551024 \end{aligned}$$

$$\text{Tang. auxl. } a' = 9.141586 = 7^\circ 52' 16'' *$$

§ g. The value of *B* is calculated from the ratio (§ 144.) between the sines of sides and of angles. $\text{Sin. } c : \text{sin. } C :: \text{sin. } b : \text{sin. } B$; $\text{sin. } c \text{ ar. co. } (\S 196.) = \text{cosec. } c$.

$$\begin{aligned} \S h. \quad c &= 74^\circ 14' & \text{cosec.} &= 0.016655 \\ C &= 69^\circ 10' & \text{sin.} &= 9.970635 \\ b &= 21^\circ 17' & \text{sin.} &= 9.559883 \end{aligned}$$

$$\text{Sin. } B = 9.547173 = 20^\circ 38' 27''$$

§ i. Now, in the other right angled triangle *A p B*, the value both of *c* and of *B* is known, and the other segment (*p B*) of the base can be determined; for (§ e.) $\text{tang. } B p = \cos. B \text{ tang. } c$.

$$\begin{aligned} \S j. \quad c &= 74^\circ 14' & \text{tang.} &= 0.549223 \\ B &= 20^\circ 38' 27'' & \text{cos.} &= 9.971187 \end{aligned}$$

$$\text{Tang. } B p = 0.520410 = 73^\circ 12' 39''$$

* Rad. (§ 98. § b.), being always equal to 1 or to 10 in logarithmic calculations, is not written down in these formulæ; its value is brought into account by applying it to the log. index of the result, as in the case above. By omitting rad. the formulæ for calculation are rendered more compact and convenient. The proper value will always be given rad. in calculation, by having the log. index of the result to consist only of one figure, as in the case above, where radius is taken into the account by simply writing 9 instead of 19 for the log. index of auxl. *a'*.

§ k. The perpendicular falling within the triangle, makes the *sum* of the segments (§ 193. § a.) equal to the base a ; therefore (§ f. & § j.) $a = 81^\circ 5' 55''$.

§ l. To find the value of A.

$$\begin{array}{l} \text{Sin. } c : \text{sin. } C :: \text{sin. } a : \text{sin. } A. \\ c = 74^\circ 14' \quad \text{cosec.} = 0.016655 \\ C = 69^\circ 10' \quad \text{sin.} = 9.970635 \\ a = 81^\circ 5' 55'' \quad \text{sin.} = 9.994738 \end{array}$$

$$\text{Sin. } A = 9.982028 = 106^\circ 22' 12''; \text{ A is greater}$$

(§ 195.) than 90° .

§ m. The value of A may also be determined by the Catholic Proposition; for (§ 171. § a.) $\cotang. C A p = \frac{\cos. b}{\cot. C} = (\text{§ 196.}) \cos. b \text{ tang. } C$. Also $\text{co-tang. } B A p = \cos. c \text{ tang. } B$.

§ n. $B A p + C A p = A$, according as the perpendicular arc A p falls within or without the triangle.

$$\begin{array}{l} \text{§ o. } C = 69^\circ 10' \quad \text{tang.} = 0.419611 \\ b = 21^\circ 17' \quad \text{cos.} = 9.969321 \end{array}$$

$$\text{Co-tang. } C A p = 0.388932 = 22^\circ 12' 51''$$

$$\begin{array}{l} B = 20^\circ 38' 27'' \quad \text{tang.} = 9.575983 \\ c = 74^\circ 14' \quad \text{cos.} = 9.434122 \end{array}$$

$$\text{Co-tang. } B A p = 9.010105 = 84^\circ 9' 21''$$

$$B A p + C A p (\text{§ c.}) = A \quad 106^\circ 22' 12''$$

§ p. The process of solution by such methods of calculation as the above, is circuitous. But cases sometimes occur when they may be used with advantage. And in order that the process, by which the required result is obtained from calculations conducted upon the principles of right angled spherical trigonometry, may be made familiar to the learner, the calculations are carried out.

§ q. By analysis and the use of a little artifice, rules are deduced, and formula constructed, for obtaining the same result from the application of the same principles to calculation, but by less tedious operations. The *auxiliary arcs* and *angles* used in such methods, are derived from the principles of the Catholic Proposition; and the methods themselves are nothing more than right angled spherical calculations, rendered less circuitous in execution by previous combinations, eliminations, and substitutions of the parts that are contained in the two right angled triangles, into which the oblique one of the problem is divided.

§ r. To find the value of A by the help of auxiliaries. Let the angle B A p be called auxiliary A''; and let auxiliary A', be the angle (C A p) which is in the right angled triangle in which two (b, C,) of the given parts of the primitive triangle are contained.

§ s. According to § m. co-tang. $A' = \cos. b \text{ tang. } C$.

§ t. Now, (§ 165. & § XLV. Alg.); $\frac{\text{Tang. } A p}{\text{rad.}} = \frac{\text{Cos. } A'}{\text{cot. } b}$; also $\frac{\text{Tang. } A p}{\text{rad.}} = \frac{\text{Cos. } A''}{\text{cot. } c}$; wherefore $\frac{\text{Cos. } A'}{\text{cot. } b} = \frac{\text{Cos. } A''}{\text{cot. } c}$; and by transposition, $\text{Cos. } A'' = \frac{\text{Cos. } A' \text{ Cot. } c}{\text{cot. } b}$

& (§ 101. § b.) $\frac{\text{Cos. } A' \text{ Cot. } c}{\text{cot. } b} = \text{Cos. } A' \text{ Cot. } c \text{ tang. } b$. Therefore, $\text{Cos. auxiliary } A'' = \cos. A' \text{ cot. } c \text{ tang. } b$. Whence the general rule for finding the value of (A) the angle opposite to the unknown side.

§ u. The product (§ s.) of tang. of the given angle, and cos. of the given side that is adjacent to it, is co-tang. of auxiliary A'. And,

§ v. The product of tang. of the same side, and co-tang. of the other given side, multiplied by cos. of auxl. A' (§ t.), is cos. of the other auxiliary A''. And the sum (§ 193. § a.), or difference (§ 193. § b.), of the two auxiliaries, gives the required angle.

§ w. $c = 74^\circ 14'$ cot. = 9.450777
 $b = 21^\circ 17'$ cos. = 9.969321 - - - tang. = 9.590562
 $C = 69^\circ 10'$ tang. = 0.419611

Cot. A' = 9.388932 = 22° 12' 51'' - cos. = 9.966506

84° 9' 21'' - Cos. A'' = 9.007845

A = 106° 22' 12'' (§ o.)

§ x. The formula for the calculation (§ w.) is arranged in the most convenient order for operation. The value of auxl. A' is evolved during the process of finding that of the other A''; and the log. tang. and cos. of b, are taken out at one opening of the tables; so, also is the value of A' and its cos. System and method in calculation should by no means be neglected. They promote accuracy and facilitate practice. It is therefore the business of every calculator to introduce system in his operations. The habit of arranging the several quantities in the most convenient order for calculation, contributes to accuracy, and makes verification more ready.

§ y. To find the value of a, by the help of auxiliaries. Call the segment C p, auxl. a', and the segment B p, auxl. a''.

§ z. From a train of reasoning analogous to that under § m., it is shown that, tang. auxl. a' = tang. b cos. C.

§ z a. Also (§ 166. & § XLV. Alg.) $\frac{\text{Rad.}}{\text{cos. } A p} = \frac{\text{Cos. } a'}{\text{cos. } b}$; and $\frac{\text{Rad.}}{\text{cos. } A p} = \frac{\text{Cos. } a''}{\text{cos. } c}$; therefore $\frac{\text{Cos. } a''}{\text{cos. } c} = \frac{\text{Cos. } a'}{\text{cos. } b}$; and by transpo-

sition $\cos. a' = \frac{\text{Cos. } a \cos. c}{\cos. b} = (\S 101. \S b.) \cos. a' \cos. c \sec. b$;
 then $a' \overset{+}{\infty} a'$ ($\S c. \& \S n.$) = a . Whence the general rule for finding
 the third side.

$\S z b.$ The product of $\cos.$ of the given angle and tang. of its adjacent side, is ($\S z.$) tang. of auxl. a' . And,

$\S z c.$ Sec. of the same side, multiplied by the *product* of $\cos.$ of the other side and $\cos.$ of auxl. a' ($\S z a.$), is $\cos.$ of the other auxl. a'' .

$\S z d.$ If the two angles adjacent to the base, be each less than 90° , the perpendicular falls within the triangle, and ($\S 193. \S a.$) the *sum* of the auxiliaries is the required part.

$\S z e.$ But if these two angles be unlike, the perpendicular falls without the triangle; and then ($\S 193. \S b.$) the *difference* of the auxiliaries is the required part.

$\S z f.$
 $b = 21^\circ 17'$ tang. = 0.590562 $c = 74^\circ 14'$ cos. = 0.434123
 $C = 69^\circ 10'$ cos. = 0.551024 - - sec. = 0.080079

Tang. $a' = 0.141586 = 7^\circ 53' 16''$ - cos. = 0.995872

73° 12' 39'' - Cos. $a'' = 0.499979$

$a = 81^\circ 5' 55''$ ($\S z d.$)

$\S z g.$ The value of the third side (a), or of its opposite angle (A), may also be determined by another method; but in this, the value of the other unknown angle (B) is necessary.

$\S z h.$ To determine the third side (a),

Is to the $\frac{1}{2}$ the difference of the two angles, (B, C);

As to the $\cos.$ of $\frac{1}{2}$ their sum;

As tang. of $\frac{1}{2}$ the sum of the two sides (b, c);

Is to tang. of $\frac{1}{2}$ the required side.

$\S z i.$ $\frac{1}{2} (C \infty B) = 24^\circ 15' 46''$ sec. = 0.040162

$\frac{1}{2} (C + B) = 44^\circ 54' 13''$ cos. = 0.850215

$\frac{1}{2} (b + c) = 47^\circ 45' 30''$ tang. = 0.041880

Tang. $\frac{1}{2} a = 0.932257 = 40^\circ 32' 58''$

2

$a = 81^\circ 5' 56''$

$\S z j.$ To find the value of A by a similar method.

Is to the $\cos.$ of $\frac{1}{2}$ the difference of the two sides (b, c);

As to the $\cos.$ of $\frac{1}{2}$ their sum;

As tang. of $\frac{1}{2}$ the sum of the two angles (A, B);

Is to co-tang. of $\frac{1}{2}$ the angle required.

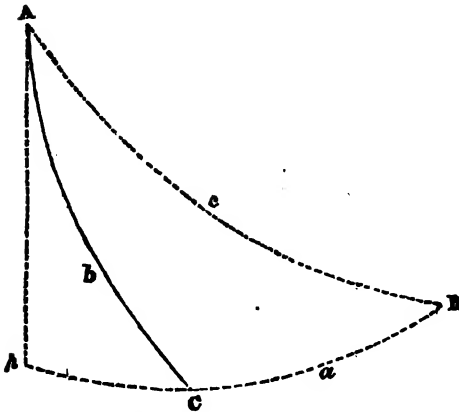
$$\begin{aligned} \S z k. \frac{1}{2}(c \pm b) &= 26^\circ 28' 30'' \text{ sec.} = 0.048114 \\ \frac{1}{2}(c + b) &= 47^\circ 45' 30'' \text{ cos.} = 0.67537 \\ \frac{1}{2}(C + B) &= 44^\circ 54' 13'' \text{ tang.} = 0.998538 \end{aligned}$$

$$\text{Co-tang. } \frac{1}{2} A = 9.874189 = 53^\circ 11' 7''$$

$$\begin{array}{r} 2 \\ \hline A = 106^\circ 22' 14'' \\ \hline \end{array}$$

These two last methods are the most common in practice.

CASE II.



§ 198. Given two angles and the side that subtends one of them,

$$\begin{aligned} B &= 29^\circ 16' \\ C &= 150^\circ 51' 50'' \\ b &= 74^\circ 46' \end{aligned}$$

To find the value of each of the other parts (A, c & a.)

§ a. The value of c (§ 144.) is determinable by a direct calculation; Sin. B : sin. b :: sin. C : sin. c.

$$\begin{aligned} B &= 29^\circ 16' & \text{co-sec.} &= 0.810802 \\ b &= 74^\circ 46' & \text{sin.} &= 0.984466 \\ C &= 150^\circ 51' 50'' & \text{sin.} &= 0.687427 \end{aligned}$$

$$\text{Sin. } c = 0.982695 = 106^\circ 4' 2''$$

§ b. Suppose the perpendicular (A p) from A, upon the side opposite to A, to fall without the triangle. Then the two known parts in the right angled triangle A p C, are b, and (§ 132.) the supplement of C. The unknown parts of the proposed oblique triangle may be computed by right angled trigonometric operations.

§ c. In the triangle A p C, (§ 197. § e.) tang. C p = cos. C tang. b. Also in the triangle p A B, tang. p B = cos. B tang. c. Then (§ 193. § b.) C p ∝ p B = a.

§ d. The value of A also, may be determined by rules of right angled trigonometry.

§ e. When the perpendicular falls without the triangle, the difference of the two auxiliary angles B A p, and C A p, is the angle A.

§ f. But if it falls within the triangle, $B A p + C A p = A$, (§ 197. § n.)

§ g. Co-tang. $p A C = \cos. b \text{ tang. } C$; and co-tang. $B A p = \cos. c \text{ tang. } B$ (§ 197. § m.). Then (§ e.) $A = B A p + C A p$.

§ h. $C = 150^\circ 51' 50''$ $\cos. = 9.941246$ (§ c.)

$b = 74^\circ 46'$ $\text{tang.} = 0.564922$

$$\text{Tang. } C p = \frac{0.506168}{0.564922} = 72^\circ 41' 3''$$

$B = 29^\circ 16'$ $\cos. = 9.940693$

$c = 106^\circ 4' 2''$ $\text{tang.} = 0.540585$

$$\text{Tang. } p B = \frac{0.481278}{0.540585} = 106^\circ 16' 15''$$

$$\text{Now } (\S c.) p B - C p = a = 35^\circ 35' 12''$$

§ i. $C = 150^\circ 51' 50''$ $\text{tang.} = 9.746181$ (§ g.)

$b = 74^\circ 46'$ $\cos. = 9.419544$

$$\text{Co-tang. } p A C = \frac{9.165725}{9.419544} = 81^\circ 40' 4''$$

$c = 106^\circ 4' 2''$ $\cos. = 9.442111$

$B = 29^\circ 16'$ $\text{tang.} = 9.748505$

$$\text{Co-tang. } B A p = \frac{9.190616}{9.748505} = 98^\circ 49' 0''$$

$$\text{Now } (\S e.) B A p - p A C = A = 17^\circ 8' 56''$$

§ j. To determine the third angle (A) by the help of auxiliaries. The angle $p A C$ is auxiliary A' ; and the angle $B A p$ is auxiliary A'' .

§ k. Co-tang. $A' = \cos. b \text{ tang. } C$ (§ g.).

§ l. And (§ 166. and § XLV. Alg.) $\frac{\text{Rad.}}{\cos. A p} = \frac{\text{Sin. } A'}{\cos. C}$; also

$\frac{\text{Rad.}}{\cos. A p} = \frac{\text{Sin. } A''}{\cos. B}$; therefore $\frac{\text{Sin. } A''}{\cos. B} = \frac{\text{Sin. } A'}{\cos. C}$; and by transposition, $\text{sin. } A'' = \frac{\text{Sin. } A' \cos. B}{\cos. C} = (\S 101. \S b.) = \text{sin. } A' \cos. B$

sec. C . Whence the general rule for finding the third angle.

§ m. The product of $\cos.$ of the given side, and tang. of its adjacent angle, (§ k.) is co-tang. of auxl. A' . And,

§ n. The product of sec. of the same angle, and sin. of auxl. A' , multiplied by $\cos.$ of the other angle, (§ b.) is sin. of auxl. A'' . And (§ 197. § n.) $A' + A''$ is the required angle.

$$\begin{array}{l} \S o. \\ C=150^{\circ} 51' 50'' \text{ tang.} = 9.746181 \\ b=74^{\circ} 40' \text{ cos.} = 9.419544 \end{array} \qquad \begin{array}{l} B=29^{\circ} 16' \text{ cos.} = 9.940693 \\ \text{sec.} = 0.058754 \end{array}$$

$$\text{Cot. } A' = 9.165725 = 81^{\circ} 40' 4'' \qquad \text{sin.} = 9.995391$$

$$98^{\circ} 49' \qquad \text{Sin. } A'' = 9.994838$$

$$A = 17^{\circ} 8' 56'' (\S e.)$$

$\S p.$ To determine the value of the side (a) opposite to the unknown angle with the help of auxiliaries. Let the segment $p C$ be auxiliary a' ; and the other segment $p B$, be auxiliary a'' ,

$\S q.$ Tang. a' ($\S c.$) = cos. C tang. b .

$\S r.$ And by a process of reasoning similar to that under Case I, ($\S 197. \S z a.$), it is shown that $\text{sin. } a'' = \text{sin. } a' \text{ tang. } C \text{ co-tang. } B$;

for ($\S 165.$) $\frac{\text{Tang. } A p}{\text{rad.}} = \frac{\text{Sin. } a'}{\text{cot. } C}$; also $\frac{\text{Tang. } A p}{\text{rad.}} = \frac{\text{Sin. } A''}{\text{cot. } B}$;

therefore, $\frac{\text{Sin. } a''}{\text{cot. } B} = \frac{\text{Sin. } a'}{\text{cot. } C}$; and by transposition, $\text{sin. } a'' =$

$\frac{\text{Sin. } A' \text{ cot. } B}{\text{cot. } C} = \text{sin. } a' \text{ cot. } B \text{ tang. } C.$ Whence the general rule

for finding the value of the side that subtends the unknown angle.

$\S s.$ The product of tang. the given side, and cos. its adjacent angle ($\S q.$) is tang. of auxl. a' . And,

$\S t.$ Tang. the same angle multiplied by the product of co-tang. the other angle, and sin. the auxl. a' , is sin. the other auxiliary a'' .

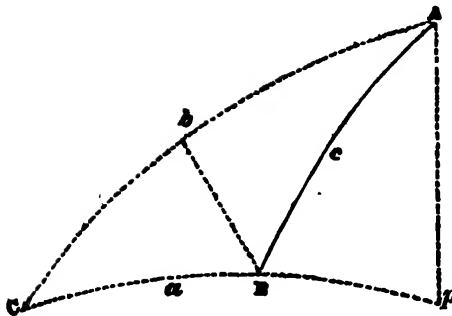
$$\begin{array}{l} \S u. \\ C=150^{\circ} 51' 50'' \text{ cos.} = 9.941246 \\ b=74^{\circ} 46' \text{ tan.} = 0.564922 \end{array} \qquad \begin{array}{l} B=29^{\circ} 16' \text{ cot.} = 0.251495 \\ \text{tang.} = 9.746181 \end{array}$$

$$\text{Tang. } a' = 0.506168 = 72^{\circ} 41' 3'' \qquad \text{sin.} = 9.979857$$

$$108^{\circ} 16' 15'' \text{ Sin. } a'' = 9.977533$$

$$a = 35^{\circ} 35' 12''$$

CASE III.



§ 199. Given two angles, and the side between them,

$$A=19^{\circ} 41'$$

$$B=134^{\circ} 17'$$

$$c=36^{\circ} 19'$$

To find the value of the other parts (a , b , & C .)

§ a . The perpendicular (Ap) being so drawn that the two given parts B , c , may

be on the same side of it, falls without the triangle proposed; and (§ 198. § e .) the auxiliary angle $B A p + A$, equals the auxiliary angle $C A p$. Also the difference of the auxiliary arcs $p B$ & $p C$ (§ 193. § b .) is the side a .

§ b . If any one of the unknown parts be determined, the two others may be found from the proportion between the sines of opposite sides and angles.

§ c . To determine the value of C by the rules of right angled trigonometry.

§ d . In the right angled triangle $B p A$, (§ 165. & § 197. § m .), co-tang. $B A p = \cos. c \text{ tang. } B$. Also, (§ 166.) $\sin. A p = \sin. B \sin. c$. And $B A p + A = C A p$.

§ e . Then in the right angled triangle $C p A$, the parts $C A p$ & $A p$ are known, and (§ 166.) $\cos. C = \sin. C A p \cos. A p$.

$$\text{§ } f. c=36^{\circ} 19' \cos. = 9.906204$$

$$B=134^{\circ} 17' \text{ tang.} = 0.010866$$

$$\text{Co-tang. } B A p = 9.917079 = 50^{\circ} 26' 15''$$

$$c \sin. = 9.772503$$

$$B \sin. = 9.854850$$

$$\text{Sin. } A p = 9.627353 = 25^{\circ} 5' 12''$$

$$B A p + A = C A p = 70^{\circ} 7' 15'' \text{ (§ } a.)$$

$$\text{§ } g. C A p = 70^{\circ} 7' 15'' \sin. = 9.973318$$

$$A p = 25^{\circ} 5' 12'' \cos. = 9.956969$$

$$\text{Cos. } C = 9.930267 = 31^{\circ} 36' 10''$$

§ h . To find the value of the third angle (C), with the help of auxiliaries. The angle $B A p$ is auxiliary A' . Co-tang. A' (§ d .) = $\cos. c \text{ tang. } B$.

§ i. And (§ 198. § l.) $\frac{\text{Cos. } A p}{\text{rad.}} = \frac{\text{Cos. } B}{\text{Sin. } A'}$; also $\frac{\text{Cos. } A p}{\text{rad.}} = \frac{\text{Cos. } C}{\text{sin. } (A + A')}$; therefore $\frac{\text{Cos. } C}{\text{sin. } (A + A')} = \frac{\text{Cos. } B}{\text{sin. } A'}$; and by transposition, $\text{Cos. } C = \frac{\text{Cos. } B \text{ sin. } (A + A')}{\text{sin. } A'} = \text{cos. } B \text{ sin. } (A + A') \text{ co-sec. } A'$.

§ j. $\text{Cos. } C = \text{cos. } B \text{ co-sec. } A' \text{ sin. } (A + A')$ when $A p$ falls without the triangle. And,

§ k. $\text{Cos. } C = \text{cos. } B \text{ co-sec. } A' \text{ sin. } (A - A')$ when $A p$ falls within it. Whence the rule.

§ l. The product of cos. the given side, and tang. of either angle (§ h.), is co-tang. of auxl. A' . And, (§ i.),

§ m. Cos. of the same angle, multiplied by the product of co-sec. of auxl. A' and sine of the sum (§ j.), or difference (§ k.) of auxl. A' and the other angle, is cos. of the required angle.

§ n.

$$c = 36^\circ 19' \text{ cos.} = 9.906204$$

$$B = 184^\circ 17' \text{ tang.} = 0.010866$$

$$\text{cos.} = 9.843984$$

$$\text{Cot. } A' = 9.917070 = 50^\circ 26' 15'' \quad \text{cosec.} = 0.112985$$

$$(A + A') = 70^\circ 7' 15'' \text{ sin.} = 9.978810$$

$$31^\circ 36' 10'' \text{ Cos. } C = 9.930287$$

To find the value of the two sides (a and b) with the help of auxiliaries.

§ o. According to a train of reasoning similar to that shown under Case I, (§ 197. § t.) $\text{cot. } b = \text{cot. } c \text{ cos. } (A + A') \text{ sec. } A'$; for

$$\frac{\text{Rad.}}{\text{Cot. } b} = \frac{\text{Cot. } c}{\text{Cos. } A'}; \text{ also } \frac{\text{Rad.}}{\text{Cot. } b} = \frac{\text{Cot. } c \text{ cos. } (A + A')}{\text{Cos. } A'}$$

$$\text{then } \frac{\text{Cot. } c \text{ cos. } (A + A')}{\text{Cos. } A'} = \text{cot. } b; \text{ and by transposition, } \text{cot. } b =$$

$$\frac{\text{Cot. } c \text{ cos. } (A + A')}{\text{cos. } A'} = \text{cot. } c \text{ cos. } (A + A') \text{ sec. } A'.$$

§ p. By drawing the perpendicular from B upon its opposite side, and calling the angle $A B b$ auxiliary B' , we obtain by a similar process of reasoning. $\text{Cot. } a = \text{cot. } c \text{ cos. } (B + B') \text{ sec. } B'$.

§ q. $\text{Cot. } B' = \text{cos. } c \text{ tang. } A$. Whence the rule for finding the value of either unknown side.

§ r. The product of cos. the given side and tang. of the angle (§ h. & § q.) opposite to the required side, is co-tang. of the required auxiliary which call A' . And (§ o. & § p.),

§ s. The product of cot. the given side and sec. of this auxl. A' ,

multiplied by cos. of the sum, or difference of A' and the angle adjacent to the required side, is co-tang. of the required side.

§ *t.* To find the value of *b* by this rule.

$$B = 134^\circ 17' \text{ tang.} = 0.010866$$

$$c = 36^\circ 19' \text{ cos.} = 9.906204 \quad - \quad - \quad - \quad \text{cot.} = 0.133700$$

$$\text{Cot. } A' = 9.917070 = 50^\circ 26' 15'' \quad - \quad \text{sec.} = 0.195916$$

$$(A + A') = 70^\circ 7' 15'' \text{ cos.} = 9.531528$$

$$54^\circ 0' 27'' \text{ Cot. } b = 9.861144$$

§ *u.* To find the value of *a* by the same rule.

$$A = 19^\circ 41' \text{ tang.} = 9.553548$$

$$c = 36^\circ 19' \text{ cos.} = 9.906204 \quad - \quad - \quad - \quad \text{cot.} = 0.133700$$

$$\text{Cot. } A' = 9.459752 = 73^\circ 55' 15'' \quad - \quad \text{sec.} = 0.557576$$

$$(B - A') = 60^\circ 21' 45'' \text{ cos.} = 9.694175$$

$$22^\circ 22' 32'' \text{ Cot. } a = 0.385451$$

§ *v.* A triangle that has a side or an angle greater than 180° must never be used in the solution of trigonometrical problems.

§ *w.* Therefore in the solution of all cases (§ *u.*) where the sum, or difference of an auxiliary, and an arc or angle, is to be brought into calculation, if the *sum* would exceed 180° , the *difference* is the proper quantity to be used.

§ *x.* The two unknown sides may also be determined by a method of calculation differing from that above, but depending on principles analogous to those under § 77. for plane triangles. This method is the most common in practice.

§ *y.* The sine of $\frac{1}{2}$ the *sum* of the two given angles (A, B);
Is to the sine of $\frac{1}{2}$ their *difference*;
As the tang. of $\frac{1}{2}$ the given side (*c*);
Is to the tang. of $\frac{1}{2}$ the *difference* of the two required sides (*a, b*). And,

§ *z.* Cos. of the *same* $\frac{1}{2}$ *sum*;
Is to the cos. of the *same* $\frac{1}{2}$ *difference*;
As the tang. of $\frac{1}{2}$ the given side;
Is to the tang. of $\frac{1}{2}$ the *sum* of the two required sides (*a, b*).

$$\begin{aligned} \S z a. \quad \frac{1}{2} (A+B) &= 76^\circ 59' \text{ co-sec.} = 0.011305 \\ \frac{1}{2} (A-B) &= 57^\circ 18' \text{ sin.} = 9.925060 \\ \frac{1}{2} c &= 18^\circ 9' 30'' \text{ tang.} = 9.515844 \end{aligned}$$

$$\text{Tang. } \frac{1}{2} (a \times b) = 9.452209 = 15^\circ 48' 58''$$

$$\begin{aligned} \frac{1}{2} (A+B) \text{ sec.} &= 0.647365 \\ \frac{1}{2} (A-B) \text{ cos.} &= 9.732587 \\ \frac{1}{2} c \text{ - tang.} &= 9.515844 \end{aligned}$$

$$\text{Tang. } \frac{1}{2} (a+b) = 9.895796 = 38^\circ 11' 29''$$

§ z b. The greater side (*b*) is opposite (§ 139.) to the greater angle; and (§ 77. § g.) $38^\circ 11' 29'' + 15^\circ 48' 58'' = 54^\circ 00' 27''$ or *b*; and $a = (38^\circ 11' 29'' - 15^\circ 48' 58'') = 22^\circ 22' 31''$.

§ z c. To determine the value of the third angle (*C*) with the help of an auxiliary arc *a'*.

§ z d. The sine of $\frac{1}{2}$ the sum } of the two given angles
Is to $\frac{1}{2}$ the sum of the sines } (*A* & *B*);
As the sine of $\frac{1}{2}$ the given side (*c*);
Is to the sine of auxl. *a'*. Then,

§ z e. The product of cos. of auxl. *a'* and sine of $\frac{1}{2}$ the sum of the two given angles, is cos. of $\frac{1}{2}$ the third angle (*C*).

$$\begin{aligned} \S z f. \quad B &= 134^\circ 17' \text{ sin.} = 9.854850 \\ A &= 19^\circ 41' \text{ sin.} = 9.527400 \end{aligned}$$

$$2) 19.382250$$

$$9.691125 = \frac{1}{2} \text{ sum of the sines of the two given angles.}$$

$$\begin{aligned} \frac{1}{2} c = 18^\circ 9' 30'' \text{ sin.} &= 9.493659 \\ \frac{1}{2} (A+B) = 76^\circ 59' \text{ co-sec.} &= 0.011305 \quad - \quad \text{sin.} = 9.988695 \end{aligned}$$

$$\text{Sin. auxl. } a' = 9.196089 = 9^\circ 2' 12'' \text{ cos.} = 9.994576$$

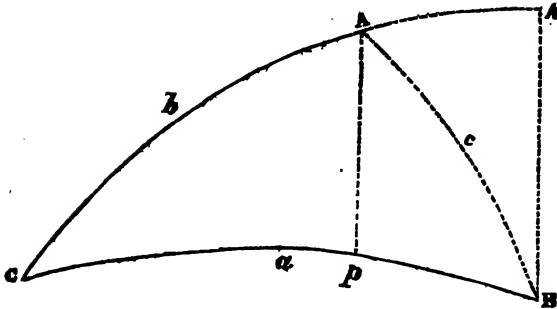
$$15^\circ 48' 5'' \text{ Cos. } \frac{1}{2} C = 9.983271$$

2

$$C = 31^\circ 36' 10''$$

CASE IV.

§ 200. Given two sides and the angle they include,



$$\begin{aligned} a &= 109^\circ 30' \\ b &= 60^\circ 00' \\ C &= 27^\circ 08' \end{aligned}$$

To find the other parts (A, B, c). This is the converse of Case III., for we have to find here what is there given.

§ a. If the perpendicular Ap fall within the triangle, the sum of the segments (Cp + pB) is equal to a; and the sum of the auxiliary angles, CAp + pAB = A.

§ b. Any one of the unknown parts of the triangle proposed, being found, the two others are determinable from the relations (§ 144.) of sides to their opposite angles.

§ c. To find B, by the Catholic Proposition. By Case II.,

$$(\S 171. \S c.) \text{ tang. } Cp = \frac{\text{Cos. } C}{\text{Cot. } b} = (\S 101. \S b.) \text{ cos. } C \text{ tang. } b; \text{ also}$$

(§ 171. § d.) $\sin. Ap = \sin. b \sin. C$. Then (§ a.) $a - Cp = pB$; and the value of Ap, and of pB, in the triangle ApB, being known, that of B (§ 155.) is determinable; $\cotang. B = (\S 172. \S b.) = \frac{\text{Sin. } pB}{\text{Tang. } Ap} = \sin. pB \cotang. Ap$.

$$\begin{aligned} \S d. \quad C &= 27^\circ 8' \quad \text{cos.} = 9.949494 \\ b &= 60^\circ 0' \quad \text{tang.} = 0.238561 \end{aligned}$$

$$\text{Tang. } Cp = \frac{0.238561}{9.949494} = 0.188055 = 57^\circ 2' 4''$$

$$\begin{aligned} C \sin. &= 9.658531 \\ b \sin. &= 9.937531 \end{aligned}$$

$$\text{Sin. } Ap = \frac{9.658531}{9.937531} = 9.596062 = 23^\circ 14' 8''$$

$$109^\circ 30' - 57^\circ 2' 4'' (\S a.) = pB = 52^\circ 27' 56''$$

§ e. p B = $52^{\circ} 27' 58''$ Sin. = 9.899267
 A p = $29^{\circ} 14' 8''$ Cotang. = 0.367204

Cotang. B = $10.266471 = 28^{\circ} 25' 54''$

§ f. To find the value of each of the unknown angles (A, B), with the help of an auxiliary arc (a'). A perpendicular drawn from either of the unknown angles (A, B) fulfils the conditions of § a. (§ 184.); suppose the perpendicular drawn from B to the fall without the triangle.

§ g. Let the distance (C p or C h) of the given angle from the perpendicular be auxiliary a' . Then whether the perpendicular fall within or without the triangle, the *difference* (§ d.) between the side upon which it falls and the auxl. a' is the other segment of that side.

§ h. First, to find the value of B, the arc C p being auxl. a' ; tang. a' (§ d.) = $\cos C \text{ tang. } b$.

§ i. According to Case II. (§ 198. § r.) $\frac{\text{Rad.}}{\text{tang. A } p} = \frac{\text{Cot. C}}{\text{Sin. } a'}$
 $\frac{\text{Cot. B}}{\text{Sin. } (a \oslash a')}$; and by transposition $\text{Cot. B} = \frac{\text{Cot. C Sin. } (a \oslash a')}{\text{Sin. } a'}$
 = $\text{cot. C sin. } (a \oslash a') \text{ cosec. } a'$.

§ j. To find the value of A. In the triangle C h B, C h is auxl. a' ; and $\text{tang. } a' = \cos. C \text{ tang. } a$.

§ k. Also, $\frac{\text{Rad.}}{\text{Tang. B } h} = \frac{\text{Cot. C}}{\text{Sin. } a'} = \frac{\text{Cot. A}}{\text{Sin. } (b \oslash a')}$; and by transposition, $\text{Cot. A} = \frac{\text{Cot. C Sin. } (b \oslash a')}{\text{Sin. } a'} = \text{cot. C sin. } (b \oslash a') \text{ cosec. } a'$.

Whence the general rule for finding either angle.

§ l. The product of $\cos.$ of the given angle (§ h. and § j.) and tang. of the side *opposite* to the required angle, is tang. of auxl. a' . And,

§ m. The *sin.* of the *difference* between auxl. a' and the side *adjacent* to the required angle, multiplied by the *product* of the *cosec.* of auxl. a' and *cotang.* of the given angle (§ i. and § h.), is *cotang.* of the required angle.

§ n. $b = 60^{\circ} 0'$ tang. = 0.238561
 $C = 27^{\circ} 6'$ cos. = 9.949494 - - cot. = 0.290963

Tang. auxl. $a' = 0.188055 = 57^{\circ} 2' 4''$ cosec. = 0.076240

$(a' \oslash a) = 52^{\circ} 27' 56''$ sin. = 9.899268

$28^{\circ} 25' 54''$ Cot. B = 0.266471

§ o. Or, to find the value of A ;
 $a = 109^\circ 30'$ tang. = 0.450851
 $C = 27^\circ 6'$ cos. = 9.949494 - - - cot. = 0.290963

Tang. $a' = 0.400345 = 111^\circ 41' 32''$ cosec. = 0.031898

 $(a' \omega b) = 51^\circ 41' 32''$ sin. = 9.894699

 $148^\circ 47' 10''$ Cot. A = 0.217560

§ p. The value of the third side (c) can be found with the help of the same auxiliary a' .

§ q. When the perpendicular falls within the triangle, $\frac{\text{Cos. } A p}{\text{Rad.}}$
 (§ 198. § l.) = $\frac{\text{Cos. } b}{\text{Cos. } a'} = \frac{\text{Cos. } c}{\text{Cos. } (a \omega a')}$; by transposition $\text{Cos. } c =$
 $\frac{\text{Cos. } b \text{ Cos. } (a \omega a')}{\text{Cos. } a'} = \text{cos. } b \text{ cos. } (a \omega a') \text{ sec. } a'$.

§ r. And when the perpendicular (Bh) falls without the triangle,
 $\frac{\text{Cos. } B h \text{ Cos. } a}{\text{Rad.}} = \frac{\text{Cos. } c}{\text{Cos. } (b \omega a')}$; by transposition, $\text{cos. } c =$
 $\frac{\text{Cos. } a \text{ Cos. } (b \omega a')}{\text{Cos. } a'} = \text{cos. } a \text{ cos. } (b \omega a') \text{ sec. } a'$.

N. B. $(a \omega a') = p B$, and $(b \omega a') = A h$.
 Hence the general rule for finding the third side.
 § s. The product of cos. of the given angle, and tangent of either given side, (§ h. & § j.), is tang. of auxl. a' .
 § t. Then (§ q. & § r.) cos. of the same given side, multiplied by the product of sec. of auxl. a' , and cos. of the difference between auxl. a' , and the other given side, is cos. of the required side.

§ u. $C = 27^\circ 6'$ cos. = 9.949494
 $b = 60^\circ 0'$ tang. = 0.238561 - - - cos. = 9.898970

Tang. $a' = 0.188055 = 57^\circ 2' 4''$ sec. = 0.264293

 $(a \omega a') = 52^\circ 27' 56''$ cos. = 9.784786

 $55^\circ 57' 24''$ Cos. c = 9.748049

§ v. Or, taking the tang. and cos. of a ;
 $C = 27^\circ 6'$ cos. = 9.949494
 $a = 102^\circ 30'$ tang. = 0.450851 - - - cos. = 9.523495

Tang. $a' = 0.400345 = 111^\circ 41' 32''$ sec. = 0.432244

 $(b \omega a') = 51^\circ 41' 32''$ cos. = 9.792312

 $55^\circ 57' 24''$ Cos. c = 9.748051

§ w. The difference between the two log. co-sines of c arises from fractions of a second (") , which are in some of the parts operated upon. These fractions may sometimes cause an error of a few seconds (") in the value of the part required.

§ x. There is a method for finding the value of the two angles (A & B) analogous to that under § y. and § z. (§ 199.), for finding two sides. In drawing up formulæ, and selecting methods, for trigonometrical calculations, attention should be paid to what is given, as well as to what is required, in the problem; and that method of solution should be adopted, which equally as correct as others, leads most directly to the result required.

§ y. To find A and B without the help of auxiliaries.

§ z. The cos. of $\frac{1}{2}$ the *sum*, } of the two sides;
Is to cos. of $\frac{1}{2}$ the *difference*, }
As co-tang. of $\frac{1}{2}$ the given angle;
Is to tang. of $\frac{1}{2}$ the *sum* of the required angles.

§ z a. The sine of $\frac{1}{2}$ the *sum*, } of the two sides;
Is to the sine of $\frac{1}{2}$ the *difference*, }
As co-tang. of $\frac{1}{2}$ the given angle;
Is to tang. of $\frac{1}{2}$ the difference of the required angles.

§ z b. $\frac{1}{2}(a+b) = 84^\circ 45' \text{ sec.} = 1.038571$
 $\frac{1}{2}(a-b) = 24^\circ 45' \text{ cos.} = 9.958154$
 $\frac{1}{2} C = 13^\circ 33' \text{ cot.} = 0.617980$

Tang. $\frac{1}{2}$ sum (A & B) = 1.614705 = $88^\circ 36' 32''$

$\frac{1}{2}(a+b)$, co-sec. = 0.001826
 $\frac{1}{2}(a-b)$, sin. = 9.621861
 $\frac{1}{2} C$ cot. = 0.617980

Tang. $\frac{1}{2}$ diff. (A & B) = 0.241667 = $60^\circ 10' 38''$

§ x c. $88^\circ 36' 32'' + 60^\circ 10' 38'' = A = 148^\circ 47' 10''$ } ($\S 77. \S h.$)
And, $88^\circ 36' 32'' - 60^\circ 10' 38'' = B = 28^\circ 25' 54''$ }

§ z d. The following method of finding the value of the third side (c) with the help of an auxl. a' , is useful and of frequent occurrence in nautical calculations.

§ z e. Half the *product* of the sines of the given sides, and *twice* the sine of half the given angle, *multiplied* by the co-sec. of $\frac{1}{2}$ the *difference* between the two sides, is the tang. of auxl. a' .

§ z f. Then the product of co-sec. of a' and said *half product* of the sines of the *three* said quantities, is the sine of *half* the required side.

§ z g.

$$a = 109^\circ 30' \sin. = 9.974347$$

$$b = 60^\circ 00' \sin. = 9.937531$$

$$\frac{1}{2} C = 13^\circ 33' \sin. \times 2 = 18.739522$$

$$2 \cdot 38.651400$$

$$19.325700$$

$$19.325700$$

$$\frac{1}{2} (a \times b) = 24^\circ 45' \text{ co-sec.} = 0.378139$$

$$\text{Tang. auxl. } a' = 9.703839 = 26^\circ 49' 22'' \text{ cosec.} = 0.345600$$

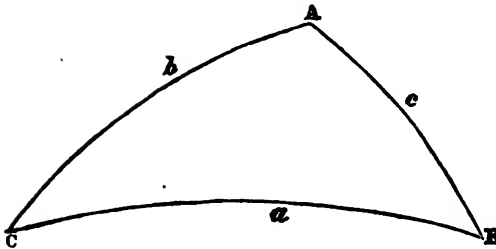
$$27^\circ 58' 42''$$

$$\text{Sin. } \frac{1}{2} c = 9.671300$$

2

$$c = 55^\circ 57' 24''$$

CASE V.



§ 201. Given the three sides

$$a = 66^\circ$$

$$b = 44^\circ$$

$$c = 30^\circ$$

To find the angles.

§ a. Neither in this, nor in the preceding case, can the value of any one of the unknown

parts be determined by the Catholic Proposition; for the proposed triangle cannot be so divided into two right angled triangles, that two of the given parts shall be contained in either of them.

§ b. This, and the case that immediately precedes it, are particularly useful to the navigator. Some of the most important problems, and those which are of most frequent occurrence in navigation, come under one or both of these cases for solution. The problems, for finding azimuths and the time of day at sea, fall under this case. This and the preceding case are both involved in the calculations for finding the *true*, from the *observed*, lunar distance. Both cases are also involved in finding the latitude by "double altitudes." And by Case IV, the lunar tables in the nautical almanac, are calculated.

§ c. In this, as under the other cases, there are several methods for finding the value of the required part. In all cases the methods, which are the best adapted to practical purposes, are given.

§ d. To find one of the angles (A). The *product* of the co-sec. of the sides that contain the required angle, *multiplied* by the *pro-*

duct of the sine of half the sum of the three sides, and sine of that half sum, less the side opposite the required angle, is double the cos. of $\frac{1}{2}$ the required angle.

§ e. Twice the tang. of $\frac{1}{2}$ the required angle, is the product of co-sec. of $\frac{1}{2}$ the sum of the three sides, and co-sec. of said $\frac{1}{2}$ sum, less the side opposite the required angle, multiplied by the product of the sines of the difference between said half sum and each of the sides that contain the required angle.

§ f. $a = 66^\circ$ (§ d.)
 $b = 44^\circ$ co-sec. = 0.158229
 $c = 30^\circ$ co-sec. = 0.301030

2)140°

$\frac{(a+b+c)}{2} = 70^\circ$ sin. = 9.972986

$\frac{(a+b+c-a)}{2} = 4^\circ$ sin. = 8.843585

Twice cos. $\frac{1}{2} A = 2)19.275830$

Cos. $\frac{1}{2} A = 9.637915 = 64^\circ 15' 5''$

2

A = 128° 30' 10''

§ g. $\frac{(a+b+c)}{2} = 70^\circ$ co-sec. = 0.027014 (§ e.)

$\frac{(a+b+c-a)}{2} = 4^\circ$ co-sec. = 1.156415

$\frac{(a+b+c-b)}{2} = 26^\circ$ sin. = 9.641842

$\frac{(a+b+c-c)}{2} = 40^\circ$ sin. = 9.808068

Twice tang. $\frac{1}{2} A = 2)20.633339$

Tang. $\frac{1}{2} A = 10.316669 = 64^\circ 15' 5''$

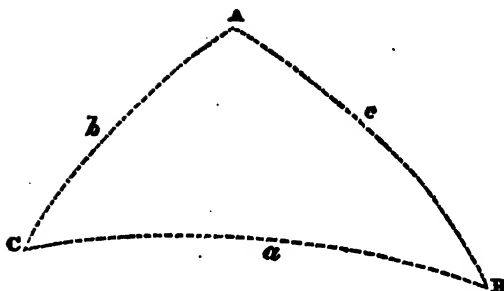
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A = 128° 30' 10''

§ h. In all cases § g. is a good solution. But when the required angle (A) is near 180°, (§ 176. § c.) solution § f. will not carry great accuracy into the result. But this seldom occurs in practice, and in most cases either solution may be adopted with success.

§ i. One of the angles being found, the two others (§ 144.) are determinable.

CASE VI.



§ 202. Given the three angles,

$$A = 114^\circ$$

$$B = 39^\circ$$

$$C = 49^\circ$$

To find the value of the sides.

§ a. This problem is but of little practical utility to the navigator, for problems in

which the three angles of a triangle are the data, seldom occur. But there are several methods by which the side required may be found; one of which, being general in its application, is thought sufficient.

§ b. The *product* of *cos.* of the difference between half the sum of the three angles, and each angle adjacent to the required side, *multiplied* by the *product* of *co-sec.* of each of said angles, is twice the *cos* of $\frac{1}{2}$ the required side.

§ c. To find the value of a .

$$\frac{(A+B+C)}{2} - B = 62^\circ \text{ co-sine} = 9.671609$$

$$\frac{(A+B+C)}{2} - C = 52^\circ \text{ co-sine} = 9.789342$$

$$B = 39^\circ \text{ co-sec.} = 0.201128$$

$$C = 49^\circ \text{ co-sec.} = 0.122220$$

$$\text{Twice cos. of } \frac{1}{2} a = 2) 19.784299$$

$$\text{Cos. } \frac{1}{2} a = 9.892149 = 38^\circ 43' 50''$$

2

$$a = 77^\circ 27' 40''$$

§ d. The value of one side being known, that of the others is determinable.

NAUTICAL ASTRONOMY.

NAUTICAL ASTRONOMY.

§ 203. That part of astronomy which treats of the motions and of the positions of the heavenly bodies, is an important branch of navigation. A knowledge of these motions and positions is highly essential to the navigator; for it is by understanding them, that methods have been devised for determining latitude and longitude at sea, by means of observations made upon the heavenly bodies.

§ 204. The figure of the earth is that of an oblate spheroid. It resembles that which would be described by the revolution of a semi-ellipse about its minor axis. It is flattened in at the poles, and elevated towards the equator.

§ 205. To suit the common purposes of navigation, the earth may be considered as a perfect sphere; the sun as the centre of the universe, and the centre of motion in the planetary system; the fixed stars may be considered to be almost stationary, and immeasurably distant from the earth and from each other; from every part of the earth's orbit, they are seen in the same relative positions, with regard to each other.

§ 206. The earth has two rotary motions; one about its axis, which produces day and night; the other in its orbit, and around the sun, which causes the seasons.

§ a. The latter is called the earth's annual, and the former its diurnal, motion.

§ 207. In its annual revolution around the sun, the earth describes the periphery of an ellipse.

§ a. The centre of the sun is in the plane, and at one of the foci of this ellipse, and the centre of the earth moves on its circumference.

§ 208. The axis of the earth is inclined, from a perpendicular to the plane of its orbit, nearly at an angle of $23^{\circ} 28'$.

§ a. If the earth's axis were perpendicular to this plane, there would be no change of seasons, or variation in the length of day and night; and at either pole there would be continual day.

§ b. It is this angle of inclination which causes the *declination* of the sun.

§ c. In a northern winter, the earth is nearer to the sun than it is in summer; but owing to the sun's declination, or the inclination of the earth's axis to the plane of the earth's orbit, the south pole is turned towards the sun during the former season, and the sun's rays striking the northern hemisphere more obliquely than they do in

summer, are in consequence, spread over a greater surface, and are therefore less effective in producing heat.

§ 209. The earth revolves from west to east. It completes one revolution on its axis in a day, and one around the sun in a year.

§ 210. The periphery of the earth's orbit, like the circumference of every re-entering curve, contains 360° .

§ a. The earth completes one revolution in its orbit, in 365d, 5h, 48m, and 48s.

§ b. Therefore, the motion of the earth in its orbit, in one day, is, as 365d, 5h, 48m, 48s is to 360° ; the ratio of which is 59' 8".

§ 211. An *astronomical, sea* or *civil*, day, is the time between two consecutive noons, or midnights.

§ 212. A *sidereal* day is the time between two consecutive transits of a star across the same meridian.

§ 213. The intervals of time from the passage of the sun and a star across, until their return to, the same meridian, are unequal. The difference between these intervals, results from the combined effect of the earth's motion about her axis, and in her orbit; for, during the interval between two consecutive transits of a star across the same meridian, the earth advances in its orbit (§ 210. § b.) nearly 59' 8"; so that the transit meridian has to go 59' 8" more than one complete revolution around the axis of the earth, before it returns to the sun.

§ a. Thus, during the time in which the earth is making one complete revolution in its orbit, a star has one transit more than the sun has, across any proposed meridian.

§ b. Sidereal accelerates upon *mean* solar time, 3m, 56s.55 in 24h, or about 9 $\frac{1}{2}$ s in an hour.

§ 214. The motion of the earth about its axis, is uniform; but the velocity of the earth in its orbit, is irregular. This irregularity is caused by the variation in the mutual action of the centrifugal and centripetal forces upon the earth.

§ a. The sidereal day is consequently of a uniform length; and the solar day varies with the velocity of the earth in its orbit.

§ b. This variation produces *equation of time*.

§ 215. *Apparent time* is the time which is deduced by calculation, from the bearing, or the altitude of the sun.

§ a. The time shown by a dial, is apparent time.

§ 216. *Mean time* is the average of apparent time.

§ a. Watches, etc., are designed to keep mean time.

§ 217. *Equation of time* is the difference between mean and apparent time.

§ 218. If the motion of the earth in its orbit, were uniform, describing equal arcs in equal times, there would always be the same length of time from one transit of the sun to another, across the same meridian; and apparent and mean time would always agree. But from noon to the succeeding noon, there is sometimes more, and sometimes less, than 24 hours.

§ a. Twenty-four hours is the average, or mean of the time, in

which, during one complete revolution of the earth in its orbit, the sun performs every one of its 365 transits across the same meridian.

§ *b*. This is the time which all good time pieces show. The time shown by them and the sun agrees only four times during a year; when this agreement takes place, then there is no equation of time.

§ 219. All astronomical calculations, such as those in the nautical almanacs, etc., are computed for astronomical time.

§ *a*. Of those in the ephemeris, some are for *mean*, and some for *apparent* astronomical time.

§ 220. An astronomical day commences when the centre of the sun is on the meridian (say of Greenwich), and ends, when it returns to the same meridian.

§ *a*. An astronomical day is divided into 24 hours, which are reckoned in succession, from 1 up to 24.

§ 221. A civil day commences at the midnight, that *precedes* the beginning of the astronomical day.

§ *a*. A civil day is divided into two equal portions; the first is A.M.; it ends at noon, when the astronomical day begins: the latter is P.M.; it ends at midnight, when the succeeding day begins. Each division consists of 12 hours.

§ *b*. Consequently, the day civil, is always 12 hours in advance of the day astronomical. Thus, Dec. 11th, 9h 34m 44s A.M. is, according to astronomical time, Dec. 10th, 21h 34m 44s; and 16 hours of civil time, or Dec. 11th, 4h P.M., is Dec. 11th, 4h, according to the astronomical time. Therefore, when the civil time is A.M. 12 hours added to it makes it astronomical time.

§ 222. The *ecliptic* is a great circle, the centre of which is that of the earth, and the plane of which coincides with the plane of the earth's orbit. Therefore these planes cut the axis of the earth at the same angle.

§ 223. The sun is always in the ecliptic.

§ *a*. Considering the earth to be stationary in its orbit, the sun appears to perform an annual revolution, moving around the earth, on the periphery of the ecliptic, and crossing the equator twice during the year.

§ 224. The ecliptic is divided into 12 arcs, of 30° each. These divisions, named and marked, are called the Signs of the Zodiac. They are—

§ *a*. Aries (♈), Taurus (♉), Gemini (♊), Cancer (♋), Leo (♌), Virgo (♍), Libra (♎), Scorpio (♏), Sagittarius (♐), Capricornus (♑), Aquarius (♒), and Pisces (♓).

§ *b*. The first six being north of the equator, are called *northern signs*.

§ *c*. The other six are south of the equator, and are called *southern signs*.

§ 225. The points in which the ecliptic (§ 223. § *a*.) crosses the equator, are called the *equinoctial points*; because, when the sun passes through these points, the days and nights are equal.

§ 226. The sun crosses the equator, and enters the first point of

Aries, about the 21st of March. The sun then passes through Aries, Taurus, and Gemini, towards the north; and about the 21st of June, it reaches its greatest northern declination at the first point of Cancer, where it appears to be stationary for a while; it is then said to be in the *summer solstice*.

§ a. The first point of Cancer, is a *solstitial point*.

§ b. Returning thence, towards the south, the sun passes through Cancer, Leo, and Virgo, and completing its tour, or the north side of the equator, it arrives, about the 23d of September, at the intersection of the ecliptic with the equator, when the day and night are again equal, and the sun is in the *autumnal equinox*.

§ c. Recrossing the equator then, the sun enters the first point of Libra, and continuing on towards the south, it *descends* through Libra, and the succeeding signs, and reaches its greatest southern declination, about the 22d of December; then it is at the first point of Capricorn, and again appears to stand still.

§ d. The sun is now in its winter solstice; and returning towards the north, it *ascends* through Capricorn, Aquarius, and Pisces, and entering the first point of Aries, completes one annual revolution, and goes on to renew the seasons.

§ 227. The time from the sun's passing the first point of Aries, until its return to that point again, is about (§ 210. § a.) 365d 5h 48m 48s.

§ a. This is called a *solar*, or a *tropical year*, and it is the year by which the seasons are regulated.

§ b. A solar, differs from a sidereal, year, about 20m 23s.

§ 228. A *sidereal year* is the time from the sun's leaving, until its return to, the same part of the heavens, or to the same fixed star.

§ a. A sidereal, is longer (§ 227. § b.) than a solar, year, on account of the *precession* of the *equinoxes*, which is a motion contrary to that of the sun through the signs.

§ b. While the sun is completing a revolution from left to right through the signs in successive order, they are moving in the contrary direction, or from right to left, and by the time the sun has returned to the first point of Aries, for instance, this point will have retrograded a little more than 51"; and, on account of this motion, the sun comes to this point sooner than it would have done, had the first point of Aries remained stationary.

§ 229. The *zodiac* is a belt in the heavens that extends 8° on each side of the ecliptic.

§ a. The orbits of all the primary planets are within the zodiac.

§ 230. The *primary planets* are Mercury, Venus, the Earth, Mars, Jupiter, and Saturn.

§ a. There are five other planets, but they cannot be seen with the naked eye. They are called *telescopic planets*.

§ 231. The points in which the orbit of a planet cuts the ecliptic, are called *nodes*.

§ a. That node is called the *ascending node* (Ω), through which

the planet passes, as it crosses the ecliptic, going from the south to the north side of it.

§ *b.* The other point of intersection is called the *descending node* (Ω).

§ 232. The *equator*, or *equinoctial line*, is a great circle, whose plane divides the earth into two equal parts.

§ *a.* These parts are called the northern and southern hemispheres.

§ *b.* The centre of the earth is the centre of the equator, and of all the great circles used in nautical astronomy.

§ 233. The *poles* (§ 123.) of the equator, also called the *Poles*, are two points on the earth, that are diametrically opposite to each other; each is 90° from the equator.

§ *a.* The one on the north side of the equator, is the *north pole*; and that on the south side, is the *south pole*.

§ 234. A straight line from one pole to the other, passes through the centre of the earth, and (§ 124.) is called the *axis* of the earth.

§ *a.* Around this axis, the earth performs its diurnal revolutions.

§ 235. From the best measurements, the equatorial appears to be about 26 miles greater than the polar diameter of the earth. And the degrees of latitude increase in length from the equator towards the poles.

§ *a.* The surface of the earth (§ 204.) being flattened in at the poles, and elevated at the equator, its meridional curvature is less near the former than it is near the latter; consequently a degree of latitude near the poles contains more fathoms, feet, etc., than one at or near the equator.

§ 236. Suppose the plane of the equator to be extended to the heavens; it there forms the *celestial equator*, and its poles, are the *poles* of the world.

§ 237. The latitude of places on the earth, is measured from the terrestrial equator; and their longitude is measured on it.

§ *a.* In the heavens, the declination of the bodies are measured from the celestial equator, and their ascension on it.

§ *b.* Declination and ascension are, in the heavens, what latitude and longitude are on the earth.

§ 238. The sun crosses the equator twice in a year.

§ *a.* While the sun is on the north side of the equator, the sun is constantly visible from the north pole, and daylight continues there until after the sun recrosses the equator, when the sun goes below the horizon, and does not rise again, until (§ 226. § *d.*) completing its southern tour, it is approaching the vernal equinox.

§ 239. The year, instead of being divided into seasons at the poles, may, more properly, be divided into day and night; for there is but one day and one night at each pole during the year.

§ *a.* At the north pole, the day commences about the 21st of March, when the sun crosses the equator, and continues until the sun recrosses the equator, which happens about the 23d of September, when night succeeds the day, and day begins at the south pole, and lasts until the sun again returns to the first point of Aries.

§ *b*. Owing to atmospheric refraction, the sun may be seen at either pole for several days previously to its rising above, and after it has gone below, the natural horizon.

§ 240. The morning and evening twilights at either pole, are together about two months and a half long.

§ *a*. When the sun crosses the *crepusculum*, the twilight begins and ends.

§ 241. *Crepusculum* is a small circle parallel to the horizon, and 18° below it.

§ 242. All small circles that are parallel to the horizon, are *almacanters*.

§ *a*. To an observer at the north, or south pole, the horizon and the equator coincide, and the parallels of declination are *almacanters*.

§ *b*. At the north pole, the *crepusculum* coincides with the parallel of the 18th degree of southern declination, which the sun crosses about the 29th of January, (near 51 days before it rises;) and on the 13th of November, (near 50 days after it has gone down).

§ *c*. On account of such long twilights, the winter nights, in high southern or northern latitudes, are rendered less gloomy than they otherwise would be.

§ 243. The earth is divided into five zones; two frigid, two temperate, and one torrid.

§ *a*. The torrid zone is the largest; it extends to the parallels of $23^\circ 28'$ north and south latitude; consequently it is $46^\circ 56'$ broad.

§ *b*. The small circles which limit it, pass through the solstitial points, and are called *tropics*, from *trepho*, (Gr.); because, when the sun reaches the parallel of declination for $23^\circ 28'$ (§ 226. § *b*. & § *d*.) it appears to turn its course, and to recede from the pole which it was approaching, and to retrograde towards the direction whence it came.

§ *c*. That parallel which is north of the equator, (§ 226.) is the *tropic* of Cancer.

§ *d*. And that on the south side of the equator, (§ 226. § *c*.), is the *tropic* of Capricorn.

§ *e*. The sun is vertical twice a year, at every place within the tropics.

§ 244. Each of the temperate zones extends over $43^\circ 4'$ of latitude.

§ *a*. That on the north side of the equator, called the north temperate zone, extends from the *tropic* of Cancer to the arctic circle.

§ *b*. And the south temperate zone extends from the *tropic* of Capricorn to the antarctic circle.

§ 245. The *arctic* and *antarctic* are small circles, parallel to the equator, and $23^\circ 28'$ from the poles.

§ *a*. The arctic is about the north, and the antarctic about the south pole.

§ *b*. They are sometimes called the *polar circles*.

§ 246. The *frigid* zones are between the polar circles and the poles.

§ 247. *Latitude* is distance on the earth, measured north or south, from the equator.

§ 248. *Meridians* are great circles, that are secondaries to the equator.

§ a. Therefore (§ 126. § a. & § 127.) they cut it perpendicularly, and intersect each other at the poles.

§ b. The arc of a meridian that is contained between a place and the equator, measures the latitude of that place.

§ c. The meridian of an observer passes through his zenith.

§ 249. *Parallels of latitude* are small circles, parallel to the equator.

§ a. All places upon the same parallel have the same latitude.

§ b. And all places that are on the same meridian, are in the same longitude.

§ 250. *Circles of longitude* are meridians.

§ 251. *Elevation of the pole* is the distance of either pole above the horizon of any observer.

§ a. This distance is measured on the meridian of the observer; it is equal to his latitude.

§ 252. *Longitude* is the distance, expressed in degrees, etc., between two meridians. It is measured on the equator, and east or west from a meridian.

§ a. A meridian from which longitude is measured, is called the *prime meridian*.

§ b. The location of this meridian is optional with topographers. The French reckon the longitude of all other places from the meridian of Paris; the Spaniards from Cadiz. The English use the meridian of the Royal Observatory at Greenwich for their prime meridian: and the Americans generally construct their charts from the meridian of Washington City.

§ c. But as long as most of the charts and the tables of the Nautical Almanac which we use, shall be constructed and calculated to the meridian of the Greenwich observatory, it will be found more expedient to reckon longitude from this, as the prime meridian.

§ 253. It is to be wished that all nations would fix upon one common prime meridian. One might be established from celestial phenomena, by which all that is arbitrary in its locality, might be made to disappear. La Place recommends the adoption of a universal first meridian, and suggests the propriety of selecting for this purpose, that meridian, upon which it was 12 o'clock when the sun entered the point of the vernal equinox in the year (1250), in which the apogee of the earth's orbit coincided with the solstitial point in Cancer. Such a universal meridian would pass about 8 miles west of Cape Mesurada on the Coast of Africa.

§ 254. The longitude of a place is measured on the arc of the equator, which is contained between the meridian of that place, and the prime meridian.

§ a. The angle at either pole, which these two meridians make with each other, is equal to the longitude.

§ b. Consequently the arc of a parallel of latitude (§ 249.), which

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is intercepted between that place and the meridian of any other place, contains, in degrees, the difference of longitude between these two places.

§ c. The difference of latitude between two places, is the arc which is contained on the meridian of either place, between the parallels of latitude of these two places.

§ 255. *Declination* is the distance of a heavenly body from the equatorial plane.

§ a. Declination is north or south, according as the body is north or south of the equator. It is measured in the heavens, as latitude (§ 248. § b.) is on the earth.

§ b. The arc of the circle of declination or of right ascension, that is contained between the equator and the centre of the body, measures its declination.

§ 256. *Circles of right ascension* or of *declination* in the heavens, correspond with meridians of terrestrial longitude.

§ a. They cut the equator at right angles, and intersect each other in the poles of the world.

§ 257. *Parallels of declination* are small circles in the heavens, that are parallel to the equator.

§ a. They correspond with parallels of latitude on the earth.

§ b. The tropics (§ 243. § c. & § d.) are the parallels of the sun's greatest declination.

§ 258. *Polar distance* is the distance of any celestial body from either pole.

§ a. It is the arc of the circle of declination that is contained between the body and either pole.

§ b. The complement of the declination of a heavenly body, expresses its polar distance.

§ 259. The *right ascension* of a body is the arc of the equator, which is between the first point of Aries and the circle of the declination of that body.

§ a. This arc is always counted in the order (§ 224. § a.) of the signs, and its dimensions are expressed in hours, minutes, and seconds.

§ 260. Right ascension in the heavens, is what longitude is on the earth. But the manner of reckoning them is different.

§ a. Longitude is reckoned from the prime meridian as far as 180° both east and west.

§ 261. Right ascension commences to be reckoned from the first point of Aries, and goes entirely around, in the direction in which the sun passes through the signs.

§ 262. The ascensional difference of the sun, is the difference between its right and oblique ascension, or between sunrise and 6 o'clock.

§ 263. *Horary angles* are those which meridians, or circles of declination, make with each other at the poles.

§ a. The *6 o'clock hour circle*, is that circle of declination (§ 256. § a.) which cuts the equator and the horizon in the east and west points.

§ b. It is secondary to the equator and to the meridian of the observer.

§ c. Any circle of declination is an *hour circle*.

§ 264. *Celestial latitude* is distance between the ecliptic and any body in the heavens; it is measured from the ecliptic, and upon a secondary to it.

§ 265. The secondaries of the ecliptic, are circles of celestial longitude.

§ a. The arc of the circle of celestial longitude, that lies between the ecliptic, and a body in the heavens, measures in degrees, etc., the latitude of that body.

§ 267. The longitude of heavenly bodies is reckoned on the ecliptic, as right ascension is on the equator (§ 259.).

§ a. It commences at the first point of Aries, and is reckoned around in the direction in which the sun passes through the signs.

§ 268. The right ascension and longitude of an object are never the same, except when the body is on the solstitial colure, which cuts both the ecliptic and the equator at right angles, and passes through their poles.

§ 269. The *solstitial colure* is that circle of declination which passes through the solstitial points (§ 226. § a. & c.).

§ 270. The *equinoctial colure* is the circle of declination, which passes through the equinoctial points (§ 226. § b.).

271. The equinoctial and the solstitial points are the four *cardinal* points of the heavens.

§ 272. The four *cardinal* points of the horizon are the east and west, and the north and south points.

§ a. The *east* and *west* are the points in which the equator and the horizon intersect each other.

§ b. The *north* and *south* are the points in which the meridian of the place cuts the horizon.

§ 273. Every observer has two horizons, one *rational*, the other *sensible*.

§ a. The eye is the centre of the latter, and is on the axis of the former.

§ 274. The *sensible horizon* is that circle which terminates the view upon an uninterrupted plane.

§ a. It is formed by the apparent meeting of the plane upon which we stand, with the concave expanse above.

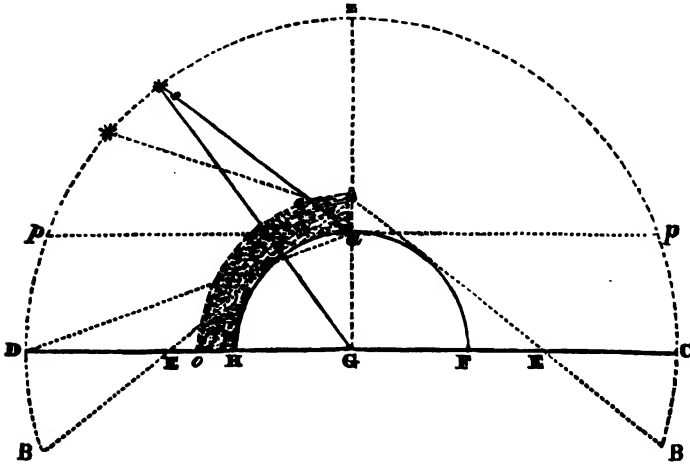
§ b. It is a small circle, and is parallel to the rational horizon.

§ c. The circle which terminates our view at sea, is the *sensible horizon*.

§ 275. The *rational horizon* is a great circle; its plane passes through the centre of the earth, and its axis through the eye of the observer. Suppose $H a F$ to be half of the earth, $D Z C$ the concave blue which bounds the vision, and $D H C$ the plane of the rational horizon. If an observer were placed at the centre (G) of the earth, the angle $D G Z$ would show the altitude of an object in the zenith, at Z , from the rational horizon. If the observer were placed at the circumference (at a) of the earth, $p a p$ would be the plane

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of his visible horizon, and the angle $p a Z (=D G Z \text{ § } 30.)$ would show the altitude of a body at Z , from the sensible horizon. But if the observer were placed at A above the surface of the earth, the angle $B A Z (>D G Z)$ would show the altitude of Z from his sensible horizon.



§ *a*. When the eye is elevated above the level of the sea, (as at A ,) the line of vision $A B$ to a point B in the sensible horizon, cuts the plane $C D$ of the rational horizon at an angle, $D E B$, which angle is called the *dip*. Consequently, when the eye is above the surface of the plane on which we stand, the sensible horizon is *below* the rational horizon.

§ *b*. The poles of the rational horizon are the zenith and nadir.

§ 276. The *zenith* is the point in the heavens which is directly over head.

§ *a*. Being a pole of the horizon, the zenith is 90° from the horizon.

§ 277. The *nadir* is the other pole of the horizon; it is in the lower part of the heavens, directly under our feet, and diametrically opposite to the zenith.

§ 278. *Azimuth* circles are secondaries (§ 127.) to the horizon; cutting it perpendicularly, they intersect each other in the zenith and nadir.

§ *a*. They are also called *vertical* circles.

§ 279. The azimuth circle which cuts the horizon in the east and west points, is called the *prime vertical*.

§ *a*. The prime vertical of every observer, is secondary to his meridian.

§ 280. The altitude, and zenith distance of a heavenly body, are measured on its azimuth circle.

§ 281. The *zenith distance* of a heavenly body is its distance from the zenith of the observer; thus, Zs (§ 275.) is the zenith distance of the star at s .

§ *a*. It is measured on the arc of the azimuth circle, which lies between the centre of that body and the zenith.

§ *b*. The *zenith distance* is *north* when the body is south of the observer.

§ *c*. And south, when the body is north of the observer.

§ 282. The *altitude* of a heavenly body, is its distance above the horizon.

§ *a*. The arc of the azimuth circle which is contained between the centre of a body and the horizon, measures the altitude of that body; thus, Ds (§ 275.) is the altitude of the star at s .

§ *b*. The complement of the altitude of a body, gives its zenith distance.

§ 283. The altitude which is taken of the sun or moon, with a quadrant or sextant, is the observed *altitude* of one of the edges, called a *limb*.

§ *a*. The apparent altitude of the *centre* is found by applying the corrections, (which are laid down in the Nautical Almanac,) for the semidiameter of the body, to the apparent altitude of its limb.

§ *b*. And the *true*, is obtained from the *apparent* altitude of the centre, by applying to the latter, corrections for the parallax and refraction, which are also known by previous calculations.

§ *c*. The apparent and the true altitude of a body, are always measured on the same azimuth circle.

§ 284. The rays of light coming from the heavenly bodies strike the atmosphere obliquely, and entering from a rarer into a denser medium, are refracted, or *bent downwards*; this causes the body whence they emanate to appear higher up in its azimuth circle than it really is. Suppose $*e$ (§ 275.) be a ray of light from $*$, and Ao to represent the atmosphere; when this ray strikes the atmosphere, it will be refracted so as to reach the eye of the observer at a , in the direction ae , which makes $*$ appear at s , above its true place.

§ *a*. Hence the apparent altitude of the sun or a star is always greater than the true altitude, unless the body be in the zenith.

§ 285. Parallax has a contrary effect; it acts in a direction opposite to that of refraction, and causes a body to appear *lower* down in its azimuth circle than it really is.

§ *a*. But the effects of parallax and refraction, though acting in contrary and opposite directions, seldom counterbalance each other, so as to make an object appear in its true place.

§ 286. The parallax of the moon is always greater than the refraction; and the moon always appears below its true place.

§ *a*. Hence the apparent, is always *less* than the true altitude of the moon.

§ 287. The *horizontal parallax* of a body, is the difference between the *true* and *apparent* place of that body, (supposing there be no refraction), when it is in the horizon.

§ *a*. The horizontal parallax is equal to the angle at the body,

which is subtended by the distance of the observer, from the centre of the earth.

§ *b*. The angle which this distance, or semidiameter, subtends, is greatest when the body is in the plane of the rational horizon; thus, the parallax of a body at *D*, (§ 275.) is the angle $G D a$; and the parallax of the same body, when at *s*, is $G s a$.

§ *c*. As the object rises above the horizon, the angle which this semidiameter subtends, is called *parallax in altitude*; it gradually decreases until the object reaches the zenith, when it vanishes.

§ *d*. The nearer the object is to the earth, the greater will this angle be at the centre. Hence the moon's parallax is greater than the sun's, and this greater than that of the fixed stars.

§ 288. The parallax of a body decreases from the horizon to the zenith, in the proportion of sine of the zenith distance (§ 281.) to radius.

§ 289. Owing to the effects of parallax and refraction upon the heavenly bodies (§ 285. § *a*.), they are never seen in their *true* places, except when in the zenith.

§ *a*. The places in which the heavenly bodies are seen, are called their *apparent places*.

§ 290. The *true place* of a heavenly body, is that place in which it would appear, if seen from the centre of the earth.

§ 291. The *azimuth* of a celestial body is the angle which is contained at the zenith, between the meridian of the place, and an arc of the azimuth circle, which passes through the centre of that body.

§ *a*. In north latitudes, the angle on the *north* side, and in south latitudes, the angle on the *south* side, of the arc of this azimuth circle, is called the azimuth.

§ *b*. Before a body crosses the meridian of the observer, its azimuth is east, and west afterwards.

§ *c*. Thus, in north latitude, an azimuth is said to be *north*, so many degrees east, if the body be east of the meridian; or *N*, so many degrees west, if it have passed the meridian.

§ *d*. And in south latitude, the azimuth is reckoned in the same manner from the south point, to the east or west.

§ 292. The *amplitude* of a celestial object, is the arc of the horizon that lies between the east or west point, and the centre of that object when it is rising or setting.

VARIATION OF THE COMPASS.

§ 293. The variation of the needle, is determined by means of azimuths, or amplitudes.

§ 294. The needle does not always point to the north and south poles. At some places it points to the east, and at others to the west, of the true north and south points.

§ *a*. Even at the same place, the polarity of the needle does not remain constant.

§ *b*. As to the direction in which the needle points, it is subject to certain periodical changes, which do not follow any known laws.

At some places, after having pointed, for several years, to the eastward of the true north, it has gradually pointed nearer to the north point, until its position lay due north and south; then crossing the direction of the meridian, the needle has continued to turn more and more to the westward of the true north, until it has attained the maximum of its deviation for that place, when, after having remained stationary for a time, it commenced its return towards its former position.

§ 295. *Variation* of the compass is the deviation of the needle, from pointing to the north and south poles.

§ 296. The points to which the needle tends, are called the *magnetic* north and south points.

§ 297. When the northern point of the needle, or the N point on the compass card, points to the eastward, or to the *right*, of the true north, the variation is *easterly*.

§ *a*. And *westerly* when the same point points to the *left*, or to the westward of the true north.

§ 298. The true direction of the magnetic north point, is found by applying the variation, when it is easterly, to the *right* of the true north; and to the *left* of the *true* north, when the variation is westerly.

§ *a*. Thus, when the variation is one point easterly, the north point of the needle, or the N point on the compass card, points N. by E. And it points N.N.W. when the variation is two points westerly.

§ *b*. Wherefore, knowing the magnetic bearing of any object, its true bearing may be determined, by applying the variation, when easterly, to the *right* of its compass bearing; and to the *left*, when the variation is westerly.

§ *c*. The true bearing of an object, that bears east per compass, is E. by S. if the variation be one point easterly; but E. by N. if the variation be one point westerly.

§ 299. The cause of variation, as well as of the attraction of the needle, towards the poles, is unknown.

§ 300. The needle is subject to the influence of another power equally mysterious in its nature; it is called *local attraction*.

§ *a*. This attraction operates on ship-board, and with different effects in different latitudes, as well as in the different directions in which the vessel may be heading,

§ *b*. Its effects upon the needle become obvious by taking the bearing of a fixed point on shore, then swinging the ship entirely around, and observing at several different points of her heading, the bearing of said fixed point.

§ *c*. The effect of local attraction upon the compass, is not often taken into consideration by navigators, although on board of vessels in some latitudes, (as in the English channel), it is said to cause the needle to deviate several degrees. The loss of fleets has been ascribed to the neglect of this attraction, on the part of navigators.

§ *d*. In conducting surveys particular attention should be paid to the effect of local attraction upon the needle.

§ 301. A *magnetic* meridian is a great circle that passes through the magnetic north and south points, and through the zenith of the observer.

A. B.

§ *a*. The needle always lies in the direction of this meridian.

§ *b*. Magnetic meridians cross each other in the magnetic poles.

§ 302. The *magnetic* equator is a secondary to all magnetic meridians.

§ 303. The *magnetic* prime vertical is a secondary to the magnetic meridian of the observer.

§ *a*. It passes through the zenith and the magnetic east and west points of the horizon.

§ 304. The *magnetic* azimuth of a celestial body, is an angle at the zenith, that is contained between the magnetic meridian of the observer and the zenith distance of the object, when its bearing is taken.

§ *a*. The magnetic azimuth should always be reckoned from the nearest pole, around towards the east, when the object is on the east side of the meridian of the observer; and to the west, after the object has crossed the meridian.

§ *b*. The advantage of reckoning the magnetic azimuth in this way, consists in having the true and magnetic azimuth always of the same name; i. e. either both east, or both west.

§ 305. The *magnetic* amplitude of a celestial body, is that arc of the horizon, which lies between the centre of the body, when the body is in the horizon, and the magnetic east or west point, according as the body is rising or setting.

§ 306. Upon the magnetic equator, the needle assumes a horizontal position.

§ *a*. To the north or south of this equator, it points *downwards*, inclining towards the nearest magnetic pole.

§ 307. The angle of this inclination of the needle, below the plane of the horizon, is called the *dip* of the needle.

§ *a*. The maximum of the dip is at the magnetic poles, and the minimum at the equator.

§ *b*. The ratio of the increment in dip, from the equator, towards the poles, has never been satisfactorily established.

§ 308. The variation of the compass is found by ascertaining the true and magnetic azimuths, or amplitudes, of any celestial object at the same moment.

§ *a*. The difference between them is the variation.

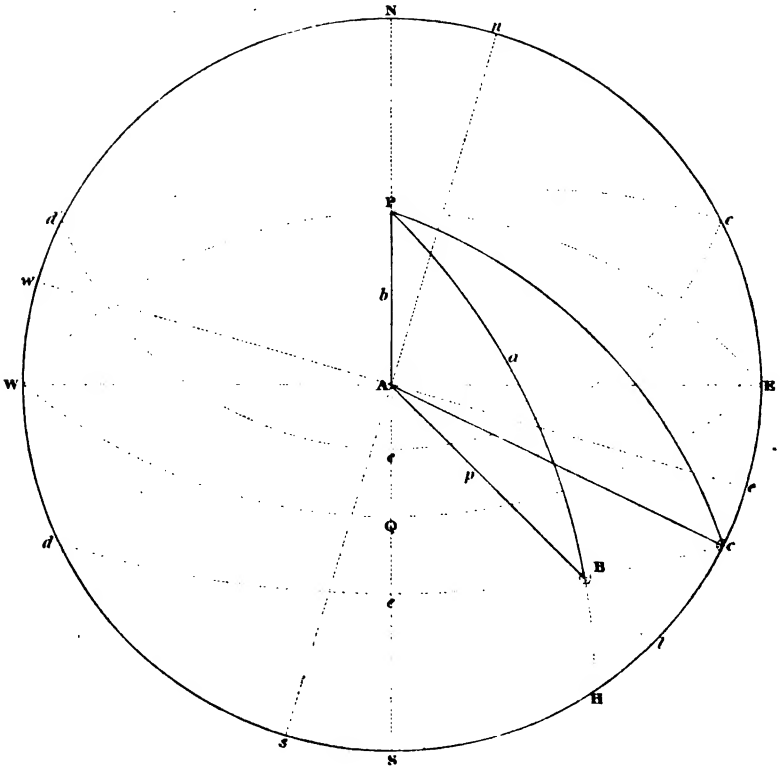
§ 309. The magnetic azimuth, or amplitude of a celestial object, is found by taking its bearing with an azimuth compass.

§ 310. The true azimuth or amplitude of a celestial object is determined by trigonometrical calculations.

§ *a*. The usual data for this operation, are the co-latitude of the observer, and the zenith, and polar, distances of the object.

§ 311. In North Latitude; the variation is easterly, if the magnetic, be less than the true, azimuth, when the object is on the east side of the meridian; or the variation is easterly, if the magnetic be the greater azimuth, when the object is west of the meridian.

§ *a*. And in South Latitude; the variation is easterly, when the magnetic is the greater azimuth in the former, and the less in the latter case.



§ 312. In each case, *mutatis mutandis*, the variation is westerly.

§ 313. *Stereographic projection* is the most useful, and being the most natural, is the most simple mode of representing a sphere, or circles of a sphere, upon a plane.

§ 314. In stereographic projection the eye is supposed to be at some point on the surface of the sphere, and to see one half of the sphere.

§ a. The circle which terminates the vision is called, in projection, the *primitive circle*.

§ b. The eye is the centre of this circle, and is the *projecting point*.

§ 315. The diagrams for the purposes of nautical astronomy, in this treatise, are projected upon the plane of the horizon.

§ 317. This Fig. is a stereographic projection upon the } Plate 2.
plane of the horizon, in lat. 40° north.

§ a. A, the projecting point, represents the centre of the horizon W N E S, as well as the zenith and the eye of the observer; P, the pole of the observer; P N (§ 251.) the elevation of the pole; W Q E (§ 232.) the equator; A Q (§ 248. § b.) the latitude of the observer; and A P the complement of his latitude; W A E (§ 279.) his prime vertical; N P S (§ 248. § c.) his meridian; A B I (§ 278.) the azimuth circle of the body whose altitude is taken; B I and A B (§ 280.) its altitude and zenith distance; *d e c* (§ 257.) the parallel of its declination; P B (§ 258. § a.) its polar distance, and P B H (§ 256. § a.) is an arc of its circle of declination; A P B (§ 263.) is the horary angle at which the body is; B A P (§ 291.) is its azimuth; N, E, S, and W, (§ 272.) are the cardinal points of the horizon; and W P E (§ 263. § a.) is the six o'clock hour circle.

§ b. It is easily conceived how it is, that, of all circles which cut the primitive circle of a projection, only that part of their circumference which is above the primitive, can be seen from the projecting point; and that 180° of every arc of these circles, that are great circles, is above (§ 125. § a.) the horizon or plane of projection.

§ c. And consequently that all of these arcs will appear in the projection to be more straightened out, or of less curvature than the primitive circle; and this curvature will be in proportion inverse to the obliquity with which their planes cut the plane of the primitive.

§ d. The less obliquely these planes cut the plane of the horizon, the more obliquely the observer at A looks upon them, and consequently the less their arcs appear to be curved, until, looking upon the edge of the planes of those which pass through his zenith, they appear as straight lines.

§ 318. In these projections, every great circle (N P S, W A E,) which passes through the zenith, is represented as a straight line.

§ a. And the length of every line (A N, A I, etc.) included between the projecting point (A) and the circumference of the primitive, measures 90° , and is considered as the quadrant of a circle.

§ 319. All of those circles which cut the plane of the primitive obliquely, and whose arcs (W Q E, P B, etc.) appear in the pro-

jection, to be of less curvature than the primitive, are called *oblique circles*.

Plate 2. { § 320. And those circles, whose arcs (N P S, W A E,) pass through the zenith (§ 318.) are called *right circles*.

§ a. With the horizon as the primitive circle; the meridian, (§ 248. § c.), prime vertical, (§ 279. § a.), and azimuth circles (§ 278.), pass through the zenith, and appear upon the plane of projection (§ 318.) as right circles.

§ 321. In lat. 40 N., the sun's declination being 20° S.; its magnetic azimuth was N. 119° 25' E. in the morning, when its true altitude was 16° 48' 25''; required the sun's true azimuth, and the variation of the compass.

§ 322. W N E S, (§ 316. § a.) represents the horizon of the observer; A, the centre of it, his zenith, and the place at which he stood to take the observation; *n A s*, the magnetic (§ 301.) meridian; and (§ 308. § a.) N A *n* is the variation.

§ a. The co-lat. (A P), the zenith, and the polar distance, (A B, P B) (§ 317. § a.), are the sides of the triangle A P B, and (§ 321.) are the given parts; and the azimuth P A B, is the required part; the value of it is determined according to Case V, (§ 201. § d.)

§ b. In the formulæ for calculation,

P D, stands for polar distance.

Z D, " zenith distance.

Co-lat. " complement of the latitude of the place of observation.

§ c. The complements of what is given (§ 321.) constitute the data of the proposed triangle.

§ d. $b = 50^\circ$ - = Co-lat.

$p = 73^\circ 11' 35'' = Z D$

$a = 110^\circ$ - = P D

To find the azimuth P A B, (§ 201. § f.)

§ e. a , or P D = 110°

p , or Z D = $73^\circ 11' 35''$ co-sec. = 0.018959

b , or Co-lat. = 50° - co-sec. = 0.115746

Sum 2) $233^\circ 11' 35''$

$\frac{1}{2}$ Sum = $116^\circ 35' 47''$ sin. = 9.951426

$\frac{1}{2}$ Sum \propto P D = $6^\circ 35' 47''$ sin. = 9.060224

2) 19.146355

Cos. $\frac{1}{2}$ P A B = $9.573177 = 68^\circ 1' 16''$

2

P A B (the true azimuth, § 291. § c.) = N $136^\circ 2' 32''$ E

B A *n* (the magnetic do. (§ 304. § a.) = N $119^\circ 25'$ E

N A *n* = Variation (§ 311.) = $16^\circ 37' 32''$ E.

§f. The difference between the true and magnetic azimuths (§ 308. § a.) (N A B—n A B=N A n) is the variation. } *Plate 2.*

§g. The true, being greater than the magnetic azimuth, the variation (§ 311.) is easterly.

§ 323. Bearing in mind, that the co-sec. sine, etc., of an arc (§ 99.), is the sec., cos., etc., of the complement of that arc, the process by which the subjoined formula is deduced from (§ 322. § e.) the one above, becomes manifest.

§ a. Calculation by this formula operates more directly upon the data, and is, perhaps, preferable in practice on account of its greater readiness.

§ b. P.D.=110°

Alt. = 16° 48' 25'' sec. = 0.018959

Lat. = 40° sec. = 0.115746

2)166° 48' 25''

$\frac{1}{2}$ Sum = 83° 24' 12'' cos. = 9.060242

($\frac{1}{2}$ S. ∞ P.D.) = 26° 35' 48'' cos. = 9.951425

2)19.146372

Cos. $\frac{1}{2}$ tr. azim. = 9.573186 = 68° 1' 15''

2

Tr. azimuth = N 136° 2' 30'' E.

§ c. This difference of 2'' in the result of the two methods, arises from the fraction of a second in $\frac{1}{2}$ sum; which, in either case, is not taken into computation.

§ d. Whence (§ 323.) the rule for calculating an azimuth. Take the difference between the P.D., and $\frac{1}{2}$ sum of the lat. alt. and P.D.; then the *product* of the cos. of this $\frac{1}{2}$ sum and of this difference, multiplied by the *product* of the sec. of the lat. and of the alt., is *double* the cos. of $\frac{1}{2}$ of the true azimuth.

AMPLITUDES.

§ 324. When the magnetic, and true amplitudes of a body, are both north, or both south, the difference between them is the variation.

§ a. But when one of the amplitudes is north and the other south, they are of different names; and their sum is the variation.

§ 325. If the amplitudes be of the same name (§ 324.), and the *true* be the less northern, or the greater southern amplitude, the variation is easterly when the object is rising; and,

§ a. The variation is also easterly, when the object is setting, and the *true* is north and the greater, or south and the less, of the two amplitudes.

T

§ 326. If "*magnetic*" be read for "*true*" in the conditions above, the variation becomes westerly.

§ 327. If the amplitudes be of different names (§ 324. § a.), and the *true* be the northern amplitude, when the object is rising, or the southern amplitude when the object is setting, the variation is westerly.

§ a. The converse of these conditions makes the variation easterly.

§ 328. The true amplitude and declination of a body, are always of the same name.

§ a. Therefore it is known by inspecting the declination, whether the true amplitude be north or south.

§ 329. Every object in the heavens, always rises and sets to the north, or to the south of the east and west points, according as its declination is north or south;

§ a. The equator (§ 272. § a.) cuts the horizon in the east and west points; and a parallel of north or south declination, being (§ 257.) parallel to the equator, must therefore cut the horizon, (if at all), to the north or south of the east or west points.

§ 330. A body (say the sun), rises and sets in the points, in which the parallel of its declination cuts the horizon.

Plate 2. { § a. Thus the sun's declination being 20° S., it rises and sets in the points (*c* & *d*) in which the parallel (*d e c*) of that declination cuts the horizon.

§ 331. To an observer who is on the equator, the sun's declination at the time of its rising or setting, is its amplitude; for,

§ a. The equator is then the prime vertical, and that arc of the horizon which (§ 292.) measures the amplitude, coincides with the arc of the circle of right ascension, which (§ 255. § b.) measures the declination.

§ 332. The sun rises in the east, and sets in the west, point, only twice in the year.

§ a. This happens at the moments in which the sun enters the first point of Aries, and the first point of Libra, for (§ 226. and § 226. § c.) the sun is then crossing the equator.

§ 333. To an observer at either pole, the horizon and equator coincide, and the sun does not set, while it is on that side of the equator, which is next to the observer. And,

§ 334. If the observer approach any point of the equator, that point will rise *above*, and the one diametrically opposite, will sink *below*, the horizon as many degrees as the observer advances from the pole. And,

§ a. The observer will not see the sun go below the horizon, until his distance from the pole is greater than the sun's declination.

§ 335. When the latitude and the sun's declination are of the same name, the sun is not seen to set as long as its declination is greater than the co-latitude.

§ 336. To find the sun's true amplitude, and thence the variation.

§ 337. At sun-rise in lat. 40° N., the sun's declination being 20° S., its magnetic amplitude was E. $9^{\circ} 53' 32''$ South; required the true amplitude, and the variation.

§ 338. The place of the sun at the time of its rising (§ 330. § a.) is at the point (*c*) in which the parallel of its declination intersects the horizon; *w A e* (§ 303.) is the magnetic prime vertical; } Plate 2.
the arc *E c* (§ 292.) is the true, and *e c* (§ 305.) the mag- }
netic, amplitude of the sun; and *P A c* is the triangle of the proposed problem, of which the sides are given, and *P A c* is the part required. The triangle (§ 142.) is quadrantal, of which the quadrantal side is *A c*, the zenith distance.

§ a. Now (§ 150. § a.), calling the quadrantal side (*A c*) radius, and the legs (*P A*, *P c*) angles, and the angle *P A c* a leg, the problem is an example under Case IV. (§ 173. § b.).

$$\begin{aligned} \text{§ b. } P A \text{ or co-lat.} &= 50^\circ \text{ cosec.} = 0.115746 \\ P c \text{ or Z. D.} &= 110^\circ \text{ cos.} = 9.534052 \end{aligned}$$

$$\text{Cos. } P A c = 9.649798 = 116^\circ 31' 4''$$

§ c. Now (§ 133. § a.) the arc *N E c* is the measure of the angle *P A c*; and the arc *E c* (§ 52. § a.) is the complement of *N E c*, and (§ 292.) the sun's true amplitude.

$$\begin{aligned} \text{§ d. } E c, \text{ the sun's true amplitude} &= E 26^\circ 31' 4'' S \\ e c, \text{ the sun's magnetic do.} &= E 9^\circ 53' 32'' S \end{aligned}$$

$$\text{The variation} = 16^\circ 37' 32'' E \text{ (§ 324.)}$$

§ e. The amplitudes are both south, and the true is the greater; wherefore (§ 325.) the variation is easterly.

§ 339. By recurring to § 323. it is evident that the process of calculation (§ 338. § b.) for determining the amplitude, may be simplified, and rendered more convenient in practice, by a similar artifice.

$$\begin{aligned} \text{§ a. Lat.} &= 40^\circ \text{ sec.} = 0.115746 \\ \text{Dec.} &= 20^\circ \text{ sin.} = 9.534052 \end{aligned}$$

$$\text{Sin. ampl.} = 9.649798 = 26^\circ 31' 4''$$

$$\text{Magnetic ampl.} = 9^\circ 53' 32''$$

$$\text{Variation} = 16^\circ 37' 32'' E$$

§ b. Whence the general rule in practice, for finding an amplitude.

The sine of the amplitude is the *product* of sec. of the lat. and sine of the dec. See Table VII.

SUNRISE.

§ 340. When the latitude of an observer, and the sun's declination, are either both north, or both south, the sun always rises before, and sets after, six o'clock.

§ 341. When the latitude and declination are of different names, the sun rises after, and sets before, six o'clock.

§ 342. When the sun rises in the east, or sets in the west, point of the horizon, the sun is (§ 272. § a.) on the equator, and (§ 263. Plate 2. § a.) in its six o'clock hour circle (W P E); and therefore rises and sets at six o'clock.

§ 343. The time of sunrise subtracted from 12 o'clock, gives the time of sunset, when the points in which the sun rises and sets, are equidistant from the point at which it crossed the meridian.

§ a. This happens when the declination at sunrise, and at sunset, is the same.

§ 344. When the sun (§ 342.) rises and sets at six o'clock, it is also in the first point of Aries, or of Libra;

§ a. As the sun's declination increases, its right ascension (§ 259.) becomes greater, and the first point of Aries farther from the east point of the horizon, when the sun is rising; and,

§ b. The interval between sunrise and 6 o'clock, also increases.

TIME.

§ 345. The time between sunrise and 12 o'clock, in any latitude, may be determined by knowing the sun's declination.

§ a. This operation consists in finding the value of the horary angle A P c, which the circle (P c) of declination, in which the sun rises, makes (§ 263.) with the meridian (N P S) of the place.

§ 346. The value of this angle, (A P c), like that of every other, is expressed in degrees; but may be converted into time, in the proportion of 360° to 24h, which (§ 218. § a.) is the mean time in which the earth performs a diurnal revolution.

§ a. $\frac{360^\circ}{24} = 15^\circ$; therefore the sun describes an horary angle of 15° in 1h; $\frac{1h}{15} = \frac{60m}{15} = 4m$; wherefore the sun describes an horary angle of 1° in 4m, of 15' in 1m, and of 15'' in 1s.

§ b. Whence the rule for converting longitude, degrees, etc., into time, and the reverse.

§ 347. Divide the degrees by 15, for hours, the product of 4 and the remainder, is minutes, (m); the quotient of the minutes (') by 15 is also minutes, (m); and the product of 4 and the remainder to this quotient, is seconds (s); also the quotient of the seconds (") by 15 is seconds (s) of time.

§ 348. The product of hours by 15, and the quotient of the minutes (m) by 4, are degrees; the product of the remainder of the minutes (m) by 15, and the quotient of the seconds (s) by 4, are minutes ('); and the product of the remaining seconds (s) by 15, is seconds (").

§ a. The two first and two last columns of Tables II., show (with the hours at the bottom or top,) the value in time of the degrees and minutes which stand nearest to them.

§ b. And these columns may be used for finding the logarithmic value of hours, minutes, and seconds, as well as for converting de-

degrees, etc., into hours, etc., and vice versa. Therefore in the solution of problems, in which a given or required part consists of hours, etc., these need not be commuted in the process of solution, into degrees, etc.; but may be operated with in calculations, under the denomination of *time*, and thus save the trouble of substituting their value in degrees, etc.

§ c. Substituting hours, etc., for degrees, etc., the logarithmic value of a horary angle may be taken from Tables II., according to the directions given under § 102. for degrees, etc. Thus, to take from the tables the log. of $4h, 20m, 40s$; $4h$ is found at the right hand *bottom* corner of the tables; $20m$ is found in the right hand (*m*) column; and above $20m, 40s$ is found in the (*s*) column on the same side; opposite to $40s$, and in the column which has the required precept at the *bottom*, is the required log.; thus, log. sine $4h 20m 40s = 9.957863$.

§ d. The time corresponding to any log. is taken from the tables according to the directions under § 103. given for taking out the degrees, etc., for a log. sine, etc. Thus, the value in time of log. sin. $9.618004 = 1h 38m 4s$. The log. 9.618004 is found in the column marked (sin.) at the *top*. In the left hand column (*s*) of the page, and opposite to 9.618004 , is $4s$; above $4s$ in the (*m*) column is $38m$, and at the top of the page on the same side is $1h$.

§ e. The small columns marked (diff.) show the difference which $1s.$ or $15''$ make in the log. sine, etc. of an arc or angle.

§ f. To convert $33^\circ 10'$ into hours, etc.; in juxtaposition with 33° is $2h$; and in the left hand columns, $10'$ is opposite to $40s$; and above $40s$, is $12m$; then $33^\circ 10' = 2h 12m 40s$.

§ g. To convert $5h 20m 28s$ into degrees, etc.; $5h$ is found at the bottom, and $20m 28s$ in the right hand columns. In juxtaposition with $5h$ is 80° , and above $20m$, but opposite to $28s$, stands $7'$ in the (*'*) column; then $5h 20m 28s = 80^\circ 7'$.

§ 349. To find the time of sunrise in lat. $40^\circ N.$, the declination being 20° south.

§ a. The triangle of the problem proposed, is the quadrantal triangle $A P c$ with the same data, and a similar } Plate 2.
process, which were used (§ 338. § b.) for finding the amplitude;

$$\S b. P c, \text{ or } P. D. = 110^\circ \cot. = 9.561066$$

$$P A, \text{ or } \text{Co-lat.} = 50^\circ \cot. = 9.923813$$

$$\text{Cos. } A P c \text{ (§ 348. § d.)} = 9.484879 = 4h 48m 52s.$$

§ c. $4h 48m 52s \propto 12h$ (§ 341.) $= 7h 11m 8s$, the apparent time of day at sunrise.

§ d. If the declination were $20^\circ N.$, $4h 48m 52s$ (§ 340.) would be the apparent time of day at sunrise, and $7h 11m 8s$ at sunset.

§ e. The angle $c P E$, (§ 263. § b.) is the complement of the horary angle $A P c$.

§ f. And being the difference between sunrise and 6 o'clock, (§ 262.) it is the ascensional difference.

§ 350. To find the ascensional difference, we have only to take

Plate 2. $\left\{ \begin{array}{l} \text{the sine, where cos. was taken above, in the operation} \\ (\S 349. \S b.) \text{ for finding the time between sunrise and noon.} \end{array} \right.$

§ a. P. Dist. = 110° co-tang. = 9.561066
 Co-lat. = 50° co-tang. = 9.923813

$$\text{Sin. } c P E = 9.484879 = 1h 11m 8s.$$

§ b. 1h 11m 8s (§ 349. § c.) is the complement of 4h 48m 52s, and (§ 349. § f.) is the ascensional difference.

§ 351. By using the trigonometric functions of the complements, the calculation (§ 350. § a.) is rendered more convenient for practice.

§ a. Dec. 20° tang. = 9.561066
 Lat. 40° tang. = 9.923813

$$\text{Sin. } c P E = 9.484879 = 1h 11m 8s.$$

Whence the rule for finding ascensional diff.

§ b. The *product* of the tang. of the lat. and dec., is the sine of the ascensional difference.

§ 352. When the latitude and declination are of the *same* name, the ascensional difference *subtracted* from 6 o'clock, gives the apparent time of day at sunrise; and,

§ 353. *Added* to 6 o'clock, when they are of *different* names, it gives the apparent time of sunrise.

§ a. Hence Table VIII. also shows the apparent time at *sunrise*, when the lat. and dec. are of the *same* name; and the apparent time of *sunset* when the lat. and dec. are of *different* names; for (§ 343.) the time at sunrise subtracted from 12h shows nearly the time at sunset. In the Table, the time from sunrise to 12 o'clock is supposed to be always equal to the time from noon till sunset.

§ 354. The length of the days and nights may be determined by means of the problem (§ 351.); for doubling the time from sunrise till noon, gives the time that the sun remains above the horizon; and,

§ a. Subtracting this time from 24, gives the length of the night, or the time that the sun is below the horizon.

THE PLANETS, MOON, ETC.

§ 355. A satellite is a body that revolves around a planet, as the centre of one of its motions, and also revolves with that planet around the sun.

§ a. The moon is the earth's satellite.

§ 356. Mercury and Venus, (§ 230.) are called *inferior* planets, because their orbits are between the earth's and the sun.

§ 357. Mars, Jupiter, Saturn, Herschel, etc., (§ 230. § a.) are *superior* planets.

§ a. Their orbits are further than the earth's is, from the sun.

§ 358. Jupiter has four satellites, which, revolving with different radii around its centre, accompany it in its revolutions around the sun.

§ a. As they revolve in their orbits from west to east, they pass behind Jupiter; and as they pass in front of it, they cross its disc. From the frequency of these passages, (called eclipses,) and the celerity of the motion of the satellites, their eclipses with Jupiter furnish excellent opportunities for ascertaining longitude, when the observer is on shore. The satellites cannot be seen with the naked eye, and are of no use for ascertaining longitude at sea, because telescopes for observing their immersions and emersions, cannot be used on ship-board on account of the motion of the vessel.

§ b. When Jupiter is nearly in conjunction with the sun, the satellites cannot be seen at all.

§ 359. The moon moves from west to east in its orbit, around the earth.

§ a. The average, or mean time in which the moon performs one revolution in its orbit, is $27d\ 7h\ 43m\ 5s$; this is called a *periodical month*.

§ b. The moon performs, in nearly the same time, one revolution on its axis, by which means the same side of the moon is always presented towards the earth.

§ 360. At the same time every day, the moon is seen about $13^{\circ}\ 10'\ 30''$ further to the east than it was the day preceding.

§ a. This gives the moon an hourly motion with regard to the sun, or fixed stars, of about $32'\ 56''$; and makes the lunar, about $49m$ longer than the solar day.

§ 361. When the moon presents a round, illuminated disc towards the earth, the latter is between the sun and the moon.

§ a. The moon is then about the point in its orbit at which it attains its greatest distance from the sun; and is said to be in *opposition* with the sun.

§ 362. At new moon, the moon is nearest to the sun, being between it and the earth.

§ a. The moon is then in *conjunction* with the sun.

§ b. The interval between two consecutive new or full moons is about $29d\ 12\frac{1}{2}h$.

§ c. This interval of time is called a *synodical month*, to distinguish it from a *periodical month* (§ 359. § a.).

§ d. The difference between the *periodical* and *synodical* months is caused by the motion of the earth in its orbit around the sun; for by the time the moon returns in its orbit to the same place in the heavens, and is again in the position with regard to the fixed stars, which it had occupied $27d\ 7h\ 43m\ 5s$ (§ 359. § a.) before, the earth will have advanced in its own orbit, and the moon will have to continue on in its orbit, about $2d$ and $5h$, before it overtakes the earth, and occupies the position, with regard to the earth and the sun, and presents the same appearance, which it did when the two months commenced together.

§ 363. The light of the moon is reflected from the sun.

§ *a*. And the different appearances of the moon's figure, called *phases*, are caused by the enlightened part of the moon being turned more or less towards the earth.

§ 364. When the moon, passing between the earth and the sun, crosses the straight line which joins their centres, the whole of its unenlightened side is turned towards the earth; then it eclipses the sun, and is seen, like an opaque circular plane, passing over the disc of the sun.

§ 365. If the plane of the moon's orbit coincided with the plane of the ecliptic, the sun would be eclipsed at every new moon.

§ *a*. But as these planes are inclined at an angle of several degrees towards each other, the new moon passes sometimes below, and sometimes above, the line of vision from the earth to the sun.

§ 366. At full moon, when the earth intercepts the line of vision from the sun to the moon, the latter passes into the shadow of the earth caused by the sun, and is eclipsed.

§ *a*. The moon is not eclipsed at every opposition, on account (§ 365. § *a*.) of the inclination of the plane of its orbit to that of the ecliptic.

§ 367. *Syzygies* are the points in which the moon is, (§ 361. § *a*. & § 362. § *a*.), at opposition and conjunction. Consequently, the moon is in syzygy at every new or full moon.

§ 368. An eclipse of the sun or moon takes place only when the moon is in syzygy, and at, or near, one of its nodes (§ 231.).

§ 369. When the moon is half way between the syzygies, it appears as a semicircular plane, with the round part turned towards the sun.

§ *a*. Only half of the enlightened surface of the moon being seen from the earth, produces the appearance of a *half moon*.

§ *b*. In this position, the moon's angular distance from the sun is about 90°, and the moon is said to be in quadrature.

§ 370. A quadrature is marked (\square), opposition (δ), and conjunction (δ).

§ *a*. There are two quadratures; one between new and full moon, the other between full and new moon.

§ 371. *Disc* is the face of the sun, of a planet, or of a satellite.

§ 372. *Digit* is the twelfth part of the breadth of a disc.

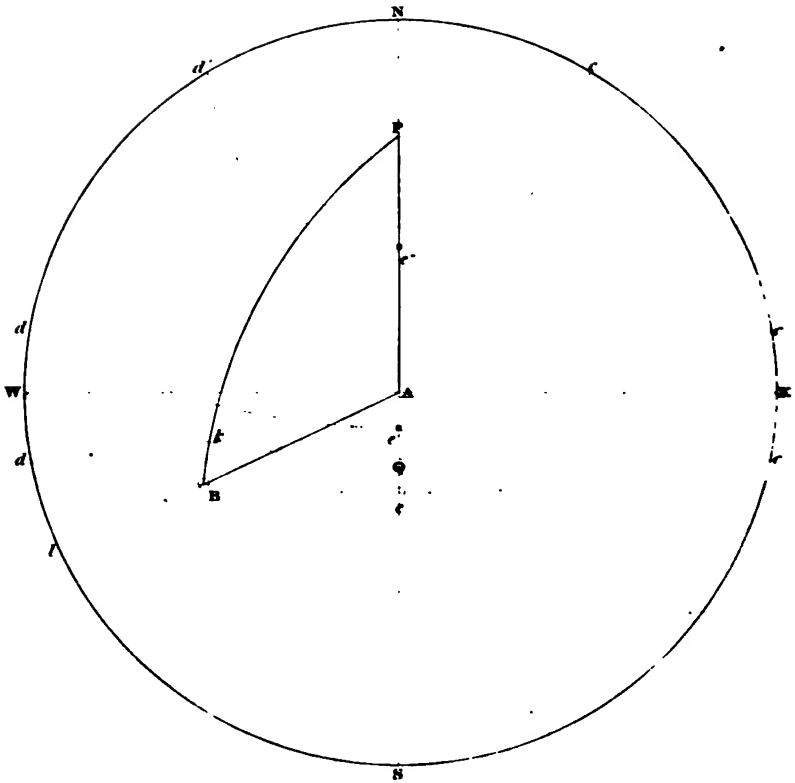
§ 373. *Apogee* is that point in the orbit of a body which is furthest from the earth.

§ 374. *Perigee* is that point in which, when a body is, it is nearest to the earth.

§ 375. *Aphelion* and *perihelion* are positions of a body with regard to the sun, similar to those of apogee and perigee with regard to the earth.

§ 376. The motion of light is progressive.

§ *a*. During the time which a ray of light requires to come from the sun to the earth, the latter advances in its orbit nearly 21''; this



causes the sun to be seen about that much always in the rear of its actual position in longitude.

§ 377. The sun's *aberration*, is the difference between the sun's longitude, at the time of the emanation of a ray of light from it, and its longitude, at the time when this ray reaches the earth.

§ a. The amount of aberration must always be added to the sun's *apparent* longitude, in order to obtain the *true* longitude of the sun.

§ b. The longitude of the sun, given in the ephemeris, is *apparent*.

§ 378. *Radius vector* is a line supposed to join the centres of a planet and the sun.

§ a. The logarithm of "the radius vector of the earth" is the logarithmic value of the earth's radius vector; the semi-axis major of the earth's orbit being taken as the base, or unity, in the logarithmic scale.

TIME OF DAY.

§ 379. In the triangle P A B, are comprised the elements necessary to the operation of finding, by trigonometrical calculation, the apparent time of day. } Plate 3.

§ a. B P A, (§ 263.), is an horary angle, and if any three parts of the triangle in which it is, be given, the value of B P A is at once determinable.

§ 381. The primitive data, in the problem for finding the time of day by an observation, are usually the latitude of the observer, and the altitude and declination of the body under observation.

§ a. The complements of which form the triangle (B A P) of the problem.

§ 382. By inspecting the triangle of the problem proposed, it appears evident, that the horary angle (A P B) expresses the interval of time that elapses between the taking of the observation, and the transit of the body, (say the sun), across the meridian of the observer.

§ a. The time thus deduced is *apparent* time.

§ 383. If the observation be made in the forenoon, the value of the horary angle must be subtracted from 12h, in order to find the apparent time of day.

§ a. But if the time be P. M. the value of the horary angle expresses the apparent time of day when the altitude was measured.

§ 384. This problem, coming under Case V., (§ 201.), may be solved by either of the methods of solution there shown.

§ a. But as it is a rule, in the application of theory to practice, (and one too, which is highly important in navigation,) to combine facility of design with readiness of execution, a solution of the problem after the formula, § f. (§ 201.), will be given, in order to show thereby a process of solution less circuitous; by which process the problem under consideration may be solved.

§ 385. In lat. 20° N.; the observed altitude of the sun's lower

limb, was $29^{\circ} 29' 50''$; and its declination was $10^{\circ} 20'$ south. To

Plate 3. $\left\{ \begin{array}{l} \text{find the time of day, the observation being made P. M.} \\ \text{\$ a. First to find the true altitude of the sun's centre.} \end{array} \right.$

$\text{\$ b. Obs. alt. Sun's L. L.} = 29^{\circ} 29' 50''$
 $\text{Sun's semi-diam.} = 15' 47''$

$\text{Sun's app. alt.} = 29^{\circ} 45' 37''$ ($\text{\$ 283. \$ a.}$)
 $\text{Refraction (\$ 284.)} = 1' 37''$ (Table IX.)
 $\text{Dip} = 4'$ ($\text{\$ 275. \$ a.}$)

$\text{Sun's true alt.} = 29^{\circ} 40'$ ($\text{\$ 283. \$ b.}$)

$\text{\$ c. To find the time of day.}$

$\text{B A, or Z, Dist.} = 60^{\circ} 20'$
 $\text{P A, or Co-lat.} = 70^{\circ} 00'$ $\text{co-sec.} = 0.027014$ ($\text{\$ 201. \$ f.}$)
 $\text{P B, or P. Dist.} = 100^{\circ} 20'$ $\text{co-sec.} = 0.007102$

$2)230^{\circ} 40'$

$\frac{1}{2} \text{ sum} = 115^{\circ} 20'$ $\text{sin.} = 9.956089$
 $(\frac{1}{2} \text{ sum} \infty \text{ Z. Dist.}) = 55^{\circ} 00'$ $\text{sin.} = 9.913364$

$2)19.903569$

$\text{Cos. } \frac{1}{2} \text{ A P B} = 9.951784 = 1h 46m \frac{1}{2}s$

2

$\text{Horary angle A P B} = 3h 32m 1s$

This time ($\text{\$ 383. \$ a.}$) is P. M.

$\text{\$ 386. The data of the problem may be operated upon, more directly, by using the lat. and alt., instead of their complements.}$

$\text{\$ a. Alt.} = 29^{\circ} 40'$
 $\text{Lat.} = 20^{\circ} 00'$ $\text{sec.} = 0.027014$
 $\text{P. D.} = 100^{\circ} 20'$ $\text{co-sec.} = 0.007102$

$2)150^{\circ} 00'$

$\frac{1}{2} \text{ sum} = 75^{\circ} 00'$ $\text{cos.} = 9.412996$
 $(\frac{1}{2} \text{ sum} \infty \text{ Alt.}) = 45^{\circ} 20'$ $\text{sin.} = 9.851997$

$2)19.299109$

$\text{Sin. } \frac{1}{2} \text{ A P B} = 9.649554 = 1h 46m \frac{1}{2}s$

2

$\text{Horary angle A P B} = 3h 32m 1s$

§ 387. Whence the general rule for practice.

Take the difference between the true alt., and half the sum of the lat., P. dist., and alt.; then the *product* of the sine of this difference, and cos. of said half sum, *multiplied* by the *product* of sec. of lat., and co-sec. of P. dist., is the sine of half the horary angle.

§ 388. The time thus found being apparent time, may be converted into mean time (§ 217.), by applying to it the equation of time, according to the precept given with the equation of time in the Nautical Almanac.

§ a. App. time (§ 385. § c.) 3h 32m 1s P. M.
Equation of time — 13m 50s

Mean time = 3h 18m 11s

§ 389. By comparing the mean time thus found, with the time shown by a chronometer, or other time-piece, when the altitude was measured, the error of the time-piece is obtained.

§ a. If this time-piece be regulated for a prime meridian, which is the case with chronometers; the difference between the true chronometrical, and the mean, time found by observation, expresses in *time* the difference in longitude between said prime meridian and the place of observation.

§ b. This difference of time being converted (§ 348. § g.) into degrees (°), minutes (′), etc., expresses the longitude of the observer.

LONGITUDE BY CHRONOMETER.

§ 390. The whole doctrine of determining longitude, consists in knowing the time of day at any two places at the same instant.

§ a. This is what is determined by every practicable method of finding longitude; whether it be by means of rockets, eclipses, occultations, lunar observations, or chronometers.

§ 391. The time of day at the prime meridian when an eclipse, occultation, or distance, occurs, is set down in the ephemeris; and the time of day at any other place when the same eclipse, etc., occurs, is known, either by well-regulated time-pieces, or by observations.

§ a. And the difference between these times, (§ 389. § b.), gives the longitude of the observer.

§ b. The longitude is west, when the *prime meridian* (Greenwich, § 252. § c.) time is in advance of the time of day at the observer.

§ c. But if the observer's time of day be in advance of the Greenwich time, his longitude is east; for it is evident that the sun must cross his meridian, before it does that of Greenwich, and consequently, that Greenwich must be to the westward.

§ 392. Chronometers are generally regulated so as to show the mean time of day at Greenwich.

§ a. But they are subject to variations from change of tempera-

ture, etc., and from other causes, and cannot be regulated so as to show at all times, the true time of day at Greenwich, or at any other prime meridian.

§ 393. The difference between the time shown by the face of a chronometer, and the mean time of day at Greenwich, is called "the error of chronometer."

§ *a.* The daily variation of this error is "the rate of chronometer."

§ 394. The rate of chronometer is found by noting the difference between mean time, and the time shown by the face of the chronometer, and after several days, noting again the difference between the mean and the chronometer time.

§ *a.* The difference between these two *differences*, (called "comparisons"), shows the time which the chronometer has gained, or lost, from the first to the last comparison.

§ *b.* And the quotient of this gain or loss, by the number of days that elapsed between the two comparisons, is the daily gain or loss, or rate, of the chronometer.

§ 395. The error (§ 393.) being applied to the chronometer time, with the precept of *plus* or *minus*, according as the chronometer be slow or fast, of Greenwich time, gives the mean time of day at Greenwich.

§ *a.* And the *rate* of the chronometer being applied to the chronometer's error yesterday, with the precept +, or —, according as the *rate* is *increasing* or *decreasing*, gives the chronometer's error for to-day.

§ 396. Before the chronometer is rated, the mean time of day at Greenwich for rating it, is found by turning the longitude of the observer, (§ 347.) into time; and,

§ *a.* Adding this time to the observer's mean time of day, if he be in west long.

§ *b.* And subtracting it from his mean time of day, if he be in east longitude.

§ 397. Ten or twelve days previous to the sailing of the vessel, will generally serve for keeping her chronometer under comparison, in order to find its error, and ascertain its rate.

§ *a.* But after sailing, the rate should be compared, (and corrected if necessary), as often as opportunities for making comparisons occur.

§ 398. To find the error and ascertain the rate of a chronometer, the observer being in lat. $40^{\circ} 42' N.$ and long. $74^{\circ} 00' W.$

§ *a.* Aug. 4th, A. M., 1834.

Chro. $12h\ 58m.$ App. alt. Sun's L. L. $30^{\circ} 17' 20''$
Sun's semi-diam. = $15' 47''$

App. alt. Sun's centr. = $30^{\circ} 33' 7''$
Refraction - = $1' 37''$

Sun's Tr. alt. = $30^{\circ} 31' 30''$

§ b.

Sun's tr. alt. = 30° 31' 30'' (§ 387.)

Lat. = 40° 42' 00'' sec. = 0.120254

Sun's P. dist. = 72° 40' 30'' co-sec. = 0.020164

2)143° 54' 00''

$\frac{1}{2}$ sum = 71° 57' 00'' cos. = 9.491147

(Alt. ∞ $\frac{1}{2}$ sum) = 41° 25' 30'' sin. = 9.820621

2)19.452186

Sin. $\frac{1}{2}$ Horary angle = 9.726093 = 2h 8m 37s +

2

12h 00m 00s (§ 383.)
 Horary angle = 4h 17m 14s - = 4h 17m 14s

Equation 7h 42m 46s App. time
 + 5m 47s (§ 388.)

Chron. 7h 48m 33s Mean time, A. M.
 12h 58m 00s (§ 398. § a.)

5h 9m 27s 1st comparison, (§ 394.)

§ c. Aug. 15, A. M. Chron. 12h 34m 25s.

App. alt. Sun's L. L. 24° 2' 10''

Sun's semi-diam. 15' 49''

App. alt. Sun's centr. 24° 17' 59''

Refraction - 2' 9'' (Table IX.)

§ d. Sun's tr. alt. 24° 15' 50''

Lat. - 40° 42' 0'' sec. = 0.120254

Sun's P. dist. 75° 51' 10'' cosec. = 0.013376

2)140° 49' 00''

$\frac{1}{2}$ Sum - 70° 24' 30'' cos. = 9.525452

($\frac{1}{2}$ sum ∞ Alt.) 46° 8' 40'' sin. = 9.857989

2)19.517071

Sin. $\frac{1}{2}$ Horary angle = 9.758535 = 2h 19m 58 $\frac{1}{2}$ s

2

Horary angle = 4h 39m 57s

	12h 0m 0s	
Horary angle	4h 39m 57s	
	7h 20m 3s	App. time
Equation	+ 4m 15s	
	7h 24m 18s	Mean time, A. M.
Chro.	12h 34m 25s	
	5h 10m 7s	2d comparison.

§ e. 2d Compr. 5h 10m 7s
 1st do. 5h 9m 27s (§ 394. § a.)

Chro. gains 40s in 11 days.

§ f. $40s \div 11 = 3\frac{7}{11}s$ (§ 394. § b.) the daily gain, or rate.

§ g. Long. of the observer $74^\circ W.$ = 4h 56m
 Time of (§ d.) last. obs. - = 7h 24m 18s

(§ 396. § a.) Greenwich time do. = 12h 20m 18s
 Chro. (§ c.) - - = 12h 34m 25s

Error (§ 393.) of chro. Aug. 15th, = 14m 7s (*fast.*)

§ h. To find long. by chronometer;

Time per watch 3h P. M. = 15h 0m 0s
 do. chron. - = 11h 15m 22s

Diff. = 3h 44m 38s Chr. slow.

Time of obs. per W. 15h 2m 30s
 Chro. slow of W. 3h 44m 38s

Chro. time of obs. = 11h 17m 52s

Formula for calculation.

Sun's Alt.	31° 18' (corrected)	
Lat.	36° 19' sec.	=0.093796
P. D.	104° 9' co-sec.	=0.013381

$$\text{Sum} = 2)171^\circ 46'$$

$$\begin{aligned} \frac{1}{2} \text{ sum} &= 85^\circ 53' \cos. &= 8.856049 \\ \text{Rem'r.} &= 54^\circ 35' \sin. &= 9.911136 \quad (\text{Alt. } \propto \frac{1}{2} \text{ Sum.}) \end{aligned}$$

$$2)18.874362$$

$$\text{Sin. } \frac{1}{2} \text{ App. time} = 9.437181 = 1^h 3^m 31\frac{1}{2}s$$

2

$$\begin{aligned} \text{App. time of obs.} &= 2^h 7^m 3s \text{ P. M.} \\ \text{Equation} &\quad - \quad + 4^m 16s \end{aligned}$$

$$\text{Mean time of obs. } 2^h 11^m 19s \text{ P. M.}$$

$$\begin{aligned} \text{Time of obs. per chro.} & 11^h 17^m 52s \\ \text{Chro. slow of Gr. time} & 1^h 3^m 8s \end{aligned}$$

$$\begin{aligned} \text{Greenwich time} & 12^h 21^m 0s \\ \text{Time of obs.} & - 14^h 11^m 19s \end{aligned}$$

$$\text{Long. in time (§ 391. § c.) } 1^h 50^m 19s = 27^\circ 34' 45'' \text{ E.}$$

§ 399. By inspecting the triangle (P B A) of the problem for finding the apparent time of day, it becomes evident, that if, of the lat., time of day, dec., alt., and azimuth, of the sun, or any other body, any three be known, the two others are determinable. } Plate 3.

§ a. The azimuth gives the angle P A B; and the apparent time of day gives the angle A P B; the altitude (B I) determines the zenith distance A B; the latitude A Q, determines P A, the co-lat., and the declination k B, determines the polar distance P B.

LATTITUDE BY MERIDIAN ALTITUDES.

§ 400. The most common method of determining latitude at sea, is by means of an altitude of a celestial body, measured when the body is on the meridian.

§ a. Then the circle of the body's declination, coincides with the meridian (N A S) of the observer.

§ 401. Suppose the body be a star; the zenith distance (A e') (§ 282. § b.) expresses the number of degrees, etc., from (A) the zenith to the star.

§ a. And the declination Q e' (§ 255. § b.) gives in the same measure, the distance of the body from the equator.

Plate 3. $\left\{ \begin{array}{l} \S b. \text{ Wherefore the difference between the declination} \\ \text{ } (Q e') \text{ and the zenith distance } (A e') \text{ gives } A Q \text{ the latitude.} \end{array} \right.$

$\S 402.$ The sun being south of the observer, and its declination $10^{\circ} 20'$ south, its meridian altitude (corrected) was $59^{\circ} 40'$.

To find the latitude of the observer,

$\S a.$ The sun's altitude ($\S 282. \S a.$) is $e S$; $90^{\circ} - e S = A e$ ($\S 282. \S b.$) the zenith distance.

$\S b.$ The observer being at A , north of the sun, makes ($\S 281. \S b.$) the zenith dist., $A e$, north.

$\S c.$ The dec. $Q e$, being taken from the zenith dist. $A e$, leaves $A Q$, the required latitude.

$\S d.$ Sun's Z. D. ($\S a.$) = $30^{\circ} 20' N.$ ($\S b.$)

Sun's dec. = $10^{\circ} 20' S.$

Lat. = $20^{\circ} 00' N.$

$\S 403.$ Suppose the object to bear south, but its dec. to be $9^{\circ} 3' N.$ and its meridian alt. $79^{\circ} 3'$.

To find the latitude,

$\S a.$ The stars alt. is $e' S$; $A S - e' S = A e'$ the *'s zenith dist.

$\S b.$ The dec. $Q e'$ being added to $e' A$ the zenith dist., makes $A Q$, which is the latitude.

$\S c.$ *'s Z. D. = $10^{\circ} 57' N.$

*'s dec. = $9^{\circ} 3' N.$

Lat. = $20^{\circ} 00' N.$

$\S 404.$ Suppose the star to be north of the observer, its declination $60^{\circ} 5' N.$, and its meridian alt. = $49^{\circ} 55'$.

To find the lat. of the observer,

$\S a.$ $N e''$ is the *'s alt. The zenith dist. ($\S 281. \S c.$) is south; $N A - N e'' = A e''$, the *'s zenith dist.

$\S b.$ The dec. $Q e'' - A e''$ (the zenith dist.), gives $A Q$, the lat.

$\S c.$ *'s dec. = $60^{\circ} 5' N.$

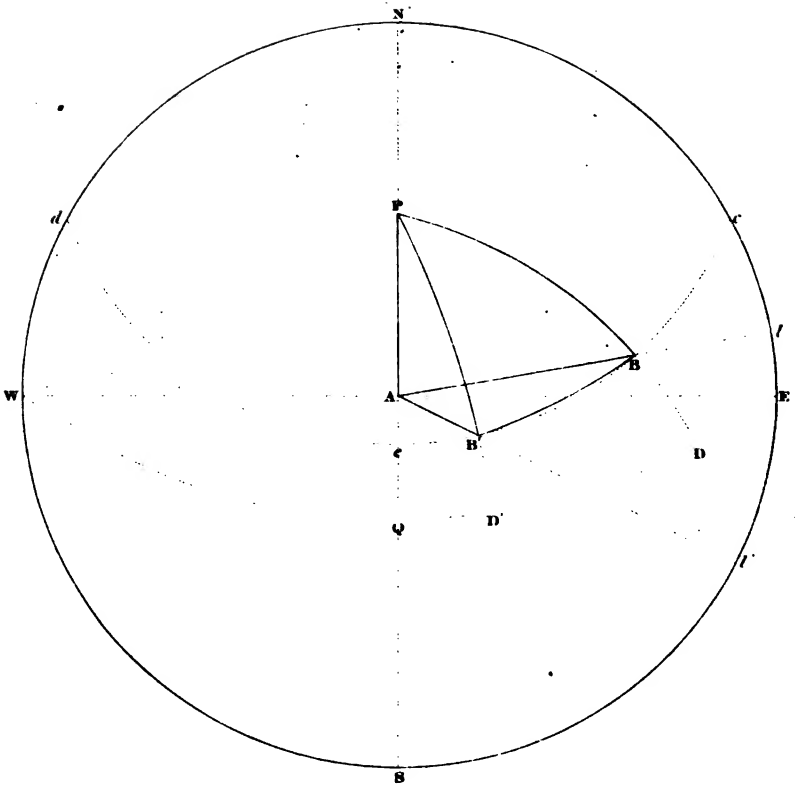
*'s Z. D. = $40^{\circ} 5' S.$

Lat. = $20^{\circ} 0' N.$

$\S 405.$ Wherefore, when the dec., and the meridian zenith dist. are both north, or both south, their *sum* is the latitude.

$\S 406.$ And, when one is north, and the other south, their *difference* is the latitude.

$\S 407.$ When the body is in the zenith, there is no zenith dist., and the dec. is the lat.



LATITUDE BY DOUBLE ALTITUDES.

§ 408. The sun, (and every other celestial object), is frequently obscured by clouds, so that a meridian alt. cannot be measured.

§ a. Under such circumstances, data sufficient for determining the lat. may be obtained by taking at two different hours of the day the sun's alt., and noting the time that elapses between the two observations.

§ 409. The time, expressed in degrees, or hours, etc. (§ 348. § b.) constitutes, with the zenith and polar distances, the required data.

§ 410. This method of finding the lat. is called "by *Double Altitudes*."

§ a. The process of calculation is tedious, but the result will give the latitude as correctly as it can be obtained through any other process of observation and calculation.

§ 412. For determining the lat. by "double altitudes," }
 B D represents the declination, and B l the alt. of the sun, } Plate 4.
 at the first observation; and B' D', and B' l' the dec. and alt., at the second observation.

§ a. A P B is the sun's horary angle at the first, and A P B', its horary angle at the second, observation.

§ b. The difference (B' P B) between these two angles, is the angular value of the time that elapses between the observations.

§ c. B B' is the arc of the great circle, which passes through the points, in which the centre of the sun was at each observation.

§ d. The other circles, arcs and angles, of the Fig., are explained under § 317. (§ a.).

§ 413. The problem of "double altitudes" involves three triangles (B' P B, A B B', & A P B') in the process of solution.

§ a. The side B' B is common to the first and second triangles; P B' to the first and third; and A B' to the second and third.

§ 414. The parts that are given in the first triangle (B' P B), are the two sides, P B and P B' (§ 412.), which are the P. dist. (§ 256.) at the first, and the second observation; and (§ 412. § b.) their contained angle B' P B = Elapsed Time.

§ a. And the parts required are, the third side B' B, and the angle P B' B.

§ 415. The value of P B' B and B' B being determined by calculation, the parts that are then known in the second triangle A B B', are B A and B' A (§ 412.); which are the zenith dist. (§ 281.) of the sun at the first and the second observation; and the common side B' B.

§ a. The part here required, is the angle A B' B.

§ 416. The difference between the angles A B' B, and P B' B (§ 414. § a.) gives the value of the angle A B' P, in the third triangle A P B'.

§ 417. Then in the third triangle, the two sides A B', P B' and their contained angle A B' P, are the given parts; and P A is the part required, and the co-lat.

§ 418. In north latitude, at 7 A. M. by a watch.

Sun's true alt. $24^{\circ} 58' 12''$

Sun's dec. $22^{\circ} 25' 00''$ N.

Plate 4. $\left\{ \begin{array}{l} \text{§ a. And at the } 10^h 15^m \text{ by the same watch,} \\ \text{Sun's true alt. } 62^{\circ} 49' 20'' \\ \text{Sun's dec. } 22^{\circ} 24' 00'' \text{ N.} \end{array} \right.$

§ b. The elapsed time is $3^h 15^m$.

§ 419. PB (1st P. D.) = $67^{\circ} 35'$

PB' (2d P. D.) = $67^{\circ} 36'$

AB (1st Z. D.) = $65^{\circ} 1' 48''$

AB' (2d Z. D.) = $27^{\circ} 10' 40''$

$B'PB$ (E. time) = $3^h 15^m$. (§ 409.)

§ 420. To find the side $B'B$ and angle $P'B'B$ (§ 414. § a.) in the first triangle $B'PB$.

§ a. 1st. To find the value of $B'B$ (§ 200. § s. & § t.).

$B'PB$ (E. T.) = $3^h 15^m$ cos. = 9.819113 (§ 348. § c.)

PB' (2d P. D.) = $67^{\circ} 36'$ tang. = 0.384923

Tang. aux. $a = 0.204036 = 57^{\circ} 59' 23''$

$PB = 67^{\circ} 35'$

$(a \cap PB) = 9^{\circ} 35' 37''$

PB' (2d P. D.) = $67^{\circ} 36'$ cos. = 9.581005

Auxl. $a = 57^{\circ} 59' 23''$ sec. = 0.275666

$(a \cap 1st P. D.) = 9^{\circ} 35' 37''$ cos. = 9.993884

Cos. $B'B = 9.850555 = 44^{\circ} 51' 30''$

§ b. To find the value of $P'B'B$ (§ 144.).

$B'B$ = $44^{\circ} 51' 30''$ co-sec. = 0.151591

$B'PB$ (E. T.) = $3^h 15^m$ sin. = 9.876125

PB (1st P. D.) = $67^{\circ} 35'$ sin. = 9.965876

Sin $P'B'B = 9.993592 = 80^{\circ} 10' 53''$

§ c. In the second triangle, to find the value of the an- } Plate 4.
gle A B' B (§ 201. § d.).

A B (1st Z. D.) = 65° 1' 48''
A B' (2d Z. D.) = 27° 10' 40'' co-sec. = 0.340319
B' B (§ a.) = 44° 51' 30'' co-sec. = 0.151591

2) 137° 3' 58''

½ Sum = 68° 31' 59'' sin. = 9.968776
(½ S. ∞ 1st Z. D.) = 3° 30' 11'' sin. = 8.786053

2) 19.246739

Cos. $\frac{A B' B}{2}$ = 9.623369 = 65° 9' 29''

2

A B' B = 130° 18' 58''

P B' B (§ b.) = 80° 10' 53''

A B' P = 50° 8' 5''

§ d. In the third triangle, to find the value of the third side A P ;
or to find the latitude (§ 200. § s. & § t.)

A B' P = 50° 8' 5'' cos. = 9.806848

P B (2d P. D.) = 67° 36' tang. = 0.884923

Tang. auxl. a = 0.191771 = 57° 15' 29''

A B' = 27° 10' 40''

(∞ A B') = 30° 4' 49''

P B' (2d P. D.) = 67° 36' cos. = 9.581005

Auxl. a = 57° 15' 29'' sec. = 0.266918

(∞ 2d Z. D.) = 30° 4' 49'' cos. = 9.937178

Cos. A P = 9.785101 sin. = lat. 37° 33' 59'' N

§ 421. In the example last quoted, the two observations are taken at the same place ; but in practice, this cannot always be done, especially at sea, when the vessel is changing her place during the time that elapses between the taking of the observations.

§ a. Therefore the 1st altitude must be corrected, in order to know what it would have been, had it been taken at the same time per watch, but at the place where the 2d obs. was made.

§ 422. When the first observation is being made, note the bearing of the sun and the course of the ship ; and after the second observation has been taken, find in a traverse table (Table IV.), the

course and distance from the place where the first, to that where the second, observation was taken.

§ a. The angle, which this distance makes with the bearing of the sun, being used in the traverse table, as a course, the correction which is to be applied to the first alt., is found under this course, and opposite to said distance, and in the column marked "*d. lat.*"

§ 423. If this angle be less than 90° , the correction thus found is additive.

§ 424. If it be greater than 90° , its supplement is the course; and,

§ a. The correction is subtractive.

§ 425. If the ship sail on the line of the sun's bearing, the distance sailed in miles, is the correction in minutes ($'$) of a degree.

§ a. It is additive when the ship sails towards the sun; and,

§ b. Subtractive when she sails from the sun.

§ 426. The corrections are found as geographical miles, but are to be applied to the altitude as minutes ($'$) of a degree.

§ 427. When the angle between the bearing of the sun, and the course which the ship makes, is 90° , there is no correction.

§ 428. The corrections thus obtained and applied, do not show *exactly* what the 1st altitude would have been at the place of the second observation; but the approximation is sufficiently close to answer the purposes of common navigation.

§ 429. The time that elapses between the taking of the observations, should not be less than 30*m*, or more than 6*h*, if avoidable, especially if the observer be at sea.

§ a. If the body under observation do not change much in declination during the "elapsed time," the *mean* of its declination at the two observations may be used as the body's declination at each observation.

§ b. This "*mean*" declination is found by taking out the declination for half of the "elapsed time," *plus* the time of the first observation.

Plate 4. } § c. When the *mean* declination is used, $PB = PB'$; and the first triangle $B'PB$ (§ 140.) is isosceles.

§ 430. The latitude at the time and place of the first observation, may be found, if required, by making APB the third triangle, and using the angles PBB' , ABB' and ABP , instead of $PB'B$, $AB'B$ and $AB'P$ (§ 420. § b. & § c.).

§ a. But generally, in navigation, the latitude of the time present, is most desirable to be known; and usually, when the problem is solved, the vessel is nearer to the place of the second, than she is to the place of the first observation.

§ b. A formula for practice in finding the latitude at sea by "double altitudes;" the mean declination being used. Under the example § 418.

Sun's mean dec.	= $22^\circ 24' 30''$,	P. D.	= $67^\circ 35' 30''$
Sun's 1st alt.	= $24^\circ 58' 12''$,	1st Z. D.	= $65^\circ 1' 48''$
Sun's 2d alt.	= $62^\circ 49' 20''$,	2d Z. D.	= $27^\circ 10' 40''$
E. time	=	3 <i>h</i> 15 <i>m</i>	

§ c. The first triangle (§ 429. § c.) is isosceles; wherefore (§ 200. § z a., § z c. & § z f.)

P. D. = 67° 35' 30'' sin. = 9.965903 sec. = 0.418842
 (½ E. time) = 1h 37m 30s sin. = 9.615642 cot. = 0.343812

Arc $\frac{B'B}{2}$ = 22° 25' 46'' Sin. = 9.581545
 2 1st Ang. 80° 12' 2'' Tang. = 0.762654

Arc B'B = 44° 51' 32'' co-sec. = 0.151587 (§ 420. § c.)

2d Z. D. = 27° 10' 40'' co-sec. = 0.340319

1st " = 65° 1' 48''

2) 137° 4' 0''

½ S. = 68° 32' 0'' sin. = 9.968777

(½ S. Z. D.) 3° 30' 12'' sin. = 8.786087

2) 19.246770

Cos. ½ 2d Angle = 9.623385 = 65° 9' 25''

2

2d Angle = 130° 18' 50''

1st " = 80° 12' 2''

3d " = 50° 6' 48''

½ (3d Angle) = 25° 3' 24'' sin. × 2 = 19.253736 (§ 200. § z f.)

P. D. = 67° 35' 30'' sin. = 9.965903

2d Z. D. = 27° 10' 40'' sin. = 9.659681

2) 38.879320

(P. D. = 2d Z. D.) = 40° 24' 50'' 19.439660

$\frac{P. D. \times 2d Z. D.}{2}$ = 20° 12' 25'' co-sec. = 0.461662

Tang. a = 9.901322 = 38° 32' 46''

a. co-sec. = 0.205412

Sin. ½ co-lat. = 9.645072 = 26° 12' 32''

2

Lat. = 37° 34' 56'' or co-lat. = 52° 25' 4''

§ d. The result by this method differs 57'' from the truth. Like every other trigonometrical calculation, in which true data are not

operated upon, it brings an error into the result; but the result is an approximation, which oscillates about the truth, and within limits that depend upon the correctness of the assumed data.

§ *c.* Some navigators solve the problem of "double altitudes" by the method proposed by a Mr. Douwes; but the correctness of the result by his method, depends upon repeated calculations, or the proximity of a *supposed*, to the correct, latitude.*

§ 431. Latitude by "double altitudes" may also be determined by taking, at the same instant, the altitudes of any two bodies, whose right ascension and declination are given.

§ 432. The method of solving this problem consists in a process of operation, precisely similar to that (§ 420.) for finding the latitude by means of two altitudes of the same body, taken at different times.

Plate 4. { § 433. The difference in right ascension of the two bodies gives an angle, which corresponds to the angular value (B' P B) of the "elapsed time," between the two observations.

§ 434. When the difference in right ascension of the two bodies, exceeds 180° or $12h$, the greater right ascension, subtracted from the less *plus* $24h$, gives the angle corresponding to B' P B.

§ 435. In this method of finding latitude, the altitude of the moon, unless the Greenwich time be known, cannot be used with certainty of success. For the moon's variation in declination and right ascension, is so rapid, that unless the Greenwich time of day, when the observations are made, be known, the proper value of the angle corresponding to B' P B, cannot be obtained.

§ 436. The stars afford the greatest facilities for the application of this problem to practice.

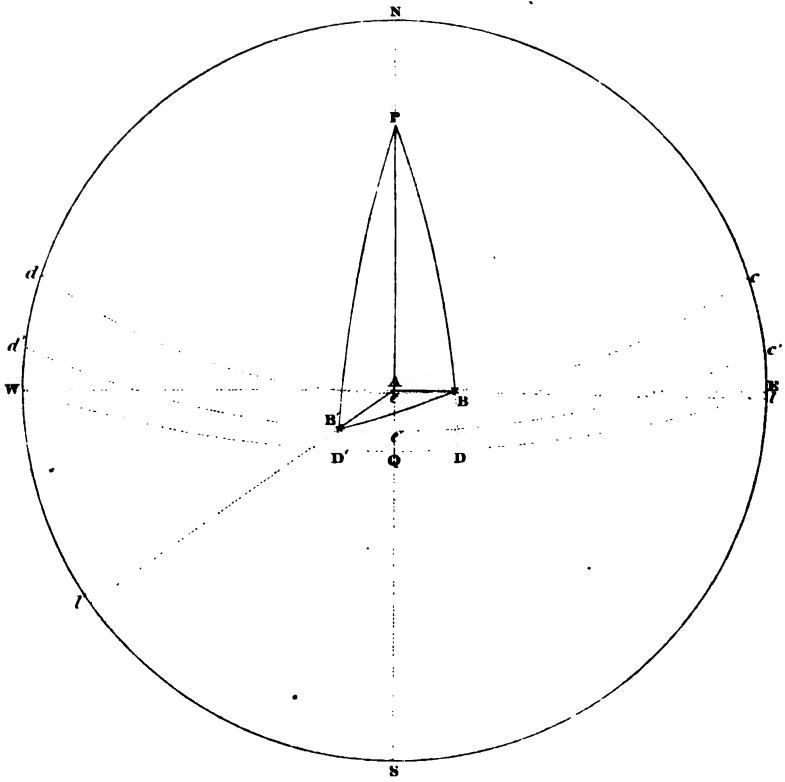
§ *a.* Their variation in declination and right ascension, being for the most part, not very rapid; consequently when the horizon is well defined, the data requisite to the solution of the problem, can be obtained by means of the stars, with sufficient accuracy.

§ 437. In north lat. June 2, 1834, Jupiter being east, and Mars west, of the meridian; their altitudes were taken at the same instant;

J.'s alt.	71°	49'	To find the latitude.
" dec.	18°	0' 40" N	
" rt. asn.	3h	27m 36s	
M.'s "	1h	9m 20s	
" dec.	5°	54' 20" N	
" alt.	70°	29' 12"	

Plate 5. { § *a.* Difference in right ascension is $2h\ 18m\ 16s$.
 { § 438. B I and B D are Jupiter's altitude and declina-

* Mr. Douwes' method is both supported and questioned by high authority. Bowditch has adopted it in his PRACTICAL NAVIGATOR. And there is a paper from M. Delambre, showing that the result by this method is not always an approximation, but that, in some cases, if repetitions be made, the result will recede from the true latitude.



tion; and $B' P'$, and $B' D'$ are Mars' alt. and dec.; and $B' P B$ } Plate 5.
 is the angular value of their difference in right ascension. }

§ 440. $B P B'$ is the first triangle in the problem, that is brought under calculation; $B' B$ and $B' B P$, are the required parts.

§ a. $B' A B$ is the second triangle, and $A B B'$ is the part in it, that is required;

§ b. $A B B' \propto B' B P = A B P$.

§ c. $A P B$ is the third triangle, and $A P$ is the part required to complete the problem.

- § 441. $P B$ (J.'s P. D.) = $71^\circ 59' 20''$
- $P B'$ (M.'s P. D.) = $84^\circ 5' 40''$
- $B' P B$ (diff. R. A.) = $2h 18m 16s$
- $A B$ (J.'s Z. D.) = $18^\circ 11' 0''$
- $A B'$ (M.'s Z. D.) = $19^\circ 30' 48''$

§ a. To find the value of $B' B$ (§ 200. § s. & § t.)

$B' P B$ (diff. R. A.) $2h 18m 16s$ cos. = 0.915646

$P B$ (J.'s P. D.) = $71^\circ 59' 20''$ tang. = 0.487937

Tang. auxl. $a = 0.403583 = 68^\circ 27' 15''$

M.'s P. D. = $84^\circ 5' 40''$

M.'s P. D. $\propto a = 15^\circ 38' 25''$

$P B$ (J.'s P. D.) = $71^\circ 59' 20''$ cos. = 0.490243

Auxl. $a = 68^\circ 27' 15''$ sec. = 0.435044

($a \propto P B'$ (M.'s P. D.) = $15^\circ 38' 25''$ cos. = 0.983614

Cos. $B' B = 0.908901 = 35^\circ 49' 42''$

§ b. To find the value of the angle $P B B'$ (§ 420. § b.)

$B B'$ $30^\circ 49' 42''$ co-sec. = 0.232578

$B P B'$ (diff. R. A.) = $7h 18m 16s$ sin. = 0.753862

$P B'$ (M.'s P. D.) = $84^\circ 5' 40''$ sin. = 0.997689

Sin. $P B B' = 0.984129 = 105^\circ 23' 52''$

Plate 5. | § c. To find the value of A B B' (§ 201. § f.)

A B' (M.'s Z.D.)	= 19° 30' 48"	
A B (J.'s Z. D.)	= 18° 11' 00"	cosec. = 0.505764
B' B	= 35° 49' 42"	" = 0.232578

$$\underline{2) 73^{\circ} 31' 30''}$$

$\frac{1}{2}$ Sum	= 36° 45' 45"	sin. = 9.777063
($\frac{1}{2}$ S α A B', M.'s Z.D.)	= 17° 14' 57"	" = 9.472066

$$\underline{2) 19.987471}$$

$$\text{Cos. } \frac{A B B'}{2} = \underline{9.993735} = 9^{\circ} 42' 30''$$

2

$$A B B' = \underline{19^{\circ} 25' 00''}$$

$$P B B' = \underline{106^{\circ} 23' 52''}$$

$$A B P = \underline{85^{\circ} 58' 52''}$$

§ d. To find the value of A P or co-lat. (§ 200. § u.)

$$A B P = 85^{\circ} 58' 52'' \quad \text{cos.} = 8.845627$$

$$P B = 71^{\circ} 59' 20'' \quad \text{tang.} = 0.487937$$

$$\text{Tang. Auxl. } a = 9.333564 = 12^{\circ} 9' 52''$$

$$A B (J.'s Z. D.) = 18^{\circ} 11' 06''$$

$$\underline{(\alpha \alpha J.'s Z. D.) = 6^{\circ} 01' 8''}$$

$$P B (J.'s P. D.) = 71^{\circ} 59' 20'' \quad \text{cos.} = 9.490243$$

$$\text{Auxl. } a = 12^{\circ} 9' 52'' \quad \text{sec.} = 0.009864$$

$$\underline{(\alpha \alpha A B, J.'s Z.D.) = 6^{\circ} 1' 8'' \quad \text{cos.} = 9.997599}$$

$$\text{Cos. } A P (\text{co-lat.}) = \underline{9.497706 S.} = \text{lat. } 18^{\circ} 30' 4'' \text{N.}$$

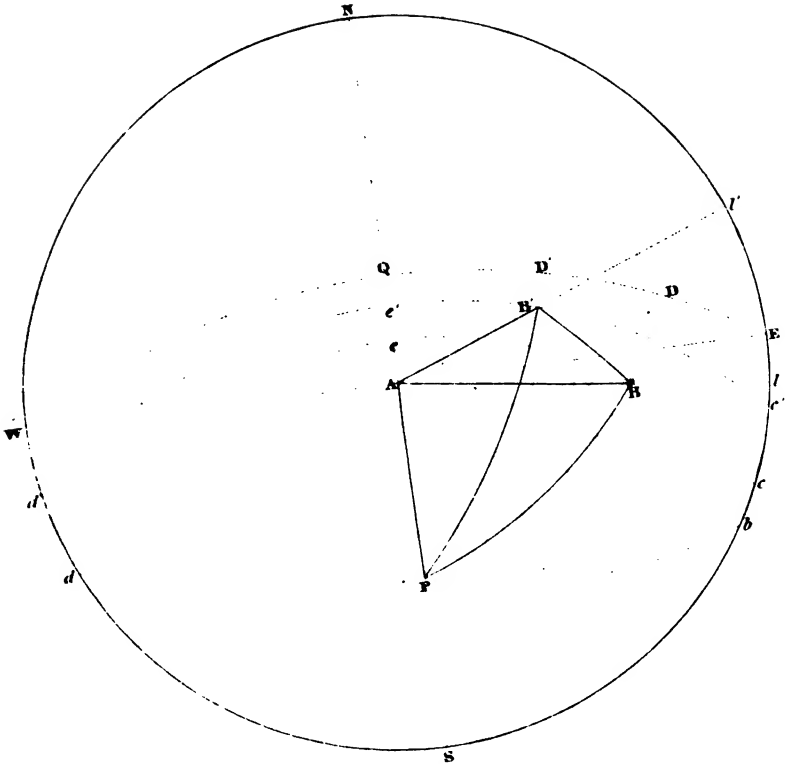
§ 442. The data for determining latitude by "double altitudes" may be obtained also by taking the altitude of two objects, each at a different time of the day.

§ a. This method may sometimes be found useful in cloudy weather, when the two objects cannot be seen at the same time, and when it is highly important to know the latitude.

§ 443. When the first object (say a star) is seen, let its altitude be taken, and the time, per watch, when the observation is made, noted down.

§ a. The bearing of the star at the same time, must also be observed, in order that the angle may be known, which is contained between this bearing, and the rhumb line, upon which the ship sails until the second observation may be made.

§ b. This angle, and the distance sailed during the interval that elapses between the taking of the two observations, are to be used



in finding the correction (§ 422. § a.) which must be applied to the first altitude, to make it what it would have been, had it been taken at the place where the second observation was taken.

§ 444. The right ascension and declination of this star must be taken out of the Nautical Almanac, also for the time at which its altitude was measured.

§ 445. When the second object (say the sun) is seen, its altitude also must be taken, and the time of its being done noted down, in order to know the time which has elapsed since the first observation.

§ 446. The right ascension and declination of the sun, when its altitude is measured, must also be found, in order to obtain the difference in right ascension of the two bodies when their altitudes were taken.

§ u. This difference is an angle formed at the pole, by the intersection of the circle of right ascension, in which the star was, when its altitude was taken, with the circle of right ascension which passed through the centre of the sun, when the alt. of it was measured.

§ 447. This angle corresponds to $B'PB$; and with the } Plate 5,
portion of *apparent time* that elapses from the taking of }
the first, till the taking of the last, observation, it constitutes the
angle which is contained between the P. distances of the two bodies;
and with the polar and zenith distances of these bodies, it also con-
stitutes the required data of the problem.

§ 448. The sum or difference of the *difference* of the two bodies in right ascension, and the portion of *mean time* from one observa-
tion to the other, give the required contained angle.

§ a. If the sun be the second object, the *apparent* instead of the *mean* time, from one observation to the other, must be taken.

§ 449. The sum of said two quantities is the value of this angle, when the first object is to the eastward of the second.

§ 450. But when the second is the more eastwardly object, the difference of the two in right ascension, *minus* the *elapsed* time, gives the value of the required polar angle.

§ 451. If, when the second is the eastern object, the elapsed time exceed the difference in right ascension of the two bodies, the second will have crossed the hour circle (§ 263. § c.) which coincides with the circle of declination, in which the first object was, when its altitude was measured.

§ 452. The excess of the elapsed time above this differ- } Plate 6.
ence in ascension, is the value of the required polar angle }
($B'PB$).

§ 453. B represents the place of Venus (the first object), when its altitude was taken.

§ a. Venus was then the morning star, and consequently to the westward of the sun, (which is the second object).

§ b. Venus' altitude was taken in the morning, when the sun was below the horizon.

§ c. Pb is an arc of the circle of right ascension in which the sun was, when Venus' altitude was taken.

Plate 6. } § *d*. $B' P b$, is the angular value of the difference in right ascension of the sun and Venus, when the altitude ($B I$) of the latter was taken; it is about $2h\ 36m\ 54s$.

§ 454. The sun's altitude, $B' I'$, was taken $4h\ 29m\ 45s$ after Venus' was measured.

§ 455. The sun had then crossed over the hour circle $P B$ (§ 263. § *c*.) in which Venus was, when its altitude was taken.

§ *a*. The sun was at B' , to the westward of this circle, when its altitude was taken.

§ 456. $B' P b$, is the horary angle, in *apparent* time, which the sun described during the interval between the taking of the altitudes.

§ *a*. Consequently the difference ($B P b$) of the sun and Venus in right ascension, when their altitudes were measured, subtracted from ($B' P b$) the angular value of the *apparent* time between the observations, gives the value of the angle $B' P B$.

§ 457. If the portion of time that elapses, between the observations, be given in mean time, it may be converted into apparent time, by applying to it the fraction which during the elapsed time, the second object's equation of time gains or loses, on mean time.

§ 458. In surveying expeditions the elapsed time should be corrected and changed into *apparent* time, in order to determine the latitude with exactness.

§ 459. But upon the open sea, this nicety in operation may be omitted, for it only advances, by a very small fraction, the accuracy of the result.

§ 460. If the altitude of the more easterly object, be taken last, and the elapsed time be equal to their difference in right ascension, the same hour circle (§ 263. § *c*.) will pass through each object when its altitude is taken; and the process of deducing the latitude will be confined to the simple operation of finding an angle with the three sides of a triangle as data; and thence in deducing, from the two sides and their contained angle, as data in a second triangle, its third side, which is the co-lat.

§ *a*. The three sides of the former of these two triangles, would be the zenith dist. of each body, and the difference between their polar distances.

§ 461. Feb. 25, 1832. Venus being the morning star, and the observer being in south latitude when he made his observations.

§ <i>a</i> . Venus' Alt.	24° 50'	(True.)
“ Dec.	20° 5' S.	
“ Rt. Ascen.	19h 59m	
“ bearing	E. $\frac{1}{4}$ S.	(§ 443. § <i>a</i> .)

Time per watch 3h 16m 19s

§ *b*. The ship then sailed N. E. $\frac{1}{4}$ N. 18 miles, when the sun was seen, and its altitude taken.

Sun's Alt.	43° 44'	(Correct. § 385. § <i>b</i> .)
Sun's Dec.	9° 16' S.	
Sun's Rt. A.	22h 30m 59s	
Time per watch	7h 46m 8s	

§ 462. The angle between the bearing of Venus (§ 461. § a.), and the ship's course, is five points, which (§ 422. § a.) with the distance sailed, (18 miles), gives 10' (§ 426.) in the column (*d. lat.*) of the traverse tables; which 10' (§ 423.) being added to Venus' altitude, makes it 25°, (what it was at the time of its being taken, but at the place where the sun's alt. was observed.)

§ 463. The watch kept *mean* time. The elapsed time (§ 457.) must therefore be corrected, in order to obtain the portion of *apparent* time, that elapsed from the first, until the second observation was taken.

§ a. During this interval there were 2s less of apparent, than of mean time, which being subtracted from the mean time elapsed, gives the elapsed apparent time.

§ 464. This correction is obtained from the ephemeris. It is the quantity which the equation of time gains or loses from the first to the second observation; and it must be applied accordingly.

§ 465. The triangulation, through which the process } Plate 6.
of finding the latitude by means of the data in this pro- }
blem is conducted, is represented in the triangles B' P B, A B' B and }
A B' P. }

$$\begin{aligned} \text{§ 466. } AB (\text{star's Z. D.}) &= 65^\circ 0' \\ PB (\text{star's P. D.}) &= 69^\circ 55' \\ AB' (\text{sun's Z. D.}) &= 46^\circ 16' \\ PB' (\text{sun's P. D.}) &= 80^\circ 44' \\ \text{Elapsed mean time} &= 4h 29m 49s \\ \text{Correction (§ 463. § a.)} & \quad \quad \quad 2s \end{aligned}$$

$$\begin{aligned} \text{Elapsed app. time} & \quad \quad \quad = 4h 29m 47s \\ \text{Diff. in R. A. of sun and star} &= 2h 31m 59s \end{aligned}$$

$$B' P B = 1h 57m 48s \quad (\text{§ 456. § a.})$$

To find the latitude.

§ a. In the triangle B' B P, to find the value of B' B. (§ 200. § n.)

$$\begin{aligned} B' P B = 1h 57m 48s \quad \cos. &= 9.939911 \\ P B' (\text{Sun's P.D.}) = 80^\circ 44' \quad \text{tang.} &= 0.787389 \end{aligned}$$

$$\text{Tang. auxl. } a = 0.727300 = 79^\circ 23' 15''$$

$$PB = 69^\circ 55' 0''$$

$$(a \oslash \text{star's P. D.}) = 9^\circ 28' 15''$$

$$PB' (\text{sun's P. D.}) = 80^\circ 44' \quad \cos. = 9.206906$$

$$\text{Auxl. } a = 79^\circ 23' 15'' \quad \sec. = 0.734791$$

$$(a \oslash PB) \quad \quad \quad = 9^\circ 28' 15'' \quad \cos. = 9.994040$$

$$\text{Cos. } B' B = 9.935737 = 30^\circ 24' 24''$$

Plate 6. $\left\{ \begin{array}{l} \S b. \text{ To find the value of } P B' B. \\ B' B = 30^\circ 24' 24'' \text{ co-sec.} = 0.295734 \\ B' P B = 1h 57m 48s \text{ sin.} = 9.691668 \\ P B (\text{star's P. D.}) = 69^\circ 55' 0'' \text{ sin.} = 9.972755 \end{array} \right.$

$$\text{Sine } P B' B = 9.960157 = 65^\circ 49' 50''$$

$\S c.$ To find $A B' B$ ($\S 201. \S f.$)

$$\begin{array}{r} A B (\text{star's Z. D.}) = 65^\circ \\ A B' (\text{sun's Z. D.}) = 46^\circ 16' \quad \text{co-sec.} = 0.141123 \\ B' B = 30^\circ 24' 24'' \quad \text{co-sec.} = 0.295734 \\ \hline 2) 141^\circ 40' 24'' \end{array}$$

$$\begin{array}{r} \frac{1}{2} \text{ sum} = 70^\circ 50' 12'' \text{ sin.} = 9.975241 \\ (\frac{1}{2} S. \omega \text{ star's Z. D.}) = 5^\circ 50' 12'' \text{ sin.} = 9.007291 \end{array}$$

$$2) 19.419389$$

$$\text{Cos. } \frac{A B' B}{2} = 9.709694 = 59^\circ 10' 10''$$

2

$$A B' B = 118^\circ 20' 20''$$

$$P B' B = 65^\circ 49' 50''$$

$$A B' P = 52^\circ 30' 30''$$

$\S d.$ To find $A P$, the co-lat.

$$\begin{array}{r} A B' P = 52^\circ 30' 30'' \text{ cos.} = 9.784365 \\ P B' (\text{Sun's P. D.}) = 80^\circ 44' \text{ tang.} = 0.787389 \end{array}$$

$$\text{Tang. auxl. } a = 0.571754 = 74^\circ 59' 37''$$

$$A B' = 46^\circ 16' 0''$$

$$(a \omega A B') = 28^\circ 43' 37''$$

$$\begin{array}{r} P B' (\text{sun's P. D.}) = 80^\circ 44' \text{ cos.} = 9.206906 \\ \text{Auxl. } a = 74^\circ 59' 37'' \text{ sec.} = 0.586823 \\ (a \omega \text{ sun's Z. D.}) = 28^\circ 43' 37'' \text{ cos.} = 9.942959 \end{array}$$

$$\text{Sin. Complement } A P, \text{ or lat.} = 9.736688 = 33^\circ 2' 59'' \text{ South.}^*$$

* This differs 1'' from the truth.

LUNARS.

§ 467. The problem of finding the *true* distance between the sun, or a star, and the moon, for the purpose of thence determining the longitude, is one of great importance in navigation.

§ 468. The necessity for the circuitry of calculation in finding the longitude by lunar observations, arises from the circumstance that the two observed bodies are not seen (§ 289.) in their true places.

§ *a.* They always appear, each in its proper azimuth circle, but higher up, or lower down in it (§ 283. § *c.*), than its true place,

§ *b.* And consequently (§ 283.) they appear either nearer to, or farther from, each other, than they really are, which makes the difference between the true, and apparent lunar distance.

§ 469. Only two triangles are involved in the process of finding, by trigonometrical calculation, the *true* from the *observed* lunar distance.

§ *a.* Of the first triangle, the three sides are given; and the angle required is that which is contained at the zenith, between the zenith distance of the moon, and of the other body.

§ *b.* Of the second triangle, this angle and its adjacent sides, are the given parts; and the third side, or true lunar distance, is the part required.

§ *c.* The three sides of the first triangle are the apparent lunar, and the app. zenith distances of the two bodies.

§ *d.* And the three sides of the second triangle, are the *true* lunar and zenith distances.

§ 470. The *lunar*, or the *true lunar distance* of the sun, or a star, is an arc of a great circle, contained between the centres of either of those objects, and the moon.

§ 471. The distance between the perfect limb (§ 283.), or the round edge, of the moon and the near limb of the sun, is the distance usually measured with a sextant.

§ 472. The *apparent* is obtained from the observed distance, by applying, with proper signs, the corrections for the error of the sextant, for the semidiameters of the two bodies, and for the augmentation of the moon's semidiameter.

§ *a.* The sun and moon's semidiameter is given in the ephemeris; the sun's for every 24*h*, and the moon's for every noon and midnight.

§ *b.* The correction for augmentation is found in Table X.

§ 473. *Augmentation* of the moon's semidiameter, is the difference between the visual angle of the moon's semidiameter at the centre, and its visual angle at the circumference, of the earth.

§ 474. The difference between these two angles, is proportional to the ratio between the length of the semidiameter of the earth, and its distance from the moon.

§ 475. On account of its great distance from the earth, the sun's semidiameter has no augmentation.

§ *a.* As the earth's semidiameter measures more in proportion to

the moon's geocentric distance, than to that distance of the sun, or of any other heavenly body, the difference of the moon's semidiameter, when seen from the circumference and from the centre of the earth, is greater than that of any other body, when seen from the same positions.

§ 476. The semidiameter of the moon when it is in the horizon, appears under nearly the same visual angle (barring refraction) that it does from the centre of the earth.

§ 477. The moon appears larger when it is in the horizon, than it does when in the zenith. This though is an optical delusion, for the moon actually subtends the largest visual angle when in the zenith; as it is then nearer to the observer than it is when in the horizon, by a little more than the semidiameter of the earth.

Plate 7. } § 478. $B' A B$ represents a diagram of the lunar problem.
 $b' b$ is the apparent, (§ 472.), and $B' B$ the true (§ 470.) lunar distance.

§ 479. The three apparent distances, ($b' b$, $b' A$, $A b$), are the given parts (§ 469. § α .) of the first triangle $b' A b$, and the angle $b' A b$ is the part required in it.

§ 480. The angle $b' A b$ is common to the second triangle $B' A B$, of which the true zenith distances $A B$, and $A B'$ are the given, and the true lunar distance $B' B$ is the required part.

§ 481. The apparent Z. D. of the sun, or of a star, (§ 284. § α .) being less, and the app. Z. D. of the moon (§ 286. § α .) being greater, than the true Z. D.; it appears from the diagram, that, when the moon's is the greater of the two Z. D., the true is always less than the apparent lunar distance.

§ α . And when the moon's is the less Z. D., the true is sometimes greater, and sometimes less, than the app. distance.

§ b . The true is the less, when the lunar distance exceeds 90° .

§ c . And it is generally the greater, when the distance is less than a quadrant, and the ratio of cos. moon's Z. D. to cos. sun or star's Z. D., is greater than the ratio of rad. to cos. of the lunar distance.

§ 482. The reason why the difference between the app. and the true L. dist., appears to be governed by the moon more than by the other body, is, that the moon being nearer to the earth, has the greater parallax, and is generally seen further from its true place in the heavens, than any other body, from which lunar distances are measured, appears from its true place.

§ 483. In N. lat., Aug. 27th, 1834.

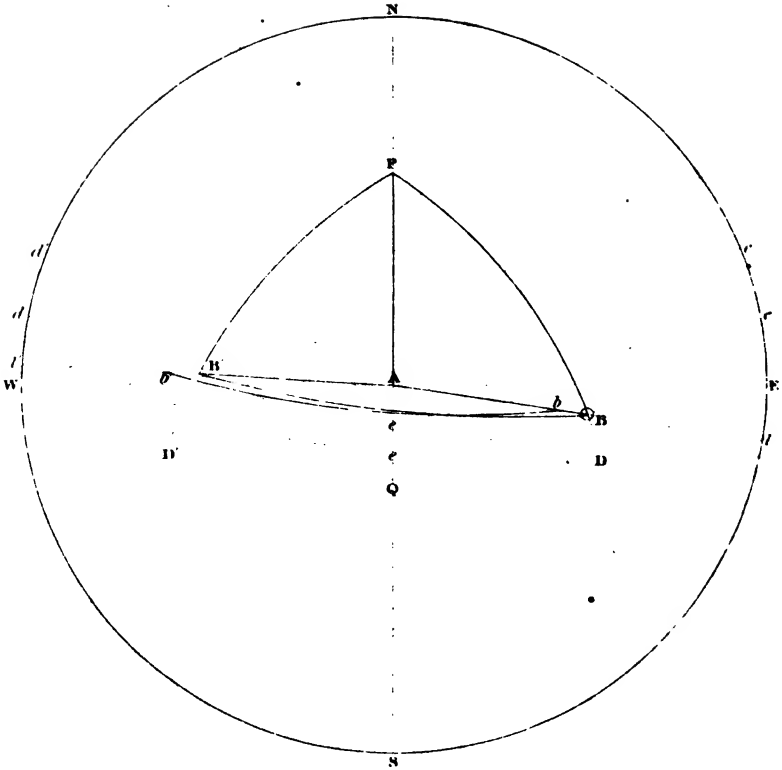
Obs. Dist. sun and moon = $89^\circ 51' 40''$

Obs. Alt. sun - = $34^\circ 4' 50''$

Obs. Alt. moon's upper limb = $54^\circ 27' 50''$

§ α . Obs. dist. sun & moon	=	$89^\circ 51' 40''$	
Sun's semidiameter	-	$15' 51''$	
Moon's " "	-	$15' 1''$	
Moon's augmentation	-	$13''$	(§ 472. § b .)
Error of sextant	-	$+ 6''$	

App. lunar dist. = $90^\circ 22' 51''$



§ <i>b</i> . Sun's obs. alt.	-	34°	4'	50"	
Sun's semidiameter	-		15'	51"	
*Dip (§ 275. § <i>a</i> .)	-		— 3'	21"	
<hr style="width: 50%; margin: auto;"/>					
Sun's app. alt.	-	34°	17'	20"	
§ <i>c</i> . Refraction for sun's alt.		— 1'	20"	(Table IX.)	
Sun's plx. in alt.			8"	(Table XI.)	
<hr style="width: 50%; margin: auto;"/>					
Sun's true alt.	-	34°	16'	8"	
<hr style="width: 50%; margin: auto;"/>					
§ <i>d</i> . Obs. alt. moon's U. L.		54°	27'	50"	
Moon's semidiameter	-		— 15'	1"	
Moon's augmentation	-		— 13"	(Table X.)	
Dip (§ 275. § <i>a</i> .)	-		— 3'	21"	
<hr style="width: 50%; margin: auto;"/>					
§ <i>e</i> . Moon's app. alt.		54°	9'	15"	cos. = 9.767605 (§ 288.)
Moon's hor. plx. (§ 287.)			55'	7"	sin. = 8.204991
<hr style="width: 50%; margin: auto;"/>					
Moon's plx. in alt. (§ 287. § <i>c</i> .)	=	32'	16"	sin.	= 7.972596
<hr style="width: 50%; margin: auto;"/>					
§ <i>f</i> . Refraction for moon's alt.	=	— 43"			
<hr style="width: 50%; margin: auto;"/>					
Correction for moon's app. alt.	=	31' 33" †			

§ *g*. To find the value of $b' A b$ (§ 201. § *f*.) in the first } } Plate 7.
triangle.

$b' b$ (app. lunar dist.)	=	90°	22'	51"	
$b' A$ (moon's app. Z. D.)	=	35°	50'	45"	co-sec. = 0.232394
$A b$ (sun's app. Z. D.)	=	55°	42'	40"	co-sec. = 0.082911
<hr style="width: 50%; margin: auto;"/>					
	2)	181° 56' 16"			
<hr style="width: 50%; margin: auto;"/>					
$\frac{1}{2}$ sum	-	90°	58'	8"	sin. = 9.999938
($\frac{1}{4}$ S. ∞ app. lunar dist.)	=	0°	35'	17"	sin. = 8.011288
<hr style="width: 50%; margin: auto;"/>					
	2)	18.326531			
<hr style="width: 50%; margin: auto;"/>					
	Cos.	81°	37'	34"	($= \frac{b' A b}{2}$)
<hr style="width: 50%; margin: auto;"/>					
		9.163265			
<hr style="width: 50%; margin: auto;"/>					
$b' A b$	=	163° 15' 8"			

* The *dip*, when the eye is 9 feet above the water, is about 3'; and 4', when the height of the eye is 17 feet. On board ship 4' is generally allowed for *dip*.

† When the nicest accuracy is required, corrections may be applied for the difference in the effects of refraction on the upper and the lower limb of the sun and moon; for the moon's parallax in different latitudes, supposing the equatorial to be greater than the polar diameter; and also a correction for the actual barometrical, and thermometrical state of the atmosphere; but at sea, greater errors are unavoidable, hence these corrections are of minor importance.

$$\begin{array}{r}
 \S h. \text{ To find } B' B \text{ (§ 200. § v.) the tr. lunar dist. (§ 480.)} \\
 b' A b = 163^\circ 15' 8'' \text{ cos.} = 9.981176 \\
 A B \text{ (Sun's tr. Z. D.)} = 55^\circ 43' 52'' \text{ tang.} = 0.166625 \\
 \text{Tang. auxl. } a = 0.147801 = 125^\circ 26' \\
 \text{Moon's tr. Z. D.} = 35^\circ 19' 12'' \\
 \hline
 90^\circ 6' 48'' \\
 \hline
 A B \text{ (Sun's tr. Z. D.)} = 55^\circ 43' 52'' \text{ cos.} = 9.750567 \\
 \text{Auxl. } a = 125^\circ 26' \text{ sec.} = 0.236755 \\
 (\text{Moon's tr. Z. D.}) = 90^\circ 6' 48'' \text{ cos.} = 7.296235 \\
 \hline
 \text{Tr. lunar dist.} = 89^\circ 53' 24'' \text{ Cos. } B' B = 7.283557 \\
 \hline
 \end{array}$$

§ 484. By referring to the Nautical Almanac, page xvli., of the ephemeris for Aug. 1834, it will be observed, that it was near noon of the 27th at the Greenwich observatory, when the moon was $89^\circ 53' 24''$ from the sun.

§ a. If the latitude of the place at which the observation (§ 483.) was made, be known, the time of day at that place, and when the observation was taken, may be found according to the rule § 387.

§ b. And the difference (§ 391. § a.) between the time of day thus found, and the time in the ephemeris, which corresponds to the calculated lunar distance, is the longitude in time.

§ 485. Distances for determining longitude are always measured from the moon, because the change of the moon's position in the heavens is more obvious than that of any other heavenly body that is visible to the naked eye.

§ 486. The moon (§ 360.) has a daily motion of about $13^\circ 10'$ in the heavens; owing to which the moon rises, culminates, and sets, later and later every day, until, having passed through all its phases, it crosses the circle of the sun's altitude, gets to the eastward of the sun, and presents the appearance of a new moon.

§ 487. At a mean, an error of $2' 12''$ in the lunar distance, will produce an error of 1° in the longitude.

§ 488. The moon changes its distance from those bodies, which lie directly in its path, more rapidly than it does from those towards or from which it moves more obliquely.

§ a. Therefore the stars from which the change of the lunar distance for the time being is the most obvious, should be preferred in taking lunar observations; for the more rapidly the distance changes, the less will the longitude be affected by a small error in the observation.

§ 489. In the Nautical Almanac; pages xiii. to xviii., of the ephemeris for each month, contain at intervals of three hours, the distance of the moon from the sun, from four of the planets, and eight of the principal fixed stars.

§ a. The small column, marked P. L. contains the proportional

logarithm of the quantity, which the distance preceding it in the left hand column, varies during the three hours between which the P. log. stands.

§ b. Table III. of this volume, contains the proportional log. for every second from 1" to 3°; or for every second of time from 1s to 3h.

§ c. Proportional logarithms are nothing more than the *difference* between the log. of 3, and the log. of the given number less than 3.

Thus, the log. of 3 - - - - = 0.4771 +
 Log. of 12'; (12' reduced to the decimal of a degree is 0.20.) Log. 0.20 - - - - } 9.3010 +

P. log. of 12', or 12m = 1.1761

§ d. This Table (III.) is always used for finding the Greenwich time when a given lunar distance occurred.

Thus, to find the time at Greenwich when the sun was distant from the moon

84° 17' 10"	}	Diff. - 20' 51" P. L. = 9362
L. Dist. at 6. 83° 56' 19"	}	Diff. for 3h = 1° 20' 9" P. L. = 3514
" at 9. 85° 18' 28"		

46m 49s = 5848

And 46m 49s added to 6h, shows the time (6h 46m 49s) at Greenwich, when the sun was distant from the moon 84° 17' 10".

§ e. This is a much shorter way than that of arriving at the same result by common logs.; thus,

As 1° 20' 9" = 1.336 (decim'ls of a deg.) Ar. co. = 9.874193
 Is to 3h - - - - - Log. = 0.477121
 So is 20' 51" = 0.3475 (do. of a deg.) Log. = 9.540955 (§ 90.)

To 46m 49s = 0.7803 " " Log. = 9.892269 (§ 93. § g.)

§ 490. In selecting a star to measure a lunar distance from, that one should be fixed upon, whose lunar distance at the time has the *least* p. log. after it in the ephemeris; for, as the greatest variations in the dist. are represented by the smallest p. logs., the lunar dist. of those bodies, which has the smallest p. log. after it, is increasing or decreasing, in a greater ratio, than that is which has after it a greater p. log.; consequently, (§ 488. § a.) those bodies are the most favourably situated for determining longitude.

§ 491. When practicable, the distance of the moon should be measured from two stars; one to the east, the other to the west of it; and the *mean* of the longitude resulting from the observations should be taken as the long. of the place.

§ a. If the sextant have an unknown error, the error will partially

Z

correct itself by its own counteracting effects in distances measured to the east and west of the moon.

§ 492. Latitude by "double altitudes" may also be determined by means of the data in the lunar problem.

Plate 7. { 493. By inspecting the lunar diagram, it is seen that the arcs (P B, P B') of the circles of declination of the sun and moon, join the extremities of the true lunar distance (B' B), forming thereby the triangle B' P B, of which the three sides are known.

§ a. Then the value of the angles P B B', and A B B' being found, their difference gives the value of A B P, which, with B A, and B P in the triangle P A B, constitutes data requisite for determining A P, the co-latitude.

§ 494. The lunar problem is one of the most useful and most beautiful problems in navigation; for besides his longitude and latitude, the navigator may deduce, from the principles involved in it, data for the solution of almost any problem in nautical astronomy.

§ 495. The three parts A P, P A B, A P B, in the fourth triangle, are determinable from the data obtained by a lunar observation.

§ a. A P is the co-lat. of the place of observation.

§ b. A P B is the horary angle, which (§ 382. § a.) shows the app. time of day at the place of observation when the latter was made. And,

§ c. The difference between this time converted into *mean* time, and the Greenwich time in the N. A. that corresponds to the lunar dist., expresses, in time, the longitude of the place of observation.

§ d. P A B is the sun's true azimuth at the time and place of observation; the difference between it, and the sun's magnetic azimuth, (§ 308. § a.) is the variation of the compass.

§ 496. By increasing the triangulation, and extending the operations, other *quæsitæ* may be added to the problem; and the length of the day and night; the time of the sun and moon's rising and setting; the hour of the moon's passing the meridian; the hour when each object bears east or west; the duration of twilight; and the amplitudes of the two objects on the day, and at the place of observation, may all be determined by means of the parts involved in the common lunar problem.

§ 497. Therefore when, on account of unknown drifts, and of gales during several days in succession of thick or cloudy weather, a ship has lost her reckoning, it may be successfully restored by a single lunar observation.

§ a. If the variation of the compass be required also at the same time in addition to the lat., time of day, and longitude, the calculation, after being conducted into the fourth triangle B A P, should be conducted by a process different from that for evolving the value of A P, the co-lat., alone.

§ b. In the latter case, the value of A P would be determined by the rule § s. and § t., under Case IV. (§ 200.)

§ c. And in the former case, the proportion between the sines of opposite sides and angles, would evolve it; for, when the three un-

Plate 7. $\left\{ \begin{array}{l} \text{known parts of this triangle (B A P) are required, the} \\ \text{most direct method of arriving at the required results,} \end{array} \right.$
 consists in applying the rules § *z.* and § *z a.*, Case IV., (§ 200.) to calculation, in order to determine first the value of the horary and azimuth angles (A P B, & P A B.)

§ *d.* In trigonometrical operations, that method of calculation should ever be adopted, which, being less elaborate, arrives more directly at the required result.

§ 498. When the latitude, etc., is required by means of a lunar observation, the operation of correcting altitudes and of finding the true lunar distance, is conducted as it is under § 483. ; and then the value of A B B' is found, before the operation passes from the second triangle B' A B.

§ *a.* The true lunar distance B' B being determined, the Greenwich time corresponding thereto, is found (§ 489. § *d.*) in the lunar tables of the Ephemeris; and then the declination and the right ascension of the sun and moon, at this time, are found in the appropriate tables of the Nautical Almanac.

§ 499. The declination of the two bodies thus found, gives (§ 258. § *b.*) the sides B' P and B P.

§ *a.* And the difference of their right ascensions (§ 433.), expresses in time the value of the angle B' P B.

§ *b.* The angle P B B' is the only part in this triangle, which is required, and it may be found without the trouble of taking from the Ephemeris, the right ascension of the two bodies.

§ 500. The true lunar distance (§ 483.) $89^{\circ} 53' 24''$ of the problem occurred (§ 484.) at Greenwich, August 27, 1834, near mean noon, when the sun's declination (page ii. of the Monthly Ephemeris) was $10^{\circ} 10' 2''$ N.; and the moon's (page v. to xii. do.) was $19^{\circ} 12' 39''$ N. Their difference in right ascension then was $6h 13m 52s$.

§ *a.* And the sun's magnetic azimuth (§ 304. § *a.*) at the time of observation, was N. $109^{\circ} 4' 30''$ East.

§ 501. The apparent and true zenith distances are found, and the calculation for determining the true lunar distance (B' B) is conducted precisely in the same manner, as it is (§ 483.) when the true distance is the only object of the problem.

§ 502. In the second triangle B' A B, to find the value of A B B'.

B' B (Tr. lunar D.)	= $89^{\circ} 53' 24''$	cosec. = 0.000001
B' A B	= $163^{\circ} 15' 8''$	sin. = 9.459632
B' A (Moon's Z. D.)	= $35^{\circ} 19' 12''$	sin. = 9.762035

Sin. A B B' = 9.221668 = $9^{\circ} 35' 24''$

Plate 7. $\left\{ \begin{array}{l} \S a. \text{ In the third triangle } B' P B, \text{ to find the value of} \\ P B B'. \end{array} \right.$

$$B' B \text{ (Tr. L. D.)} = 89^\circ 53' 24'' \quad \text{cosec.} = 0.000001$$

$$B' P B \text{ (Diff. R. A.)} = 6h 13m 52s \quad \text{sin.} = 9.999205$$

$$B' P \text{ (Moon's P.D.)} = 70^\circ 47' 21'' \quad \text{sin.} = 9.975116$$

$$\text{Sin. } P B B' = 9.974322 = 70^\circ 29' 27''$$

$$A B B' = 9^\circ 35' 24''^A$$

$$A B P = 60^\circ 54' 3''$$

$\S b.$ To find, in the fourth triangle, the sun's azimuth $P A B$, and the horary angle $A P B$ ($\S 200.$ $\S z b.$)

$$\frac{AB + BP}{2} = 67^\circ 46' 55'' \quad \text{sec.} = 0.422357 \quad \text{cosec.} = 0.033505$$

$$\frac{AB \oslash BP}{2} = 12^\circ 3' 3'' \quad \text{cos.} = 9.990322 \quad \text{sin.} = 9.319687$$

$$\frac{ABP}{2} = 30^\circ 27' 1'' \quad \text{cot.} = 0.230713 \quad \text{cot.} = 0.230713$$

$$\text{Tang. } \frac{PAB + APB}{2} = 77^\circ 11' 38'' = 0.643392$$

$$\text{Tang. } \frac{PAB \oslash APB}{2} = 20^\circ 59' 17'' \quad - \quad - \quad = 9.583905$$

$$A P B \text{ (H. Ang.)} = 56^\circ 12' 21'' \text{ (}\S 77. \S h.)$$

$$P A B \text{ (Azm.)} = N. 98^\circ 10' 55'' \text{ E.}$$

$$\S c. APB \text{ (H. Ang.)} = 56^\circ 12' 21'' = 3h 44m 49s = (\S 348. \S g.)$$

$$8h 15m 11s \text{ A.M.}$$

$$1m 25s = \text{Equation of time, (Naut. Al., p. ii. Mon. Eph.)}$$

$$8h 16m 36s \text{ A.M.} = \text{Mean time of day at the place of observation.}$$

$\S d.$ To find $A P$, the co-lat.

$$A P B \text{ (Hry. Ang.)} = 56^\circ 12' 21'' \quad \text{cosec.} = 0.080377$$

$$A B \text{ (Sun's Z. D.)} = 55^\circ 43' 52'' \quad \text{sin.} = 9.917192$$

$$A B P = 60^\circ 54' 3'' \quad \text{sin.} = 9.941402$$

$$\text{Cos.} = 29^\circ 40' 8'' \text{ N. Lat.} = \text{Sin. } A P \text{ (co-lat.)} = 9.938971$$

$$\S e. \text{ Sun's Mag. Azm. (}\S 500. \S a.) \text{ N. } 109^\circ 4' 30'' \text{ E.}$$

$$\text{Sun's Azm. (}\S b.) \text{ N. } 98^\circ 10' 55'' \text{ E.}$$

$$\text{Variation (}\S 312.) = 10^\circ 53' 35'' \text{ Westerly.}$$

$\S f.$ Time at Greenwich ($\S 484.$)
 when the obs. was made, $12h\ 00m\ 17s$ P.M. } Plate 7.
 Time ($\S c.$) at the place of observation, $8h\ 16m\ 36s$ A.M.

Long. of the obsr. ($\S 391. \S b.$) W. $3h\ 43m\ 41s = 55^{\circ}\ 55'\ 15''$

$\S 503.$ If the latitude and time of day, without the variation, be required from a lunar observation, the process of calculation may be varied from that above for finding the azimuth, time, and latitude; and may be made more direct.

$\S a.$ But in either case, the order and method of calculation are the same, in the process of arriving at the value of ABP in the fourth triangle.

$\S 504.$ The true lunar distance ($B'B$) must always be determined, before the correct $P.$ distances, ($P'B, P'B'$) and the difference ($B'PB$) in right ascension ($\S 498. \S a.$) of the two bodies, can be known.

$\S a.$ These parts must always be taken from the Ephemeris, and for the time in it, which corresponds to the true lunar distance.

$\S 505.$ Having found, in the second triangle, the required angle $AB'B'$; then having taken ($\S 498. \S a.$) from the Ephemeris, the proper $P. D.,$ etc., and calculated ($\S 502. \S a.$) the value of the angle ($P'B'B'$) required in the third triangle; the value of ABP is obtained, and the lat. and time of day may thence be deduced by the following formula.

$\S 506.$ Formula of calculation for finding the latitude and time of day by a lunar observation.

To find the lat. ($\S 200. \S n.$)

	Contd. Angle (ABP)	cos. = *.*****
Sun's Z. D. (AB)	cos. = *.*****	tang. = *.*****
 		<hr/>
Auxl. a	sec. = *.*****	Tang. auxl. a = *.*****
($a \propto$ Sun's $P. D.$)	cos. = *.*****	<hr/>
 		<hr/>
Cos. co-lat. (AP)	= *.*****	

To find the time of day, ($\S 144.$)

Co-lat. (AP)	co-sec. = *.*****	
Contd. Angle (ABP)	sin. = *.*****	
Sun's Z. D. (AB)	sin. = *.*****	
 		<hr/>
Sin. Horary Angle (APB)	= *.*****	

$\S 507.$ If the second and third triangles fall on opposite sides of the lunar distance $B'B$, then the sum of the angles ($AB'B'$ & $P'B'B'$) of the second and third triangles, gives the value of the required angle (PBA) of the fourth triangle. } Plate 8.

$\S 508.$ When the polar distance of each body is greater than the co-latitude of the observer, then the second and third triangles fall upon the same side of the lunar distance $B'B$, } Plate 7.

Plate 7. } and the difference between the angles (A B B', P B B') is
 } the value of the angle (A B P) sought in the fourth triangle.

§ 509. It may always be known, whether the sum or difference of the angles of the second and third triangle, will give the angle required to be known in the fourth triangle, by observing the inclination of the plane in which the sextant is held to bring the sun and moon in contact.

§ a. If the sextant be inclined to the plane of the horizon, towards the pole of the observer, then his co-lat. is greater than the P. dist. of either body, and the second and third triangles fall upon different sides of the lunar distance, and the sum of the two angles is the required angle of the fourth triangle.

§ 510. May 16, (P. M.), 1834.

Obs. dist. sun & moon 97° 16' 28''

Obs. alt. sun's L. L. = 41° 3' 59''

Obs. alt. moon's U. L. = 37° 58' 24''

Required the latitude and longitude of the place of observation ?

§ a. To apply the preliminary corrections.

1st. Obs. lunar dist.	97° 16' 28''	
Sun's semidiam.	-	15' 50'' (N. A., p. ii., M. E.)
Moon's "	-	16' 6'' (" iii. ")
Moon's augmentation	-	10''

App. lunar dist. = 97° 48' 34''

2d. Obs. alt. sun's L. L.	41° 3' 59''	
Sun's semidiameter	-	15' 50''
Sun's app. alt.	-	41° 19' 49''
3d. Sun's refraction	-	-1' 11''
Sun's plx. in alt.	-	7''
Sun's true alt.	41° 18' 45''	Z. D. 48° 41' 15''

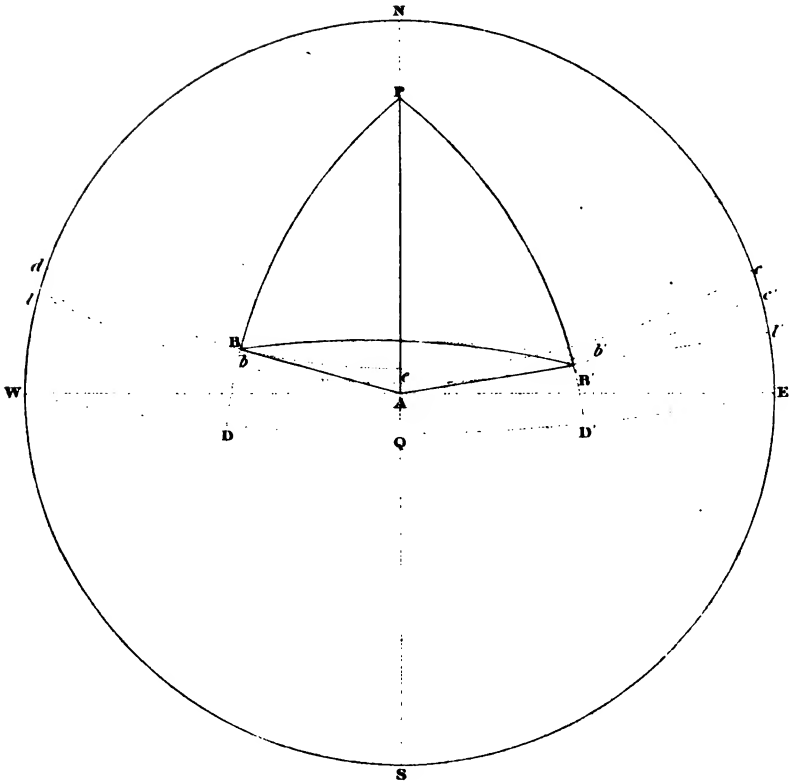
4th. Obs. alt. moon's U. L.	37° 58' 24''	
Moon's semidiameter	-	-16' 6''
Moon's augmentation	-	-10''
Moon's app. alt.	-	37° 42' 8''
Moon's hor. plx.	-	59' 4''
Moon's plx. in alt.	-	46' 44''
Moon's refraction	-	-1' 14''

cos. = 9.898286

sin. = 8.235047

Sine = 8.133333

Moon's true alt. 38° 27' 38'' Z. D. 51° 32' 22''



§ *b*. To find the tr. lunar dist.

1st. To find *b A b'* in the first triangle, (§ 201. § *f*.) } Plate 8.
b b' (Ap. lunar dist.) = 97° 48' 34"
A b (sun's ap. Z. D.) = 48° 40' 11" co-sec. = 0.124410
A b' (moon's ap. Z. D.) = 52° 17' 52" co-sec. = 0.101714

2) 198° 46' 37"

$\frac{1}{2}$ sum = 99° 23' 18" sin. = 9.994144
 ($\frac{1}{2}$ S. ∞ *b b'*) = 1° 34' 44" sin. = 8.440173

2) 18.660441

Cos. $\frac{b A b'}{2}$ = 77° 38' 55" = 9.330220

2d. To find *B B'* }
 (§ 200. § *u*.) } *b A b'* = 155° 17' 50" cos. = 9.958319
A B = 48° 41' 15" cos. = 9.819653 - tang. = 0.056056

Auxl. *a* sec. = 0.157822 Tang. *a* 134° 3' 7" = 0.014375

51° 32' 22" (M.'s Z. D.)

(*a* ∞ M.'s Z. D.) cos. = 9.114977 - 82° 30' 45" (*A B' \infty a*)

Cos. *B B'* = 9.092452 = 97° 6' 25" (Tr. l'r dist.)

When this distance (97° 6' 25") occurred, it was 9 P. M. at Greenwich of the given day (N. A. p. xv., Monthly Ephemeris.)

§ *c*. To take (§ 498. § *a*.) the other requisite data from the ephemeris.

Moon's dec. 14° 59' 6" N. P. D. 75° 0' 54" (N. A., p. viii. M. E.)

Sun's dec. 19° 8' 28" N. P. D. 70° 51' 32" (" ii. ")

Moon's Rt. A. 10^h 25^m 15^s - (" viii. ")

Sun's " 3^h 52^m 25^s - (" ii. ")

Polar angle = 6^h 52^m 50^s = *B P B'*.

§ *d*. To find the required angle *B' B A* of the second triangle.

B B' (lunar dist.) = 97° 6' 25" cosec. = 0.003350

B A B' 155° 17' 50" sine = 9.621084

A B' (moon's Z. D.) = 51° 32' 22" sine = 9.893782

Sin. *B' B A* = 9.518216 = 19° 15' 18"

Plate 8. } § e. To find the required angle B' B P of the third
 triangle
 B B' (lunar dist.) = 97° 6' 25" co-sec. = 0.003350
 B' P B (pl'r. angl.) = 6h 52m 50" sin. = 9.988356
 P B' (moon's P. D.) = 75° 0' 54" sin. = 9.984975

§ f. 4th triangle B A P; } Sin. B' B P = 71° 23' 30" = 9.976681
 to find the co-lat. } B' B A = 19° 15' 18" (§ 506.)

P B A = 90° 38' 48" cos. = 8.052549
 A B = 48° 41' 15" cos. = 9.819653 - tang. = 0.056056

Auxl. α - sec. = 0.000036 Tang. α 179° 15' 51" = 8.108605
 70° 51' 32" (S.'s P. D.)

(α ∞ sun's P. D.) cos. = 9.499325 108° 24' 19" (A B ∞ a)

Cos. A P, or sin. lat. = 9.319014 = 12° 1' 55" N. sec. = 0.009647

To find the Hr. } P B A = 90° 38' 49" sin. = 9.999973
 angle A P B. } A B (sun's Z. D.) = 48° 41' 15" sin. = 9.875710

Sin. A P B 3h 20m 41s = 9.885330

Equation = 3m 56s

Mean time = 3h 16m 45s P. M.

Gr. time (§ b.) = 9h P. M.

W. Long. (§ 391. § b.) 5h 43m 15s = 85° 48' 45"

§ 511. Examples under § 483. and § 510. are drawn out in the solution, in order to familiarize the problem to the student. But such a process of calculation as is there shown, is not desirable in practice; especially that part of it which has been made to come under Case IV., (§ 200. § n.) on account of the perplexity in determining whether the value of the auxl. α be greater or less than 90°.

§ a. Therefore, in practice, those methods of calculation are generally preferred, which give the trigonometric function of *half* the value of the required part; such as § a. (§ 386.), for *half* the value of a required arc or angle should never exceed 90°.

§ 512. The learner has now become familiar with the triangulation, which is brought under calculation in the process of finding longitude, etc., from a lunar observation; he will therefore readily comprehend the manner by which such an operation is simplified, and reduced to the following methods and formulæ for practice.

§ 513. The process of correcting the obs. distance and altitudes is fully shown under § 483. and § 510. This part of the operation

is the same in every method for calculating the true from an obs. lunar distance.

§ 514. An improved method for finding the true from an observed lunar dist.*

§ a. Aug. 27th, 1834.

App. dist. sun & moon = $88^{\circ} 26' 52''$

Sun's app. alt. - $47^{\circ} 52' 12''$

Sun's refraction - $53''$

Sun's pl'x. in alt. $7''$

Sun's tr. alt. - $47^{\circ} 51' 26''$

Moon's app. alt. - $40^{\circ} 52' 58''$

cos. = 9.878550

Moon's hor. pl'x. - $55' 0''$

sin. = 8.204070

Moon's pl'x. in alt. - $41' 35''$

sine = 8.082620

Moon's refraction - $1' 7''$

Moon's true alt. = $41^{\circ} 33' 26''$

§ b. App. lunar D. $88^{\circ} 26' 52''$

Sun's app. alt. - $47^{\circ} 52' 12''$

sec. = 0.178397

Moon's " - $40^{\circ} 52' 58''$

sec. = 0.121449

$2) 177^{\circ} 12' 2''$

$\frac{1}{2}$ sum = $88^{\circ} 36' 1''$ cos. = 8.367876

($\frac{1}{2}$ S. ∞ app. lunar D.) = $0^{\circ} 9' 9''$ cos. = 9.999998

Sun's tr. alt. - $47^{\circ} 51' 26''$

cos. = 9.826710

Moon's " - $41^{\circ} 33' 26''$

cos. = 9.874073

$2) 38.383503$

Sun's + Mon's alt. $2) 89^{\circ} 24' 52''$

19.191751

Sun's + Moon's Alt.

= $44^{\circ} 42' 26''$ cos. = 9.851693†

2

Sin. auxl. a = $9.340058 = 12^{\circ} 38' 20''$

Auxl. a cos. = $9.989847†$

Sin. Tr. lunar dist. = $9.841040 = 43^{\circ} 54' 25''$

2

2

True lunar dist. = $87^{\circ} 48' 50''$

* Vide KELLY'S SPHERICS, p. 195.

§ c. (N. A., p. xvii., M. E.) 3h P. M.

Sun's lunar dist. $88^{\circ} 29' 8''$ P. L. = 3277

True lunar dist. = $87^{\circ} 48' 50''$

$40^{\circ} 18''$ P. L. = 6500 3h P. M.

3223 = 1h 25m 42s

Greenwich time = 4h 25m 42s P. M.

LATITUDE BY THE NORTH STAR.

§ 515. In northern latitudes, the latitude may be found at any time of the night, by an altitude of the north star.

§ 516. If the north star were situated in the pole of the world, its altitude would always show the elevation of the north pole; and (§ 251. § a.) consequently to the observer, his latitude.

§ 517. But as this star revolves in a very small circle around the pole, the elevation of the pole may always be determined by applying to the star's altitude at any time, the corrections opposite to that time in the subjoined tables (A & B).

§ 518. The corrections in Table A, and those after (*'s alt. +) in the hour columns of Table B, are *always* to be added to the star's alt. They are expressed in seconds (").

§ 519. And the other corrections in Table B, must be applied to the star's altitude, according to the precept, which is at the head of the hour column in which the *sidereal* time is found.

§ 520. To find the lat. by an alt. of the north star; the altitude being corrected for dip, refraction, etc.

§ a Subtract 1' from the star's alt.

§ b. For the supposed time of night when the observation is made, take the corresponding sidereal time; the correction found in Table B and opposite to this time, being applied, according to the precept (§ 519.) at the head of its column, to the star's alt., gives the approximate latitude.

§ c. And the corrections under the hour in Table B, and opposite the hour, but under the "approximate latitude," in Table A, being added to the approximate latitude, give the lat. of the observer.

§ 521. May 4, 1834, long. $85^{\circ} W$. the altitude of the north star, at 7h P. M., was $35^{\circ} 29' 43''$. To find the lat.

§ a. Supposed time - - 7h 0m 0s P. M.

Longitude in time - - 5h 40m 0s

Time at Greenwich - - 12h 40m 0s (§ 396. § a.)

Sidereal time, May 4 - - 2h 47m 34s (N. A., p. ii., M. E.)

Supposed time - - 7h

Acceleration of S. T. for 12h 40m 2m 5s (§ 213. § b.)

Sidereal time of obs. - - = 9h 49m 39s

$\S b.$	Star's alt. $35^{\circ} 29' 43''$	
	<u>1'</u>	($\S 520. \S a.$)
	$35^{\circ} 28' 43''$	
Oppos. $9h 49m 39s$ (Table B) correction +	$1^{\circ} 3' 38''$	
	<u> </u>	
	Apprx. lat. $36^{\circ} 32' 21''$	
Under $9h$, *'s alt. + (Table B)	-	66''
Oppos. $9h 49m 39s$ and under 40° (Table A)	-	<u>32''</u>
	Latitude $36^{\circ} 33' 59''$ N.	

$\S 522.$ Jan. 14, 1834. Long. 45° East $9h$ P. M.; the altitude of the north star was $41^{\circ} 11'$. To find the lat.

$\S a.$ Supposed time	-	$9h$	
Longitude in time	-	<u>$3h$</u>	
Time at Greenwich	-	$6h$	($\S 396. \S b.$)
Sidereal time, Jan. 14	-	$19h 23m 53s$	(N. A., p. ii., M.E.)
Supposed time of obs.	-	$9h$	
Acceleration of S. T. for $6h$	-	<u>$59s$</u>	($\S 213. \S b.$)
Sidereal time of obs.	-	$28h 34m 52s$	$-24h = 4h 34m 52s$

$\S b.$ Star's altitude	-	$41^{\circ} 11'$	
	-	<u>1'</u>	
	-	$41^{\circ} 10'$	
Opposit. $4h 34m 52s$ (Table B)	Cor.	<u>$-56' 15''$</u>	
Approx. lat.	-	$40^{\circ} 13' 45''$	
Under $4h$, *'s alt. + (Table B)	Cor.	$56''$	
Oppos. $4h 34m 52s$, and under 41° (Table A)	-	<u>$41''$</u>	
	-	Latitude $40^{\circ} 15' 22''$ N.	

$\S 523.$ Aug. 8, 1834, longitude 150° East. The alt. of the north star at 4 A. M. was $49^{\circ} 33'$. To find the latitude.

$\S a.$ Supposed time $4h$ A. M.	-	$= 16h$	
Longitude in time	-	<u>$= 10h$</u>	($\S 221. \S b.$)
Time at Greenwich	-	$= 6h$	
Sidereal time, Aug. 8	-	$9h 6m 3s$	
Supposed time of obs.	-	$16h$	
Acceleration of S. time for $6h$	-	<u>$59s$</u>	
Sidereal time of observation	-	$25h 7m 2s$	$= 1h 7m 2s.$

§ b. Star's altitude	-	-	-	-	51° 34'
					1'
					51° 33'
Oppos. 1h 7m 2s (Table B)	Cor.				-1° 41' 54''
Approx. latitude					50° 1' 6''
Under 1h, *'s alt.+(Table B)	Cor.				58''
Oppos. 1h and under 50° (T. A)	"				0''
					Latitude 50° 2' 4''

§ 524. An error of several degrees in the longitude, and of several minutes in the time of taking the star's altitude, brings only a small error into the latitude, therefore the ship's time, and longitude by dead reckoning, may always be used for finding the latitude at sea by the polar star.

*Tables for finding the Latitude by the alt. of the North Star.**

TABLE A.

Sidereal Time.	APPROXIMATE LATITUDE.							Sidereal Time.
	15°	30°	40°	50°	60°	65°	70°	
h m	"	"	"	"	"	"	"	h m
0.	1	3	4	6	9	12	15	12
30	0	1	1	2	2	3	4	30
1.	0	0	0	0	0	0	0	13.
30	0	1	1	2	2	3	3	30
2.	1	3	4	6	9	11	14	14.
30	3	7	9	13	13	24	31	30
3.	5	11	16	23	34	42	53	15.
30	7	17	24	34	50	62	79	30
4.	10	22	33	46	67	83	106	16.
30	13	28	41	58	85	109	134	30
5.	15	34	49	70	101	128	160	17.
30	18	38	56	79	115	145	182	30
6.	19	42	61	87	126	159	199	18.
30	20	44	65	91	133	168	210	30
7.	21	45	64	93	135	170	214	19.
30	21	44	64	91	133	168	211	30
8.	20	42	60	87	127	159	200	20.
30	18	38	50	79	116	146	183	30
9.	15	44	43	70	102	129	161	21.
30	13	28	32	59	86	109	136	30
10.	10	23	29	47	69	84	109	22.
30	8	17	27	35	50	62	80	30
11.	5	11	17	23	34	42	54	23.
30	3	7	10	13	20	25	32	30
12.	1''	3''	4''	6''	9''	12''	15''	24.
Sidereal Time.	15°	30°	40°	50°	60°	65°	70°	Sidereal Time.

* Vide N. A. for 1834, p. 483.

TABLE B.

Sidereal Time.			Corrections.	Sidereal Time.			Corrections.
H	M	H		H	M	H	
1	0	12	1° 31' 19"	6	0	18	0° 24' 43"
	10		32 12		10		20 44
	20		33 1		20		16 41
	30		33 39		30		12 36
	40		34 7		40		8 31
	50		34 24		50		4 24
1	0	13	1° 34' 30"	7	0	19	0° 01' 6"
	10		34 25		10	7	3 51
	20		34 10		20		7 58
	30		33 44		30		12 4
	40		33 7		40		16 8
	50		32 19		50		20 11
2	0	14	1° 31' 21"	20	0	8	0° 24' 12"
	10		29 13		10		28 9
	20		28 54		20		32 4
	30		27 25		30		35 55
	40		25 46		40		39 41
	50		23 57		50		43 23
3	0	15	1° 21' 59"	21	0	9	0° 47' 1"
	10		19 51		10		50 33
	20		17 34		20		53 59
	30		15 8		30		57 19
	40		12 34		40	1°	0 32
	50		9 52		50		3 38
4	0	16	1° 7' 1"	22	0	10	1° 6' 38"
	10		4 3		10		9 29
	20		0 57		20		12 13
	30	0°	57 45		30		14 48
	40		54 26		40		17 15
	50		51 0		50		19 33
5	0	17	0° 47' 29"	23	0	11	1° 21' 42"
	10		43 53		10		23 42
	20		40 11		20		26 32
	30		36 25		30		27 12
	40		32 35		40		28 42
	50		28 41		50		30 3
6	0	18	1° 24' 43"	24	0	12	1° 31' 19"
H	M	H	Corrections.	H	M	H	Corrections.

TIDES.

§ 525. The flux and reflux of the waters, known under the name of **TIDES**, are caused by the attractive forces of the sun and moon exerted upon the ocean.

§ 526. The moon being nearer to the earth than the sun is, has a greater effect than the sun upon the tides.

§ a. The action of the moon upon the tides, is about three times greater than that of the sun.

§ 527. When the attractive forces of the sun and moon act in conjunction, they produce the highest tides.

§ a. When this is the case, the moon (§ 367.) is in syzygy.

§ b. And the tides caused about this time are called *spring* tides.

§ 528. The tides have the least rise and fall, when the attractive forces act perpendicularly to each other.

§ a. And this is the case, when the moon (§ 369. § b.) is in quadrature. Then the tides are called *neap* tides.

§ 529. That portion of the ocean which is immediately under, and nearest to, the sun or moon, is more attracted by either than the centre of the earth is. This portion then, has a tendency to approach the attracting body, and rises up, until its tendency is counteracted by the attraction of gravitation towards the centre of the earth.

§ a. About twelve hours afterwards, this portion of the ocean, owing to the earth's diurnal motion (§ 206. § a.), is at the furthest point from the sun or moon; and the waters about it, owing to their tendency to restore the equilibrium, which is disturbed by the effects of the attraction of the sun or moon on the opposite side of the earth, rise up and make high tide again.

§ b. Hence, during the time in which the moon is performing one revolution around the earth (§ 362. § d.), the tide rises twice, and falls twice, at the same place.

§ 530. The attractive forces of the sun and moon upon equal portions of the surface of the sea, being (§ 526. § a.) about 3 to 1 in favour of the latter; if the solar tide at any place be 2 feet and the lunar 6, and the moon be in syzygy, the two tides will happen at the same time, and (§ 527. § b.) make a spring tide of 8 feet.

§ a. Then, as the lunar (§ 360. § a.) is longer than a solar day, the following solar will happen earlier than the succeeding lunar tide, by the difference between half a solar and half a lunar day.

§ b. Thus, the lunar tide continues to retard upon the solar tide, until the moon quadrates; when the high lunar and low solar tides coincide in time, and (§ 528. § a.) we have a neap tide of four feet rise.

§ c. After this, the lunar still continues to retard upon the solar tide, until the moon arrives in the other syzygy; when the two high tides again happen at the same time, and bring about other spring tides.

§ *d.* At this second spring tide, the moon has completed half of a revolution in its orbit, and has lost one tide upon the sun; therefore, while the moon is completing one entire revolution through its phases, there are two more solar, than lunar, tides.

§ 531. The attraction of the moon being more partial, and its effect upon small portions of the earth's surface being more obvious than that of the sun, the combined tides are governed more by the moon than by the sun.

§ 532. The effects of the moon's attraction upon the earth, tend (§ 529.) to create high tide, both in that part of the ocean immediately under, and nearest to, the moon, and in that part diametrically opposite; and were not the motion of the waters resisted and obstructed, there would be high tide at the moment when the moon crosses either the superior or the inferior meridian.

§ *a.* But, at some places, the passage of the moon across the meridian, precedes, by several hours, the time of high water.

§ *b.* This retardation of the tides might be explained, by attributing the retarding power to the effects of the resistance, which the shores, unequal depths of ocean, etc., offer to the mass of moving waters, were it not, that the waters do not rise up into spring tides, when the moon is in syzygy, and when the attractive forces of the sun and moon are combined in their action upon the waters; but the highest tides in every synodical month (§ 362. § *c.*), is generally about the third tide after the passage of the moon through either syzygy.

§ *c.* And the least neap tide, is generally the third tide after the moon quadrates.

§ 533. Observations show, that the tides require a little longer time to ebb, than they do to flow.

§ *a.* This difference observable at sea, becomes more obvious in rivers of strong currents.

§ 534. Upon the smaller seas, and sheets of water, such as the Lakes, the Mediterranean, etc., the effect of the lunar and solar attractions, are partially counteracted by the circumscribed limits to their action; and hence but little, or no tide is produced there.

§ 335. There are a variety of causes, (and some of a local nature), which, acting together, tend, some to retard at one place, and others to hasten at another place, the hour of high tide on the full and change days of the moon; so that, there are no general rules for determining beyond certain limits of approximation, the time of high or low water at any place.

§ 536. The greatest interval between two consecutive tides, generally happens about a day and a half after the moon has quadrated; when the tides, (§ 532. § *c.*) being at the minimum of their rise and fall, are the weakest.

§ *a.* And the least retardation of one tide after another, is the 2d upon the 3d, or the 3d upon the 4th tide, after the moon's latest passage through a syzygy; the tides, having then (§ 532. § *b.*) the greatest rise and fall, are strongest.

§ 537. On an average, the tides rise and fall once, in about 12 $\frac{1}{2}$ 48m.

· § a. Consequently every tide retards, at a mean, about 24m, upon the one which it succeeds; and each tide happens about 48m later than it did the day before.

§ 538. Were there not so wide a difference in the tides, between the minimum and maximum of their retardation, the hour of high or low water, at any place and on any day, might be found by adding to the time of high water on full or change days, the product of 48m, and the number of days from the latest full or new moon, to the day proposed.

§ a. About the time of neap tides, there is sometimes a difference of more than an hour and three quarters in the time of high water, on two successive days.

§ b. And a similar difference about the time of spring tides, is frequently less than half an hour.

§ 539. The annexed table (C) shows the retardation of the *mean* tides, upon the hour of high tide on full and change days, for every day between the full and change of the moon.

§ a. Therefore, to find the time of mean high water on any day, we have only to look in the Nautical Almanac (page xii. M. Ephemeris), for the day of the latest new, or full moon, in order to find its age (in days) since the last change in syzygy; the time, in the annexed table (C), found opposite to this age, and added to the time of high water at the place proposed, on full and change days, gives the time of mean high tide for the day when it is so required.

§ b. The hour of high tide at any port, on full and change days, is called the *establishment* of the port.^a

§ c. And the *establishment of the port* for any one place is always the same; for on every full and change day throughout the year, high tides are considered to take place there at the same hour of the day.

§ 540. A table for finding the time of high or low water, at any place, the establishment of the port (§ 539. § b.) being known.

TABLE C.

From \bullet to \circ		Days.	From \circ to \bullet	
Corrections.			Corrections.	
H	M		H	M
	21	$\frac{1}{2}$		20
	41	1		41
1	0	$1\frac{1}{2}$	1	5
1	19	2	1	28
1	38	$2\frac{1}{2}$	1	47
1	58	3	2	6
2	18	$3\frac{1}{2}$	2	26
2	38	4	2	46
2	58	$4\frac{1}{2}$	3	5
3	19	5	3	24
3	40	$5\frac{1}{2}$	3	45
4	3	6	4	6
4	26	$6\frac{1}{2}$	4	28
4	51	7	4	50
5	15	$7\frac{1}{2}$	5	15
5	41	8	5	44
6	14	$8\frac{1}{2}$	6	14
6	53	9	6	48
7	30	$9\frac{1}{2}$	7	28
8	5	10	8	10
8	38	$10\frac{1}{2}$	8	47
9	7	11	9	20
9	29	$11\frac{1}{2}$	9	48
9	51	12	10	14
10	18	$12\frac{1}{2}$	10	35
10	48	13	10	35
11	10	$13\frac{1}{2}$	11	16
11	30	14	11	38
11	51	$14\frac{1}{2}$	11	59
12	12	15	12	20

§ a. To find the time of high water at Charleston, Aug. 29, 1834. The establishment of the port being 7h 5m.

§ b. The last change in syzygy (§ 367.) was full moon, Aug. 18th, 20h (N. A., p. 12, M. E.); consequently the moon's age is 10 days.

§ c. Opposite to 10d, and in the column, from \circ to \bullet , is 8h 10m, which, added to the establishment of the port, gives 15h 25m, the hour of high water on the day proposed.

§ d. To find the time of high water at Portland, Feb. 14, 1834; the establishment of the port being 10h 45m

§ e. The last change in syzygy was new moon February 3; consequently the age of the moon is six days.

§ f. Opposite to 6d, and in the column, from \bullet to \circ , is 4h 3m, which, added to 10h 45m, gives 14h 48m, the time of high water.

§ 541. This method does not show the precise hour of high water; but it approximates the true time of high water, within limits which will serve on all ordinary occasions.

§ a. The time of low water may be found by adding 6h 15m to the time of high water on the day proposed.

* The arguments \bullet to \circ , mean from *New* to *Full* moon; and \circ to \bullet , from *Full* to *New* moon.

NAVIGATION.

NAVIGATION.

§ 542. The geographical situation of places, is designated by their distance, north or south, from the equator; and by their distance, east or west, from a prime meridian (§ 252. § c.).

§ a. These distances (§ 248. § b. & § 252.) are known by the name of Latitude and Longitude of the places.

§ 543. It has been shown in Nautical Astronomy, how the latitude and longitude of places may be determined by means of observations made upon the sun, moon, or stars; but in practice these means are not always at hand.

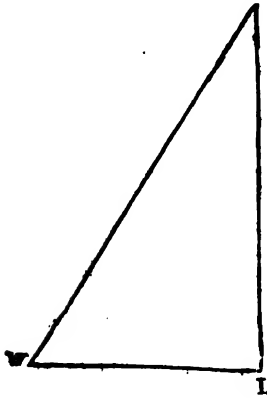
§ a. It remains then to be shown, how the geographical position of places may be determined, by knowing in certain respects, their relative situation with regard to each other.

§ 544. Washington city is to the southward and westward of Baltimore. The difference of latitude between them (§ 254. § c.) is the meridional arc, which is contained between Baltimore and the parallel of the latitude of Washington.

§ a. And the arc of this parallel which is between Washington and the meridian of Baltimore, (§ 254. § b.), is the difference in longitude between the two places.

§ b. These two arcs are perpendicular to each other; for the meridian of Baltimore (§ 248. § a.) cuts the equator at right angles, and the parallel of Washington (§ 249.) is parallel to the equator.

§ c. Now, if a line (W B) be drawn direct from Washington to Baltimore, it will connect their difference of longitude (W L) and of latitude (B L), and form a right angled triangle (W L B), of which the hypotenuse (W B) is the distance, and the legs, the difference of latitude and of longitude, between Washington and Baltimore.



§ d. The two acute angles (W & B) show the bearings of each place from the other. The angle W is the course from Washington to Baltimore, and the angle B is the course from Baltimore to Washington.

§ e. Hence if, of the Dist., Diff. Lat., Diff. Long., and Course, between two places, any two be given, the others are determinable.

§ 545. Now, the section of a loxodromic curve, which a ship traces while she is sailing upon a given course, is the hypotenuse of a right angled triangle, and corresponds to W B.

§ a. Then if the course and distance, which a ship sails in any given time, be known, we have the hypotenuse and an acute angle of a right angled triangle, given; and the other parts are determinable.

§ b. Therefore, knowing the latitude and longitude of the place from which a ship takes her departure, and knowing the course and distance which she sails from that place, in a specified time, we may determine by right angled trigonometrical calculations, the latitude and longitude of the ship, at the end of that time.

§ 546. The sides of all such triangles are curved lines; for the earth (§ 204.) is a spheroid, therefore all triangles upon its surface are spherical triangles, and are properly subject to the rules of spherical trigonometry, for investigation.

§ a. But the laborious operations of obtaining results from these triangles, when brought under calculation, as spherical triangles, would be inconvenient in practice; and they are not necessary in the ordinary purposes of navigation.

§ b. Therefore all such triangles are considered in navigation as right angled plane triangles, and are brought under calculation according to the rules of plane trigonometry.

§ 547. A degree ($^{\circ}$) of all great circles, such as the equator, meridians of longitude, etc., whose radii and the earth's semidiameter are the same, consists of 60 *nautical miles*, or of about $69\frac{1}{2}$ statute miles; so that, calling the number of minutes ($'$), miles, which is contained in an arc of one of these great circles, we have the value of this arc considered as a straight line.

§ a. Wherefore, knowing the distance and the difference of latitude between any two places, the difference of longitude between them, may be determined also in *nautical miles*, by right angled, plane trigonometrical calculations founded upon § 75.

§ b. The difference of longitude thus found, is called departure.

§ 548. *Departure* is the difference in longitude between any two places, expressed in *miles*.

§ a. And the departure in any latitude, may be converted into its corresponding minutes ($'$) of longitude by means of the principles established under § 76.

§ 549. It is evident, that, if in mathematical calculations, curved lines be treated as right lines, there must be an error in the result; and this error increases with the number of degrees contained in such curved lines.

§ 550. The result then, obtained by calculating the sides of a spherical triangle, as though they were the straight lines of a plane triangle, is only an approximation; and the smaller the sides, the closer is the approximation.

§ a. This method therefore is particularly applicable in small distances, and short runs; it is used for working up "dead reckoning", and for calculating the distance, the difference of latitude and

of longitude, and the course, from one place to another; any two of which four quantities being known, (§ 544. § e.), the others are determinable.

§ 551. The diagram shows by a mere inspection, the } solution of every problem which can occur in right angled } Plate 1.
plane trigonometry, provided the hypotenuse of the triangle involved, do not exceed 300 miles, or 300 units of any measure.

§ a. This diagram may answer the purpose of Tables IV. & V.; and it also shows the principles upon which they are constructed; for the number in the Dist. column of these tables, is the length of the hypotenuse of a right angled triangle, whose acute angles contain, one the number of degrees at the top, and the other those at the bottom of the page, and the value of whose legs is set down opposite to that of the hypotenuse, and in the columns marked Dep. and D. Lat.

§ 552. This diagram, then, shows the solution of every case, or problem in loxodromic sailing; for it shows the dimensions of the triangle involved in every problem, provided the distance or hypotenuse do not exceed 300 miles.

§ a. If it exceed 300 miles, the solution may be obtained by dividing the given side or sides, by 2 or 3, or by any other divisor, which will reduce the triangle of the problem within the limits of the diagram; then the value of the side or sides, obtained, by using said quotient in the diagram, being multiplied by the same divisor (§ 70.), gives the value of the required side or sides.

§ 553. The angle B A D is 45° .

§ a. The acute angles of a right angled triangle (§ 32. § d.) being complementary to each other, the less must always be located at A.

§ b. And the leg adjacent to it, must be called the *base*, and be a part of A D.

§ c. And the leg opposite to it, must be called the *perpendicular*.

§ d. All the straight lines standing upon A D, are alluded to as *perpendiculars*.

§ e. B C is the greatest *perpendicular* in the diagram. Equal portions of it are transferred by means of the parallels *b d*, to the graduated arc B D, for admeasurement.

§ f. The graduated arc B D is thus made to answer the purpose of a graduated *perpendicular*, for the arc contains the same divisions which the *perpendicular* B C would have shown, had it been graduated to the scale of A B and A D.

§ g. The height of every *perpendicular* is therefore shown on that portion of the graduated arc, which is between the *base* (§ b.) and that parallel, under which the *perpendicular* stands. Thus the height of the *perpendicular* *b C*, transferred by the parallel *b d*, to the graduated arc, is $D d = 11.3$.

§ 554. The arcs in the diagram show the angular value of any course, provided it be not more than 4 Points, or 45° .

§ a. If the course be greater than this, they show the angular value of its complement.

§ b. The arcs, 3, 4, 5, etc., also show the length of the *base* and

Plate 1. } hypotenuse of the triangle proposed; while the scale on
 the graduated arc (§ 553. § g.) shows the value of the *perpendicular* ($b c$) in the several triangles.

§ c. The section of any arc, which is contained between the *base*, and the lines $A e$, shows the angular value of the courses which are marked *upon* these lines; and the angular value of the complements (§ 553. § a.) of the courses, marked *under* these lines.

§ d. The marks on the arcs show the value of these arcs in degrees.

§ e. The even numbers (2° , 4° , 6° , etc.) of degrees are marked on the concave side of the arcs; and the odd numbers (1° , 3° , 5° , etc.) are marked on the convex side.

§ f. The degrees (46° , 47° , 48° , etc.) that stand on the outside of the graduated arc, are the complements of those (42° , 43° , 44° , etc.) on the inside of the arc.

§ g. The entire length of every arc (§ 553.) is 45° .

§ 555. The dotted lines, $A e'$, represent quarters of Points; the broken lines $A e$, represent halves of Points; and the continuous lines $A e$ whole Points.

§ a. The distance on a given course is shown on the lines $A e$, at the end of which lines that course is found; then the portion of that line ($A e$) which is contained between A , and the arc 3, 4, etc., which is marked with the distance proposed, is hypotenuse to the triangle required.

§ b. Thus 24 miles on a $2\frac{1}{2}$ Point course, is the distance $A b$ on the broken line $A e$, at the end of which $2\frac{1}{2}$ is marked; and $A C b$ is the right angled triangle, of which the course and the distance proposed constitute an angle and the hypotenuse.

§ c. The two legs of this triangle are the *base* $A C$, and the *perpendicular* $b C$ which stands under the parallel $b d$.

§ d. The graduated scale on $A D$ shows the value of $A C=21.1$, the *D. lat.*; and the portion ($D d$) of the scale on the graduated arc, between the *base* and the parallel $b d$ (§ 553. § g.), shows the height of the *perpendicular* $b C=11.3$, the *Dep.*

§ 556. Now as the distance which a ship may sail upon any loxodromic course, is considered (§ 546. § b.) as the hypotenuse of a right angled plane triangle, if the distance sailed upon such course can be found in the diagram, the departure and difference of latitude, which correspond to that course and distance, may be found by means of the graduated *base* and arc, $A D$ and $B D$.

§ a. Or, if any two of these quantities; viz., Course, Dist., Dep., and Diff. Lat., be known, the other two are determinable by means of the diagram.

§ 557. A ship sails N. W. $\frac{1}{2}$ N. 19 miles, What departure and difference of latitude does she make?

§ a. The given course is found *upon* the dotted line $A e'$, and that portion ($A b$) of this line, which is between A and the 19th arc, is the hypotenuse of the triangle ($A c b$) involved.

§ b. The *perpendicular* ($b c$) intersects the *base* at 14.1, which is the difference of latitude.

§ c. And the parallel, from the top of the *perpendicular*, } intersects the graduated arc at 12.7, which is the departure. } Plate 1.

§ 558. A ship sails N. W. by N. and makes 12.7 miles of departure, What distance and diff. lat. does she make?

§ a. The parallel from 12.7 on the graduated arc, intersects the N. W. by N. line, in the point, (nearly), where the 23d arc crosses it; then nearly 23 (22.9) miles in the distance sailed.

§ b. And the *perpendicular* from this point of intersection, falls upon the *base*, and shows the difference of latitude required, to be 19.1 miles.

§ 559. A ship sails N. by W. $\frac{1}{2}$ W. and makes 19.8 miles diff. lat., What departure and distance does she make?

§ a. The *perpendicular* at 19.8 on the *base*, being traced up, is found to intersect the N. by W. $\frac{1}{2}$ W. line on the 21st arc; then 21 miles is the distance sailed.

§ b. And the parallel from this point of intersection, being traced to the graduated arc, shows the height of the perpendicular 7 miles, which is the departure required.

§ 560. A ship, after sailing 18 miles, finds that she has made 16.2 miles diff. lat., What course did she sail, and what departure has she made?

§ a. The *perpendicular* from 16.2 on the *base*, intersects the 18th arc on the dotted line marked $2\frac{1}{2}$ Points, which is the course required.

§ b. And the parallel from this point of intersection, shows, on the graduated arc, the height of the perpendicular to be 7.8 miles, which is the departure required.

§ 561. A ship sails 28 miles, and finds that she has made 15.6 miles departure, What course and diff. lat. has she made?

§ a. The parallel from 15.6 on the graduated arc, cuts the 28th arc, on the line marked 3 Points, which is the course sailed.

§ b. And the *perpendicular* from this point of intersection, falls upon the *base* at 23.3 miles, which is the diff. lat. required.

§ 562. The departure between two places is 8.7 miles, and the diff. lat. between them is 28.6 miles, What is the course, and the distance from one place to the other?

§ a. The *perpendicular* from 28.6 on the *base*, intersects the 30th arc on the broken line, $1\frac{1}{2}$, which shows the required course and distance to be $1\frac{1}{2}$ Points, and 30 miles.

§ 563. If the distance sailed be not more than a mile, or if the hypothenuse of the triangle proposed, be not greater than 3, the divisions on the scales must be decimated; then the figures 1, 2, 3, 4, 5, etc., stand for $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, $\frac{5}{10}$, etc., and the subdivisions stand for 100ths.

§ a. Wherefore, when the distance is not more than 3 miles, the distance from A, to the 10th arc, is 1 mile; to the 20th, 2 miles; and to the 30th, 3 miles; and all the numbers in the diagram stand for 10ths.

§ 564. What departure and difference of the lat. would a ship make by sailing 2 miles N. W. $\frac{1}{2}$ N.?

C c

Plate 1. } § a. The parallel and *perpendicular* which would pass through the point in which the 20th arc intersects the N. W. $\frac{1}{2}$ N. line, measured by the eye, would cut the graduated arc and base in 12.7 and 15.4.

§ b. And using only one decimal place (§ 563.), the nearest would be 1.5 diff. lat. and 1.3 departure; but the exact is 1.54 and 1.27 diff. lat. and dep.

§ 565. When the lines in the diagram do not pass through the point required by the conditions of the triangle involved, the imaginary lines required, may be traced by the eye with all the precision which is necessary; and after a little practice, with the utmost accuracy and facility.

§ a. Thus, the distance of one place from another, is N. by W. $27\frac{1}{2}$ miles; the triangle involved in this problem, being filled up by the eye, the imaginary parallel and *perpendicular*, cut the graduated arc and base in 5.3 and 27; which shows the departure between the two places to be 5.3 miles, and the diff. lat. 27 miles.

§ 566. If the distance sailed on a single course exceed 30 miles, or the hypotenuse of the triangle involved, be greater than 30, all the numbers in the diagram must be increased tenfold.

§ a. Then the arcs must be reckoned as being 10 miles a part; the numbers 1, 2, 3, 4, 5, etc., will stand for 10, 20, 30, 40, 50, etc.; and the distance between these numbers being divided, every one in 10 equal parts, every one of these subdivisions must be counted as 1 mile.

§ 567. A ship sails N. W. 240 miles; what departure and diff. lat. does she make?

§ a. The parallel and *perpendicular* from the intersection of the 24th arc (now counted (§ 566.) 240), with the N. W. line, cuts the graduated arc and base (§ 41.) at equal distances, and the departure and diff. lat. (§ 14.) are the same.

§ b. Departure = 169.5. diff. lat. = 169.5. This is $\frac{2}{5}$ of a mile less than the result by logarithmic calculation. This difference is owing to the smallness of the scale upon which the diagram is projected, and not to any defect in the principles involved in the projection of the diagram.

§ 568. In the examples quoted above, the courses are *on* the lines, and the diff. lat., except (§ 567.) when the course consists of 4 Points, is always greater than the departure.

§ 569. The courses and figures *under* the lines, (§ 554. § e.) are the complements of those *on* the line above them; and when the course is found *under* the line, the departure is the longest leg of the triangle involved, and must be read off upon the base A D; and the diff. lat. is taken from the graduated arc B D.

§ a. A ship sails E. $\frac{1}{2}$ N. 28 miles. What departure and diff. lat. does she make?

§ b. The parallel and *perpendicular* which pass through the point where the 28th arc cuts the dotted line, *under* which the given course is marked, meet the graduated arc and base in 1.3 and 27.9+.

§ c. Then (§ 569.) the departure = 27.9. and the diff. lat. = 1.3.

§ 570. The four cardinal points of the horizon (§ 272.) } Plate 1.
 divide it into four quadrants, each containing 8 Points. }

§ a. Wherefore a Point = $11^{\circ} 15'$, the quotient of 90° by 8.

§ 571. The Points, when expressed by numbers, begin at the north and south, and are reckoned in numerical order, to the east and west points.

§ a. Hence, it is easy to conceive that a vessel that sails N. 3 Points W., or N. W. by N., would make more *northing* than *westing*, and that her diff. lat. would be greater than her departure.

§ b. And conversely, that if a vessel sail on any course between 4 and 8 Points, e. g., N. 5 Points W., or N. W. by W., she would make more *westing* than *northing*, and that consequently her departure would be greater than her diff. lat.

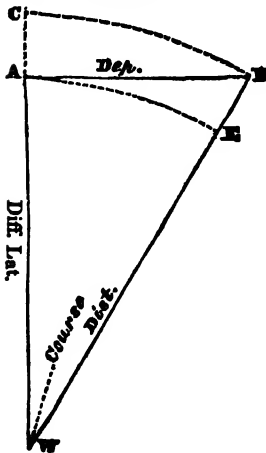
§ 572. From what has already been advanced in explanation of the diagram, it appears that the triangle involved in the trigonometrical solution of any problem in *loxodromic* sailing, is a right angled plane triangle.

§ a. *Loxodromic* sailing,* is so called from the sort of curve which the track of a ship forms, when she is sailing upon any course not exactly due east or west, north or south.

§ b. *Loxodromic* curves are spirals, which continually approach the poles, but can never reach them.

§ c. And any course, not due east or west, north or south, is called a *loxodromic* course.†

§ 573. A number of formulæ, varied in the trigonometric functions to be used, may be drawn up for the solutions of the several cases in *loxodromic* sailing.



§ a. The terms in these formulæ depend upon the side of the triangle, whether it be the distance, the departure, or diff. lat., which is made radius.

§ 574. Baltimore is 35 miles N. N. E. $\frac{1}{4}$ E. from Washington. The formula for finding by calculation the departure and diff. lat. between the two places, depends upon the trigonometrical construction which is given to the triangle involved.

§ a. If the hypotenuse be made radius, to an arc B C, the dep. becomes (§ 54.) sine to the course, and the diff. lat. its co-sine.

§ b. Then, (§ 75.), rad. : dist. :: sin. of course : dep. :: cos. course : diff. lat.

* The subdivisions of the *sailings*, viz., Plane, Traverse, Parallel, and Middle Latitude Sailing, are not here preserved. These distinctions are by no means necessary for the purpose of facilitating the navigator's calculations; they have therefore been generalized under the term of *loxodromic sailing*.

† E. by N. is a *loxodromic* course. Suppose a vessel could sail E. by N.

§ c. Hence (§ 74.) are deduced the following formulæ; the three first being the given terms, and the fourth, the unknown and required term, of the proportion.

§ d. Sin. course : dep. : : rad. : dist. (§ 75. § b.).

§ e. Dist. : rad. : : dep. : sin. course.

§ f. Cos. course : diff. lat. : : rad. : dist.

§ g. Dist. : rad. : : diff. lat. : : cos. course.

§ h. Cos. course : diff. lat. : : sin. course : dep.

§ i. Dist. = 35. log. = - - - 1.544068
Course = 2½ pts. (§ 571.) sin. (Table D) = 9.711049

Log. dep. = 1.255117 = 18 miles.

§ j. Dist. 35. log. = - - - 1.544068
Course, 2½ pts. cos. = - - - 9.933350

Log. diff. lat. = 1.477418 = 30 miles.

§ 575. If the triangle be constructed upon the diff. lat. as radius of the arc A E, then the dist. becomes (§ 57.) secant, and the dep. (§ 56.) tangent, to the course.

§ a. And (§ 75.) sec. course : dist. : : rad. : diff. lat. : : tang. course : dep.

§ b. Wherefore, the diff. lat. being taken as radius, the following formulæ are derived either (§ 75. § c.) immediately from the construction of the triangle, or (§ 67. § e.) from the relation of the terms, as they are expressed in the proportion § a.

§ c. Diff. lat. : rad. : : dist. : sec. course.

§ d. Rad. : diff. lat. : : sec. course : dist.

§ e. Diff. lat. : rad. : : dep. : tang. course.

§ f. Tang. course : dep. : : rad. : diff. lat.

§ g. Tang. course : dep. : : sec. course : dist.

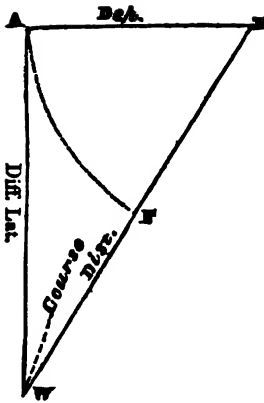
§ h. Course 2½ pts. cos. = 9.933350 (§ 101. § b.)
Dist. 35 miles, log. = 1.544068

Log. diff. lat. = 1.477418 = 30 miles.

§ i. Course 2½ pts. cos. = 9.933350
Dist. 35 miles, log. = 1.544068
Course 2½ pts. tang. = 9.777700

Log. dep. = 1.255118 = 18 miles.

without any obstruction, or without ever deviating from that course; the N. pole would then be always *one Point* forward of her beam. It is difficult to imagine how a vessel could ever arrive at such a point.



§ 576. Or if the triangle be constructed upon the dep., as radius of the arc A F, the dist. becomes secant, and the diff. lat. the tangent, of B, the complement (§ 32. § d.) of the course.

§ a. Wherefore (§ 75. § c.) co-sec. course : dist. :: rad. : dep. :: co-tang. course : diff. lat.

§ b. Other formulæ of proportion, similar to those under § 575., might be drawn out here, but these for the most part would be a repetition of those, as the expression 4×6 is a repetition of the form 6×4 of multiplication. Indeed several sets of the proportions under § 574. & § 575., are similar repetitions of each other.

§ c. Course $2\frac{1}{2}$ pts. $\sin.$ = 9.711049 (§ 101. § b.)
 Dist. 35 miles, $\log.$ = 1.544068

$$\text{Log. dep.} = 1.255117 = 18 \text{ miles.}$$

§ d. Course $2\frac{1}{2}$ pts. $\sin.$ = 9.711049
 Dist. 35 miles, $\log.$ = 1.544068
 Course $2\frac{1}{2}$ pts. co-tang. = 0.222300

$$\text{Log. diff. lat.} = 1.477417 = 30 \text{ miles.}$$

§ 577. The principles of the above trigonometrical construction are also developed in the solution of problems } Plate 1.
 by the diagram.

§ a. Thus, to find the course and dist. which correspond to 12 miles diff. lat., and 5 miles dep.

§ b. The *perpendicular* which cuts the *base* in 12, is tangent to the 12th arc, and A 12 is its radius.

§ c. The parallel from 5, on the graduated arc, cuts this *perpendicular* on the 13th arc, at its intersection with the 2 Point line, and A 13 is secant to the course (2 Points) and the dist. required.

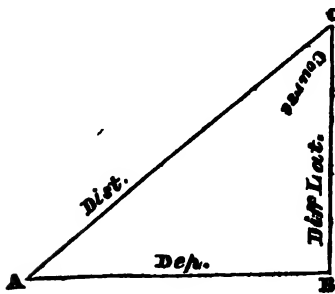
§ d. If the dist. A 13, on the 2 Point line be called radius of the 13th arc, then the portion of the *perpendicular*, between the point of this intersection, is sine of the course, and A 12 on the *base*, is its co-sine.

§ 578. Table of log. sines, etc., of the Points of the Compass.

TABLE D.

Points.	Sine.	Co-sec.	Co-sin.	Sec.	Tang.	Co-Tang.	
$\frac{1}{2}$	8.690795	1.309205	9.999477	0.000523	8.691319	1.308681	$7\frac{1}{2}$
$\frac{3}{4}$.991302	.008698	.997904	.002096	.993398	.006602	$7\frac{1}{4}$
$\frac{1}{2}$	9.166520	0.833480	.995274	.004726	9.171246	0.828754	$7\frac{1}{8}$
1	.290236	.709764	.991574	.008426	.298662	.701338	7
$1\frac{1}{2}$.385572	.614428	.986787	.013213	.398784	.601216	$6\frac{3}{4}$
$1\frac{1}{4}$.462824	.537176	.980885	.019115	.481939	.518061	$6\frac{3}{8}$
$1\frac{1}{2}$.527488	.472512	.973840	.026160	.553647	.446353	$6\frac{1}{4}$
2	.582840	.417160	.965615	.034385	.617224	.382776	6
$2\frac{1}{4}$.630992	.369008	.956163	.043837	.674829	.325171	$5\frac{3}{4}$
$2\frac{1}{2}$.673387	.326613	.945430	.054570	.727957	.272043	$5\frac{1}{2}$
$2\frac{3}{4}$.711049	.288951	.933350	.066650	.777700	.222300	$5\frac{1}{4}$
3	.744739	.255261	.919846	.080154	.824893	.175107	5
$3\frac{1}{4}$.775037	.224963	.904828	.095172	.870199	.129801	$4\frac{3}{4}$
$3\frac{1}{2}$.802359	.197641	.888185	.111815	.914173	.085827	$4\frac{1}{2}$
$3\frac{3}{4}$.827083	.172917	.869790	.130210	.957294	.042706	$4\frac{1}{4}$
4	.849485	.150515	.849485	.150515	0.000000	0.000000	4
	Co-sine.	Sec.	Sine.	Co-sec.	Co-tang.	Tang.	Points.

§ 579. In every case, except when the dep. and diff. lat. constitute the two given parts, the solution of the problem proposed, may be obtained by means of the proportion (§ 74.) between the sides of a triangle and sines of their opposite angles.



§ a. In the triangle A B C (§ 74.) right angled at B, (§ 74.) dist. : sin. B = 90° :: dep. : sin. course; but the sine of 90 (§ 62.) is radius, wherefore we have again the formula § e., (§ 574.), viz., dist. : rad. :: dep.

: sin. course, and a repetition of others under § 574.

§ 580. Some one of the following formulæ (rejecting or borrowing 10 for rad., (§ 197. § f.), in the index of the second member of the equation), may be used in every case where two of the four quantities, dist., course, dep. and diff. lat., are given.

§ a. Dep. (§ 574. § b.) = dist. × sin. course.

§ b. Dep. (§ 575. § e.) = diff. lat. × tang. course.

§ c. Diff. lat. (§ 574. § j.) = dist. × cos. course.

§ d. Diff. lat. (§ 575. § f.) = $\frac{\text{Dep.}}{\text{tang. course}}$ (§ 101. § b.) = dep. × cot.

course.

§ e. Dist. (§ 575. § d.) = diff. lat. \times sec. course.

§ f. Dist. (§ 576. § a.) = dep. \times co-sec. course.

§ g. Sin. course (§ 574. § e.) = $\frac{\text{Dep.}}{\text{dist.}}$

§ h. Cos. course (§ 574. § g.) = $\frac{\text{Diff. lat.}}{\text{dist.}}$

§ i. Tang. course (§ 575. § e.) = $\frac{\text{Dep.}}{\text{diff. lat.}}$

§ j. These formulæ will always be found convenient in practice.

OF TURNING DEPARTURE INTO DIFF. LONG.

§ 581. If the difference of longitude between two places (§ 548.) be expressed in miles, it is called departure.

§ a. A minute of longitude is no where, except at the equator, equal to a mile of departure; for the distance between any two meridians of longitude (§ 248. § a.) is greatest at the equator, and the least about the poles.

§ b. Therefore the dep. between any two meridians is greater at the equator than in any latitude.

§ c. Furthermore, as all meridians (§ 248. § a.) intersect at the poles, the dep. between any two becomes less and less, as you approach the poles, where there can be no dep.; whereas the difference of longitude, between these meridians in any latitude, (§ 254. § a.) is the same, being expressed by the same number of degrees ($^{\circ}$), minutes ($'$), etc.

§ 582. Suppose two places to be situated in the same latitude, the arc of this parallel of latitude, contained between them, (§ 254. § b.) is their difference in longitude.

§ a. And parallels of latitude (§ 249.) are small circles.

§ b. Suppose also an arc of a great circle (§ 122.) to be drawn from one of these places to the other, this arc (§ 122. § a.) would be the shortest distance between the two places, and the minutes ($'$) contained in it, would show (§ 547.) the departure between them.*

§ 583. These two arcs are contained between the same two points, and (§ 76.) the arcs are to each other, as the radius of the great circle is to the radius of the small circle.

§ a. The rad. of such a great circle, as well as of the equator, etc., (§ 122.) is semidiameter to the earth.

§ b. And the radius of the small circle, or the parallel, (§ 130. § a.) is the cos. of its latitude.

§ 584. Then, calling the semidiameter of the earth, unity, or rad., we have (§ 76.) for any latitude, diff. long. : dep. :: rad. : cos. lat.

§ a. Also cos. lat. : dep. :: rad. : diff. long.

b. And rad. : diff. long. :: cos. lat. : dep.

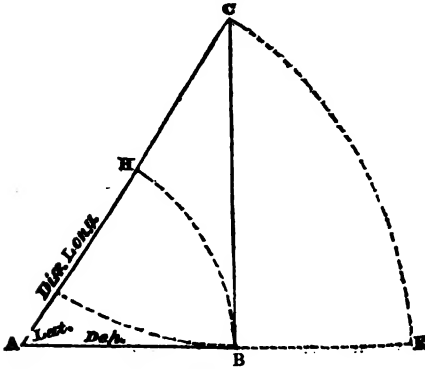
* Hence it appears that the *course* between two places, is never the shortest distance between them, unless they be either upon the equator, or upon the same meridian.

§ c. Wherefore, if any two of these three quantities, (rad. is always known), be given, the other and remaining one is determinable.

§ 585. The terms of these proportions represent quantities, comprised in the elements of a right angled plane triangle.

§ a. Wherefore, if the figure which furnishes such proportions, be considered a plane triangle; it will be right angled, the latitude will be one of its acute angles; the diff. long., the hypotenuse; and the dep. will be one of the legs of such triangle.

§ b. The result obtained from calculations conducted upon such principles, (§ 549.) must be only an approximation.



§ 586. Suppose a ship sailed E. $15\frac{1}{2}$ miles on the parallel of $56^{\circ} 15'$ south latitude, her departure would be $15\frac{1}{2}$ miles, the distance sailed.

§ a. And in the right angled triangle ABC, the angle $A = 56^{\circ} 15'$, the lat.; the leg $AB = 15\frac{1}{2}$, the dep.; and the hypotenuse (§ 585. § a.) is the diff. long.

§ b. Now, if the diff. long. be made radius, of the arc CE; CB becomes its sine, and AB its co-sine, whence (§ 75.) arise the proportions, under § 584.

§ c. Rad. : diff. long. : : cos. lat. : dep.

§ d. Diff. long. : rad. : : dep. : cos. lat.

§ e. Cos. lat. : dep. : : rad. : diff. long.

§ f. To find the corresponding diff. long. by calculation (§ e.).

Lat. $56^{\circ} 15'$ sec. = 0.255261 (§ 101. § b.)

Dep. 15.5 log. = 1.190332

Log. diff. long. = 1.445593 = 27.9 minutes (')

§ 587. If the dep. (AB) be made radius, to the arc BH, then the diff. long. becomes secant to the lat., and we have (§ 75. § c.)

§ a. Rad. : dep. : : sec. lat. : diff. long.

§ b. Sec. lat. : diff. long. : : rad. : dep.

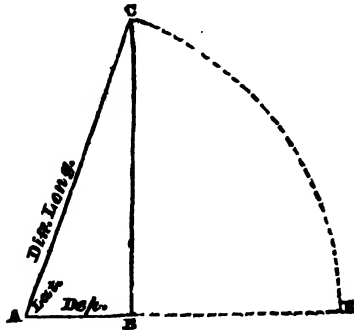
§ c. Dep. : rad. : : diff. long. : sec. lat.

§ 588. Now, if the other leg (CB) be made radius, the dep. becomes co-tang. (§ 57. § b.), and the diff. long. the co-sec., of the latitude, whence (§ 75.) arise the following sets of proportions.

§ a. Co-tang. lat. : dep. : : co-sec. lat. : diff. long.

§ b. Co-sec. lat. : diff. long. : co-tang. lat. : dep.

§ 589. To find the number of miles, or the dep., which correspond to 1° or 60' on the parallel of 67° 30'.



§ a. The diff. long. being made radius, we have, (§ 586. § c.),
 Rad. : diff. long. :: cos. lat. : dep.

$$\begin{aligned} \text{Diff. long. } 60 \text{ log.} &= 1.778151 \\ \text{Lat. } 67^\circ 30' \text{ cos.} &= 9.582840 \end{aligned}$$

$$\text{Log. dep.} = \underline{\underline{1.360991}} = 22.9$$

§ 590. C (§ 32. § d.) is the complement of A, and is equal to 22° 30'. C B is the cos. of C, and shows the number of miles contained in 1° of long. on the parallel of lat. 22° 30'.

$$\begin{aligned} \text{§ a. Diff. long. } 60 \text{ log.} &= 1.778151 \text{ (§ 589. § a.)} \\ \text{Lat. } 22^\circ 30' \text{ cos.} &= 9.965615 \end{aligned}$$

$$\text{Log. dep.} = \underline{\underline{1.743766}} = 55.4 \text{ miles,}$$

which is equal to 1° of long. in lat. 22° 30'.

§ b. Now (§ 75. § b.) $CB : \cos. C :: AB : \cos. A$. AB shows the number of miles equal to 1° of long. in lat. 67° 30', and CB shows the number of miles, that make 1° of long. in lat. 22° 30', the complement of 67° 30'.

§ c. Wherefore, in any lat., the miles in a degree of long. are to the cos. of that lat., as the miles contained by a deg. of long. in any other lat., are to the cos. of this lat. And alternately.

§ 591. As a right angled plane triangle (§ 585,) is in- } Plate 1.
 volved in the solution of the several problems under § 586. }
 § 587. § 588., etc.; the solution of every one of such problems may be obtained by means of the diagram.

§ a. The diff. long. (§ 585. § a.) is the hypotenuse; and the leg adjacent to the angle which in degrees (°), is equal to the latitude in which the dep. or diff. long. is made, is the dep.

§ 592. Of dep., diff. long. and lat., either is determined on the diagram, by means of the two others, in the same manner, that any

D D

Plate 1. } one of the three quantities, viz., diff. lat., dist. and course,
 } is determined (§ 556. § a.) by means of any two of these.

§ 593. When the latitude in which the diff. long. or dep. is made, is less than 45° , the degrees of lat. are to be found on the *inside* of the graduated arc; and the dep. on the *base*.

§ a. But when the latitude is *greater* than 45° , the degrees of it are to be found on the *outside*, and the dep. on, the graduated *arc*.

§ b. Thus, 26' of long. is equal to 24.5 miles, in lat. 19° ; and in lat. 71° , it is equal to 8.4 miles.

§ c. The *perpendicular* from the mark for the 19° th on the 26th arc, cuts the *base* at 24.5, which shows the dep. for 26' of long. in lat. 19° .

§ d. The parallel from 71° on the 26th arc, cuts the graduated arc at 8.4, which is the dep. (§ 593. § a.) for 26' of long. in lat. 71° .

§ 594. A ship in lat. 27° , makes 20.5 miles dep. To find the diff. long.

§ a. The diff. lat. is found on the *inside* (§ 593.) of the graduated arc, and the dep. on the *base*.

§ b. The *perpendicular* from 20.5 on the *base*, intersects 27° on the 23d arc, then 23' is the required diff. long.

§ 595. A ship in lat. 60° , makes 12 miles dep. To find the diff. long.

§ a. The degrees of the lat. (§ 593. § a.) are found on the *outside*, and the dep. on the graduated arc.

§ b. The parallel from 12, intersects 60° on the 24th arc; 24' is the diff. long.

§ 596. The diff. long. between two places in lat. 33° is 27'. To find the dist. between them.

§ a. The lat. is found on the *inside* (§ 593.) of the graduated arc, and the dep. on the *base*.

§ b. The *perpendicular* from 33° on the 27th arc, falls upon the *base* at 22.6, the dist. in miles between the two places.

§ 597. The diff. long. between two places in lat. 54° , is 24'. To find the dist. between them.

§ a. The degrees of lat. are on the *outside*, and the dist. (§ 593. § a.) is to be found on the graduated arc.

§ b. The parallel from 54° on the 24th arc, cuts the graduated arc at 14.1, the dist. required.

§ 598. The dep. between two places on the same parallel, is 18.4 miles, and the diff. long. is 20'. To find their lat.

§ a. The *perpendicular* from 18.4 cuts the 20th arc at 23° , the lat. required.

§ b. The dep. being found on the *base*, the degrees (§ 593. § a.) are taken from the *inside* of the arc.

§ c. But if the dep. were found on the graduated arc, then the degrees on the *outside* of the arc, would show the lat.

§ 599. The diff. long. being 20', and the dep. between two places in the same latitude being 6.8 miles; to find their lat.

§ e. The parallel from 7.8, cuts the 20th arc at 67° , { Plate 1.
which (§ 598. § c.) is the lat.

b. When the minutes (') of long. exceed 30', every division on the several graduated scales, (§ 566.) must be counted as a mile, or as a minute (').

§ c. And when the minutes (') of long. do not exceed 3', the divisions on the graduated scales (§ 563.) must be decimated.

§ 600. Of the diff. long., dep. and lat., either one may be found, the others being given, by means of Table V.

§ a. The degrees at the top of the table, are those which are *inside* of the arc; and those at the bottom of the table, and on the *outside* of the arc, are the same.

§ b. The numbers in the column marked (dist.) stand for minutes (') of longitude; they show the value of the hypotenuse of a triangle, whose legs are equal to the quantities in the next two columns; and whose acute angles contain, one the degrees at the *top*, and the other those at the *bottom* of the page.

§ c. When the lat. is found at the top of the page, the miles which are equal to any number of minutes of longitude, are opposite to them in the column marked, at the top, (d. lat.)

§ d. Thus, in lat. 41° , 60' of long. = 45.3 miles.

§ e. If the degrees of lat. be found at the bottom of the page, then the miles which are equal to any number of minutes (') of long., are found opposite to them in the column marked (d. lat.), at the bottom.

§ f. Thus, in lat. 71° , 60' of long. = 19.5 miles.

§ 601. When a ship sails due east, or west, she sails either upon a parallel of lat., or upon the equator, and the whole distance sailed is dep.; which may be converted into diff. long. by the formula § f. (§ 566.), or by means of the diagram (Plate I.), or Table V.; either of the two latter means is most convenient in practice.

§ 602. When a ship sails due North or South, she sails upon the arc of a meridian, and the whole distance sailed is diff. lat.

§ 603. It is frequently the case, that a ship sails during the day, on more courses than one, making a zig-zag track.

§ a. In such cases, it would be a very tedious operation, after finding the diff. lat. and dep. for every course and distance, to convert the dep. for every such course and dist. into diff. long. by a separate operation.

§ b. Such circuitry is avoided by taking the whole amount of dep., and converting it into diff. long. by a single inspection of the diagram, or of Table V.

§ 604. Suppose a ship sails from lat. 64° N., 216 miles N.E. by N.; she makes 120 miles of dep. and 179.6, say 180 or 3° diff. lat.

§ a. Now this dep., or the diff. long. is not all made upon the parallel of 64° , '5', '6', or '7'; or upon any one of the intermediate parallels, but upon all of them together.

§ b. And in order to convert, by a single operation, this dep. into long., and to find the diff. long. which the ship has made by sailing this course and distance, the whole dep. is supposed to be

made upon that parallel ($65^{\circ} 30'$) which is midway between the lat. left (64°), and that (67°) arrived at.

§ c. This lat. ($65^{\circ} 30'$) is called the *middle latitude*; it is *half the diff. lat. added to lat. left*; or to that arrived at, if the latter be the less lat.

§ 605. If the number of miles in a degree of long. on the different parallels, as you approach the poles, decreased, as the miles from the equator to those parallels increase, this method would show the true diff. long.

§ a. But the number of miles in a degree of long. at different parallels, is (§ 590. § c.) as the cosines of the lat. of such parallels.

§ b. Wherefore, the result given by this method is only an approximate one.

§ c. But in working up "dead reckoning," "days' works," etc., this method is generally used; it is used in all cases, except in the computation of great distances.

§ 606. On ship-board at the expiration of every hour, sometimes of every two hours, the course and distance sailed during that time are marked opposite each other, on the *log slate*.

§ a. And at the end of every sea day, or at every noon, the courses and distances sailed in the last 24 hours, are "taken off," for the purpose of "working up;" and the whole distance sailed on every separate course is set down, opposite to that course, in a *traverse table*.

Plate 1. } § b. The diff. lat. and the dep. for every course and dist., are taken, either from the diagram, or from Table IV., and set down opposite to their proper course and dist. in the "traverse table."

§ c. The dep. and diff. lat. for every course and dist., are then added up; the sums show the whole dep. and diff. lat.

§ d. And by means of the whole dep. and diff. lat., the course and dist. "made good," are found on the diagram (§ 577. § a.), or are taken from Table IV. or V.

§ e. The "middle lat." (604. § c.) is then found; and by means of it and the dep., the diff. long. is found (§ 594.) on the diagram, or in Table V.

Fig. A.

Course.	Dist.	Diff. Lat.		Depart.	
		N.	S.	E.	W.
N. by E.	20	19.6	- -	3.9	- -
S. W.	12	- -	8.5	- -	8.5

§ 607. Fig. A is the formula of a "traverse table."

§ a. The diff. lat. is written in the column marked N. or S. according as the course has *nothing*, or *southing* in it. Thus, the diff. lat. for 20 miles N. by E. is put in the N. column, and the diff. lat. for 12 miles S. W. is written in the S. column.

§ b. The dep. must be placed in the column E. or W., according as the course has *easting*, or *westing* in

it. Thus the dep. for the dist. 20 N. by E., is written in the E. column, and the dep. for the dist. 12, S.W. is written in the W. column.

§ 608. The dep. and diff. lat. for any dist. less than 300 miles, and for any course, are found in Table IV., in a manner similar to that by which they are found on the diagram.

§ a. If the course be less than 4 Points, it is to be found at the top of the page, and the diff. lat. must be taken from the column marked at the top (d. lat.), and over which the course is found.

§ b. Thus, to take from Table IV. the dep. and diff. lat. for 120 miles S. $\frac{1}{2}$ E.; find S. $\frac{1}{2}$ E. at the top of the page, and 120 in the dist. column; then in the two columns, at the head of which S. $\frac{1}{2}$ E. is found, and opposite to 120, are the diff. lat. 119.4, and the dep. 11.8.

§ c. But if the course be more than 4 Points, it is to be found at the bottom of the page, and the precept at the foot of the column must be the guide for taking out dep. and diff. lat.

§ d. Thus to take from Table IV. the dep. and diff. lat. for 30 miles E.N.E.; find E.N.E. at the bottom of the page, and 30 in the dist. column, then in the two columns, at the foot of which E.N.E. is found, are 27.7 (dep.) and 11.5 (diff. lat.) opposite to 30.

§ e. If the dist. exceed 300 miles, some aliquot part of it is taken as the 2d, 3d, etc., and the dep. and diff. lat. opposite to this quotient being increased by 2, 3, etc., as much as the dist. was reduced, give the required dep. and diff. lat. Thus to find the dep. and diff. lat. for 400 miles N.N.W.; take 200, the half of it; then, under N.N.W. and opposite 200, stands 184.8 in the d. lat. column, and 76.5 in the dep. do.; each of which being multiplied by 2, gives the required diff. lat. 369.6 or $6^{\circ} 9' 36''$, and the dep. 153 miles.

§ f. Table V. is used in the same way for finding dep., diff. lat., etc., when the course is given in degrees. Thus the dep. for 18 miles N. 30° E. is 9 miles; and the diff. lat. for 40 miles S. 74° E. is 11 miles.

TRAVERSE TABLE.

Compass Courses.	Courses Corrected.	Dist.	Diff. Lat.		Dep.	
			N.	S.	E.	W.
N.N.E.	N.E. by N.	14	11.6	—	7.8	—
S.S.E.	S. by E.	18	—	17.7	3.5	—
South	S. by W.	40	—	39.2	—	7.8
S.½E.	S.½W.	19	—	18.9	—	1.9
N.W.½N.	N.N.W.½W.	9	7.9	—	—	4.3
N. by W.	North	6	6	—	—	—
W. by S.	West	3	—	—	—	3
East	E. by S.	2	—	.4	2	—
N.E.	N.E. by E.	60	33.3	—	49.9	—
S.W. by S.	S.W.	16	—	11.3	—	11.3
W.½S.	W.½N.	7	.3	—	—	7.
W.½N.	W. by N.½N.	4	1	—	—	3.9
Lat. sailed from 41° 13' N.			60.1	67.5	68.2	39.1
Diff. Lat. - - - 27' 24" S.				60.1	69.1	
Lat. in - - - 40° 48' 36" N.						
Mid. Lat. (§ 604. § c.) 41°				27.4	24.1	
Long. sailed from 70° 15' W.						
Diff. Long. 32' E.						
Long. in 69° 43' W.						
			Variation 1 Point E.			

§ 609. The courses in the 1st column of the "traverse table" are the courses sailed per compass.

§ a. Those in the 2d column are those corrected (§ 298. § b.) for variation, which is 1 Point E.

§ b. The diff. lat. and dep. in their columns, answer to the dist. and *corrected courses* to which they are opposite.

§ d. The diff. lat. 27' 4 S. and the dep. 24.1 E. show that the ship, by sailing the several courses and distances in the table, went to the Southward and Eastward.

§ d. The dep. 24.1 and diff. lat. 27.4 are found opposite 36 (Table V.) and *under* 42°, which shows the course and dist. "made good," (§ 606. § d.) to be S. 42° E., 36 miles.

§ e. Under 41° (Table V.) 24.1 in the d. lat. column, stands opposite to 32 in the dist. column; the diff. long. then is 32' East.

§ 610. The manner in which "dead reckoning," "days' works," etc., are worked up, and kept at sea, whether by means of the Tables IV. and V., or the diagram, may be learned from the subjoined formulæ. } Plate 1.

§ a. Aug. 28, lat. and long., 62° N. and 47° W.

§ b. Variation W. 1½ Pt., which (§ 298. § b.) is to be allowed towards the left.

Aug. 24.

Compass Courses.	Courses Corrected.	Dist.	Diff. Lat.		Dep.	
			N.	S.	E.	W.
S. E.	S. E. by E. ¼ E.	16		7.5	14.1	
E. ¼ N.	E. N. E.	12	4.6		11.1	
E. by N.	N. E. by E. ¼ E.	14	6.6		12.3	
N. E.	N. N. E. ¼ E.	20	17.6		9.4	
N. N. E.	N. ¼ E.	18	17.9		1.8	
N. by E. ¼ E.	North	66	66			
North.	N. by W. ¼ W.	40	38.3			11.6
S. W.	S. S. W. ¼ W.	26		22.9		12.3
Yesterday's lat. (§ a.) 62° N.			151.	30.4	48.7	23.9
Diff. lat. - - - 2° 0' 36" N.			30.4		23.9	
Lat. in - - - 64° 0' 36" N.						
Mid. lat. - - - 63° 0'						
Yesterday's long. 47° 0' W.			120.6		24.8	
Diff. long. - - - 55' E.						
Long. in - - - 46° 5' W.						
Course and dist. "made good" N. 12° E. 123 miles.						

§ c. Above 63°, the mid. lat. (Table V.) 24.8* in the d. lat. column, stands opposite to 55 in the dist. column; the diff. long. then is 55' E.

§ d. And the dep. 24.8 and diff. lat. 120.6 are found opposite to each other under 12°, (Table V.), and opposite to 123, which shows the course and distance made good.

* In this and all similar cases, the tabular number which is nearest to the given number is always used, when the given number cannot be found. Thus, in the present example, the given number 24.8 cannot be found in the d. lat. column over 63°; 25 is the tabular number nearest to 24.8; and 55' is opposite 25.

AUG. 25.

Compass Courses.	Courses Corrected.	Dist.	Diff. Lat.		Dep.	
			N.	S.	E.	W.
East	E. $\frac{1}{2}$ S.	30		4.4	29.7	
E.S.E.	S.E. by E. $\frac{1}{2}$ E.	18		9.3	15.4	
S. $\frac{1}{2}$ E.	South	16		16		
S.S.W.	S.S.W. $\frac{1}{2}$ W.	19		16.3		9.8
S.W. by S.	S.W. $\frac{1}{2}$ S.	25		18.5		16.8
W.S.W.	W. by S. $\frac{1}{2}$ S.	22		5.3		31.3
W. $\frac{1}{2}$ S.	West	17				17
West	W. $\frac{1}{2}$ N.	30	4.4			29.7
W.N.W.	N. W. by W. $\frac{1}{2}$ W.	29	14.9			24.9
Yesterday's lat. $64^{\circ} 0' 38''$ N.			19.3	69.8	45.1	119.5
Diff. lat. - $50' 30''$ S.				19.3		45.1
Lat. in - $63^{\circ} 10' 6''$				60.5		74.4
Mid. lat. - 64°						
Yesterday's long. $46^{\circ} 5' W.$						
Diff. long. - $2^{\circ} 50' W.$						
Long. in - $49^{\circ} 55' W.$						
			Variation $\frac{1}{2}$ Point E.			
Course and dist. sailed S. 56° W. 90 miles.						

§ f. To find the bearing and dist. of a place in Lat. $63^{\circ} 20' N.$, and long. $49^{\circ} 10' W.$

§ g.

Lat. of place	$63^{\circ} 20'$
Lat. in	$63^{\circ} 10'$
Diff. lat.	<u>10 miles.</u>
Long. of place	49.10
Long. in	48 49
Diff. long.	<u>$=21'$</u>

§ h. 21 being found in the dist. column (Table V.) over 63° the lat., shows opposite to it 9.5 in the d. lat. column; 9.5 then is the dep. between the ship and the place.

§ i. The diff. lat. 10, and dep. 9.5 are found together under 43° (Table V.) and opposite 14; the course and dist. then from the ship to the place, is N. 43° W. 14 miles.

§ j. April 9, lat. $39^{\circ} 50' N.$, long. $70^{\circ} 10' W.$; bound to New York.

APRIL 10.

Compass Courses.	Courses Corrected.	Dist.	Diff. Lat.	Dep.
			N.	W.
W.N.W.	W. by N. $\frac{1}{4}N.$	30	8.7	28.7
W. by N. $\frac{1}{4}N.$	W. by N. $\frac{1}{4}N.$	25	6.1	24.3
W. by N. $\frac{1}{2}N.$	W. by N.	18	3.5	17.7
W. by N.	W. $\frac{1}{4}N.$	29	2.8	28.9
W. by N. $\frac{1}{4}N.$	W. $\frac{1}{2}N.$	26	3.8	25.7
W. $\frac{1}{2}N.$	W. $\frac{1}{4}N.$	9	.4	9
W. $\frac{1}{4}N.$	West	5	.0	5
Yesterday's lat. $39^{\circ} 50' N.$			25.3	139.3
Diff. lat. - $25' 18'' N.$				
Lat. in - $40^{\circ} 15' 18''$				
Mid. lat. 40°				
Yesterday's long. $70^{\circ} 10' W.$			Variation $\frac{1}{2}$ Point W.	
Diff. long. - $3^{\circ} 2' W.$				
Long. in - $73^{\circ} 12' W.$				
Course and dist. sailed N. $80^{\circ} W.$ 143 miles.				

§ k. To find the bearing and dist. of Sandy Hook Light House.

S. L. House, lat. $40^{\circ} 28' N.$

Lat. ship - $40^{\circ} 15' N.$

Diff. long. $13'$

Long. S. L. House, $74^{\circ} 1' W.$

Long. ship - $73^{\circ} 12' W.$

Diff. long. $49'$

49' of long. in lat. 40° is equal to 37.5 miles. (Table V.)

§ l. 13 and 37.5 are found together over 71° (Table V.), and opposite 40. The dist. then of the Light House from the ship, is N. $71^{\circ} W.$ 40 miles.

MERCATOR'S SAILING.

§ 611. Dep. : rad. : : diff. long. : sec. lat. (§ 587. § c.). Upon the principles involved in this proportion, the charts, called Mercator's, are constructed.

§ a. In these charts, the meridians of longitude, after they cross the equator, instead of approaching each other, till they reach the parallel of any latitude, in the ratio (§ 486. § b.) of rad. to the cos. of that lat., are drawn parallel to each other.

§ b. And the parallels of lat., say at 1° a part, instead of being at equal distances from each other, are drawn in the ratio of rad. to the sec. of their latitude. By which means, places on a sphere, are represented on a plane with their proper relative positions.

§ 612. In this manner of representing portions of the surface of a sphere upon planes, the parallels of latitude are lengthened out, and the meridians of longitude are expanded.

§ a. So that, if the 1st degree of lat., from the equator, be divided on the chart into 60 equal parts, every degree as they succeed each other in numerical order will contain a greater number of these parts, than the 1st, or than that which precedes it; thus, the 61^{st} of lat. contains 126 of these parts, and the 31^{st} , 70 of the same parts.

§ 613. These parts are called *meridional* parts.

§ a. Table VI. shows the number of meridional parts from the equator to any degree and minute within the parallel of the 84^{th} of lat.

§ b. The number of meridional parts between any two parallels, is called the *meridional* diff. lat. of those parallels.

§ c. Thus, the mer. diff. lat. between $13^\circ 10'$ and $18^\circ 10'$, is 312.

The mer. parts of $18^\circ 10' = 1109$	}	Table VI.
do. $13^\circ 10' = 797$		

Mer. diff. lat. = 312

§ 614. Now, if any section of the earth's surface be represented on a plane projected according to Mercator's plan, the *true* (nearly), instead (§ 550.) of the *approximate*, distance, etc., between any two places, may be determined by *plane* trigonometrical operations.

§ a. To do this, the Mer. diff. lat., as well as the actual diff. lat. between places, must be used.

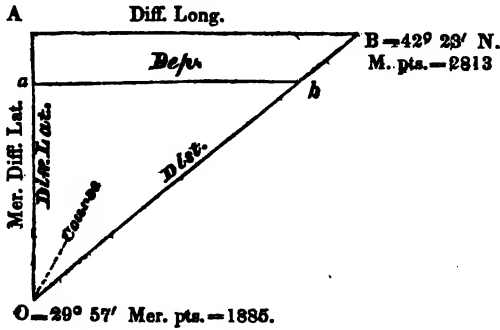
§ 615. Mercator's sailing is useful to the navigator, chiefly in enabling him to find, by plane trigonometrical calculations, the number of miles between places that are at a great distance from each other.

§ a. The several cases in loxodromic sailing, may also be accurately solved, according to the principles of Mercator's sailing; but the Mer. diff. lat. must be used, as well as the actual diff. lat.

§ b. Methods of applying the principles of Mercator's sailing to the solution of cases in loxodromic sailing, will be shown; but the

application of them to practice, will be left for the amusement of the learner.

§ 616. A B O represents the relative position of Boston and New Orleans in lat. and long., according to the principles of Mercator's sailing.



§ a. And the triangle $a b O$ shows the bearing, dist., etc., according to loxodromic sailing.

§ b. The meridional lat. of Boston is 2813, and that of New Orleans, is 1885 (Table VI.); and the difference between these (§ 613. § b.) is (O A) the Mer. diff. lat.; and A B is the diff. long.; A O B is the course from New Orleans to Boston.

§ c. According to the loxodromic plan of constructing the triangle, $a O$ is the actual diff. lat. between the two places, and $a b$ is the dep.

§ d. Whether the problem proposed be solved according to the loxodromic or Mercator plan, the actual diff. lat. between the two places is the same.

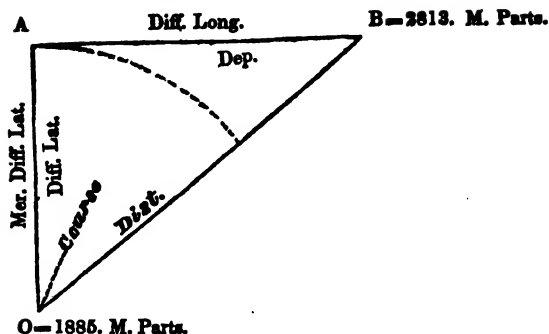
§ e. But the dep. and dist. determined by the former method (§ 550.) are not the true dep. and dist. between the two places.

§ 617. As the angles $a b O$ and A B O (§ 616. § d.) are equal, $a b$ (§ 30. § d.) is parallel to A B; whence (§ 73. § d.) arise the relation between the sides of a triangle in loxodromic sailing when compared with the sides of a similar triangle in Mercator's sailing.

§ a. Diff. lat. : dep. : : Mer. diff. lat. : diff. long.; also inversely and alternately.

§ b. The relations between the other parts of the two triangles may be established according to the principles derived from § 72. & § 73. The learner may arrange them.

§ 618. If the Mer. diff. of lat. (O A) between New Orleans and Boston be made radius, several sets of proportions (§ 75.) will appear among the different parts of the triangle involved.



§ a. But the most useful proportion (and in fact almost the only one which occurs in practice), in Mercator's sailing, is that by which the *true* course and dist. between any two places are evolved. I shall give the terms of this proportion, and leave the others to be arranged by the learner.

§ 619. The Mer. diff. lat. (§ 618:) being radius ; Mer. diff. lat. : rad. :: diff. long. : tang. course.

§ a. The diff. long. thus used, must in all cases be converted into minutes, which must be used in the log. Table of Numbers, as miles.

§ b. To find the course from New Orleans to Boston.

Boston	lat.	42° 23'	M. pts.	=2813
N. Orleans	"	29° 57'	"	1885
		12° 26'	"	
		60	"	928 = Mer. diff. lat.

Diff. lat. = 746 miles.

N. Orleans,	long.	90° 9' W.
Boston	"	71° 4' W.
		19° 5'
		60

Diff. long. = 1145 minutes (').

Now, (§ 619.) 928 : rad. :: 1145 : tang. course.

928	log. (Ar. co.)	=7.032452
1145	log. (§ b.)	=3.058806

Tang. course = 0.091258 N. 50° 58' 34" E.

§ c. Then calling O A, the actual diff. lat., A B becomes dep., and retaining O A, as rad., we have (§ 574. § d.) rad. : diff. lat. :: sec. course : dist.

Diff. lat. 746 miles log.=2.872739
 Course N. 50° 58' 34" E. sec.=0.200906

Log. dist.=3.073645=1184.8 miles.

§ d. Problems in Mercator's sailing may also be solved either by Tables IV. & V., or by the diagram (Plate 1). Thus the mer. diff. lat., and the diff. long., being used on the diagram, or in the tables as diff. lat. and dep., show the course. Then with this course the dist. is found opposite to the actual diff. lat.

§ e. Philadelphia lat.	39° 57' N.	M. parts	-	2610
Washington city lat.	38° 53' N.	"	-	2536
	1° 4'	M. diff. lat.=		83
	60			60

Diff. lat.=64 miles.

Washington long.	77° 2' W.
Philadelphia "	75° 9' W.

	1° 53'
	60

Diff. long.=113 minutes (').

113 in the dep. column stands opposite to 83 in the d. lat. column; (Table V.) 54° is at the bottom of the page. The course then from Philadelphia to Washington is S. 54° W. On the same page 64 in the d. lat. column stands opposite to 109 miles, which is the dist. between the two places.

SURVEYING.

§ 620. In conducting the survey of a coast, harbour, etc., the first object should be to ascertain the geographical position of some point connected with the survey, and the next to establish a *base line*.

§ 621. The most advantageous location for the base line, must be determined by the surveyor himself; and in this, he should be governed by circumstances, such as the nature of the ground about the place to be surveyed, and the place itself.

§ a. A level piece of ground should be selected for the base line, and the line should terminate at points, whence some of the prominent points, headlands, etc., of the place (say a harbour) under survey can be seen.

§ 622. The length of the base line must be ascertained by actual measurement, and the direction in which it lies must be established by observations, and noted down.

§ a. Then, knowing the length of the base line, it serves as a

given side of a triangle, either for determining the length of other lines, or for finding the distance between either end of it, and any point visible from each end of the base.

§ *b*. For if the bearing of such point be taken from each end of the base, a triangle may be formed, in which the angles and the base are known, wherefore (§ 106. § *a*.) the two sides are determinable.

Plate 9. { § 623. The figure in the annexed plate is the *profile* of a harbour, that is being surveyed. A B is the base line, A C B, A D B, etc., are triangles constructed upon it; the angles at A, B, C, D, etc., are measured with a sextant or a theodolite, and the length of the lines B D, B C, A C, etc., is determined by trigonometrical calculation. The principal triangles used, are represented by the broken lines A X, A D, etc.

§ *a*. So that triangles may be constructed from one line to another, until sufficient data are obtained for determining, by trigonometrical calculation, the position of every point of the survey.

§ *b*. The line (X Y) of verification is determined, as to its length, both by trigonometrical calculation, and by measurement, in order to establish the accuracy of the survey.

§ 624. The position of the prominent points C, D, E, F, etc., is determined to a greater degree of nicety in this way, than it can be by taking cross bearings, etc., and the intersection of lines.

§ 625. After the *triangulation* of the survey is completed, and the points C, D, E, F, etc., are laid down upon the chart to the requisite scale; the intermediate spaces, G L, L H, etc., of the shore, may be filled up by the eye. A little practice will enable one to sketch these intermediate spaces with all necessary accuracy.

§ *a*. The position though, of every prominent point, such as light houses, rocks, hills, castles, etc., should be established by actual calculation.

§ *b*. In taking the angles, all the angles of every triangle should be measured whenever it can be done. This affords one means of detecting errors during the work of triangulating; for if the sum (§ 32.) of the three angles of a triangle be not 180° according to the measurement, an error is known to exist.

§ *c*. Also the sum of all the angles around the same point (§ 26.) should be 360° ; and, the sum of all the angles which lie within any measured angle, should be equal (§ 24. § *d*.) to the whole angle.

§ 626. After the work of triangulation is completed, or when it is temporarily suspended, the trigonometrical calculations for the part already triangulated, may be made.

§ 627. The soundings of the harbour should be accurately taken, and correctly laid down upon the chart.

§ *b*. The most practicable way of doing this, is to take a row boat, which will pull equal distances in equal times; establish the position of the point from which she sets out, and let her pull directly for a given point, sounding and noting the soundings at every interval of one, two, three, or more, minutes, as she goes along. When

she arrives at the end of this line, establish the position of this end also, draw the line on the chart, divide it into as many equal parts as there were soundings taken, and write along the line every one at its proper point.

§ *b*. The surest way of making a straight course in the boat, is to bring two points on shore in the same line of bearing, and then pull for them, keeping them in that line. For instance; a boat at *a*, would pull in a straight line to *E*, if she kept a tree, or a steeple, or any object at *b*, in a line with *E*; then the soundings taken would all be on the line *a E*; and the position at *a* being determined; taking the angles *B a E* & *E B a*, the length of *a E* may be known, and the soundings laid down.

§ 628. The soundings should be given for low water; wherefore the time of the tide when the soundings are being taken, should be noted, in order to correct them for low water.

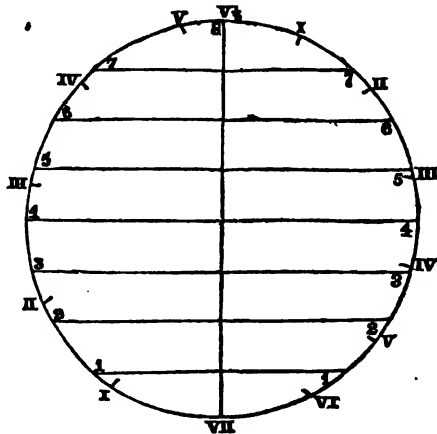
§ *a*. The corrections to be applied may be obtained, either according to the rate of the rise and fall of the tide, as established by previous observation, or by means of a figure (§ *e*.), thus;

§ *b*. Say that the rise and fall of the tide is 8 feet, and that the flood tide lasts $5\frac{1}{2}$ hours, and the ebb, 7.

§ *c*. Let the diameter of a circle be divided into 8 equal parts, by the chords 1, 2, 3, etc. Also let one semicircumference of this circle be divided into $V\frac{1}{2}$ equal parts or hours, to represent the time of flooding; and let the other semicircumference be divided into VII. equal arcs to represent the hours during ebb.

§ *d*. The point in which each of these chords cuts the circle, shows the correction in feet which is to be applied, when the tide is that number of hours, flood or ebb. Thus, when the tide is $III\frac{1}{2}$ flood, the correction to be applied is $4\frac{1}{2}$ feet; and when the tide is III. hours ebb, the correction is 5 feet.*

§ *e*.



* These corrections are always subtractive.

§ 629. The sort of bottom should also be noted down, i. e. mud, rock, sand, etc.

§ 630. The position, extent, etc., of all the hidden dangers, such as rocks, reefs, shoals, banks, and of every other object of importance to the navigator, should be ascertained and laid down.

§ 631. Surveys of harbours are frequently taken by measuring first, a base line, then drawing this line upon the paper at the proportional length to the scale upon which the chart is to be constructed, and then establishing the position of the principal points in the survey, by the intersection of the lines of their bearing from different points.

§ a. But the difficulty of measuring with accuracy the proper angles upon paper, makes this method of taking a survey more liable to inaccuracies, than the triangulating plan.

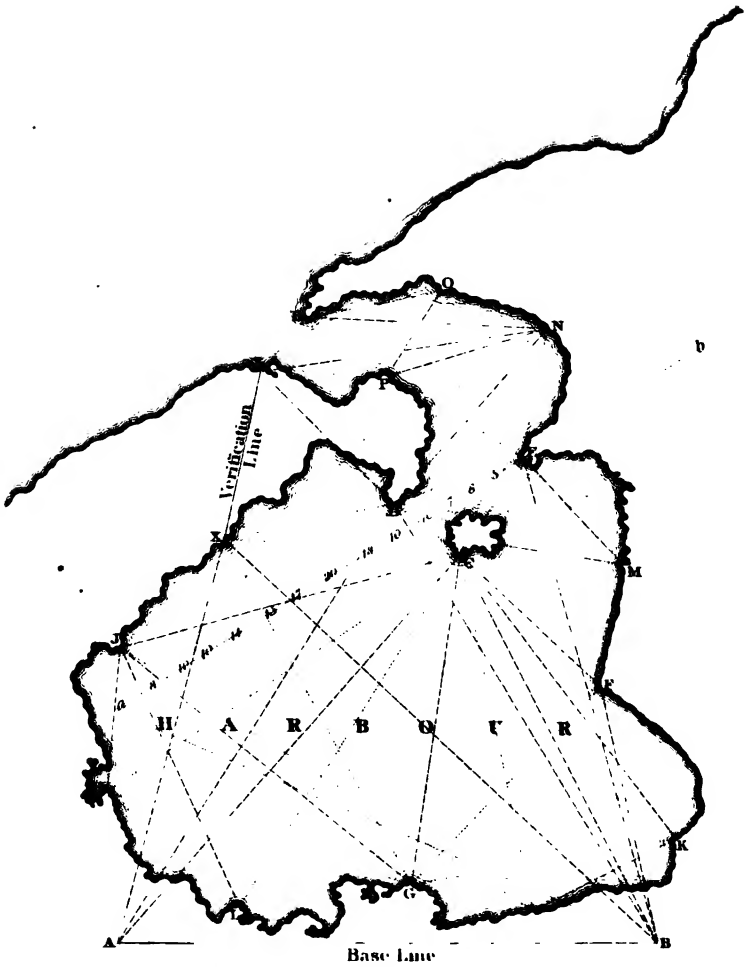


TABLE I.

LOGARITHMS OF NUMBERS

TABLE I.

LOGARITHMS OF NUMBERS.

N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0.000000	25	1.397940	50	1.698970	75	1.875061
2	0.301030	26	1.414973	51	1.707570	76	1.880814
3	0.477121	27	1.431363	52	1.716003	77	1.886491
4	0.602060	28	1.447158	53	1.724276	78	1.892014
		29	1.462398	54	1.732393	79	1.897627
5	0.698970						
6	0.778151	30	1.477121	55	1.740363	80	1.903090
7	0.845098	31	1.491362	56	1.748188	81	1.908484
8	0.903090	32	1.505150	57	1.755875	82	1.913814
9	0.954242	33	1.518514	58	1.763426	83	1.919178
		34	1.531479	59	1.770852	84	1.924579
10	1.000000						
11	1.041393	35	1.544068	60	1.778151	85	1.929919
12	1.079181	36	1.553092	61	1.785330	86	1.934499
13	1.113743	37	1.558202	62	1.792392	87	1.939319
14	1.146128	38	1.570784	63	1.799340	88	1.944483
		39	1.581064	64	1.806180	89	1.949390
15	1.176091						
16	1.204120	40	1.609060	65	1.812913	90	1.954943
17	1.230449	41	1.612784	66	1.819544	91	1.957041
18	1.255272	42	1.623949	67	1.826075	92	1.963786
19	1.278754	43	1.633409	68	1.832509	93	1.968483
		44	1.643453	69	1.838849	94	1.973128
20	1.301030						
21	1.322219	45	1.653212	70	1.845098	95	1.977724
22	1.342423	46	1.662758	71	1.851258	96	1.982271
23	1.361728	47	1.672098	72	1.857322	97	1.986772
24	1.380211	48	1.681241	73	1.863292	98	1.991226
		49	1.690196	74	1.869222	99	1.995635

TABLE I.—LOG. OF NOS.

N.	0	1	2	3	4	5	6	7	8	9	
100	00	0000	0434	0868	1301	1734	2166	2598	3030	3460	3891
01		4321	4751	5181	5609	6038	6466	6894	7321	7748	8174
02		8600	9026	9451	9876						
	01					0300	0724	1147	1570	1993	2415
03		2837	3259	3680	4100	4521	4940	5360	5779	6197	6616
04		7033	7451	7868	8284	8700	9116	9532	9947		
	02									0361	0776
105		1189	1603	2016	2428	2841	3253	3664	4075	4486	4896
(6		5306	5715	6124	6533	6942	7350	7757	8164	8571	8978
07		9384	9789								
	03		0185	0600	1004	1409	1812	2216	2619	3021	3421
08		3424	3826	4228	4628	5028	5430	5830	6230	6629	7028
09		7427	7825	8223	8620	9017	9414	9811			
	04							0207	0602	0996	
110		1393	1787	2182	2576	2969	3362	3755	4148	4540	4932
11		5323	5714	6105	6495	6885	7275	7664	8053	8442	8830
12		9218	9606								
	05			0380	0766	1152	1538	1924	2309	2694	3079
13		3078	3463	3846	4230	4613	4996	5378	5761	6142	6524
14		6905	7286	7666	8046	8426	8806	9185	9563	9942	
	06										0360
115		0698	1075	1453	1829	2206	2582	2958	3333	3709	4083
16		4438	4832	5206	5580	5953	6326	6699	7071	7443	7815
17		8186	8557	8928	9298						
	07					0638	0407	0777	1145	1514	1882
18		1893	2250	2617	2985	3352	3718	4085	4451	4816	5182
19		5547	5912	6276	6640	7004	7368	7731	8094	8457	8819
120		9181	9543	9905							
	08			0266	0636	0967	1347	1707	2067	2426	2786
21		2725	3144	3503	3861	4219	4576	4934	5291	5647	6004
22		6320	6716	7071	7427	7781	8136	8491	8845	9198	9552
23		9905									
	09		0252	0611	0963	1315	1667	2018	2370	2721	3071
24		3422	3772	4122	4471	4820	5169	5518	5867	6215	6562
125		6910	7257	7604	7951	8297	8644	8990	9335	9681	
	10										0696
26		0370	0715	1059	1403	1747	2091	2434	2777	3119	3462
27		3804	4146	4487	4828	5169	5510	5851	6191	6531	6871
28		7210	7549	7888	8227	8565	8903	9241	9578	9916	
	11										0260
29		0520	0896	1263	1628	1994	2370	2746	3121	3495	3869
130		3043	4277	4611	4944	5278	5610	5943	6276	6608	6940
31		7271	7603	7934	8265	8595	8926	9256	9586	9915	
	12										0245
32		0574	0903	1232	1560	1888	2216	2543	2871	3198	3525
33		3852	4178	4504	4830	5156	5481	5807	6131	6456	6781
34		7105	7429	7753	8076	8399	8722	9045	9368	9690	
	13										0012
135		0334	0655	0977	1298	1619	1939	2260	2580	2900	3219
36		3539	3858	4177	4496	4814	5133	5451	5769	6086	6403
37		6721	7037	7354	7671	7987	8303	8618	8934	9249	9564
38		9879									
	14		0184	0496	0809	1126	1450	1768	2077	2380	2702
39		3015	3327	3639	3951	4263	4574	4885	5196	5507	5818
140		6128	6439	6748	7058	7367	7676	7985	8294	8603	8911
41		9219	9527	9835							
	15				0142	0449	0756	1063	1370	1676	1982
42		2928	3234	3540	3845	4150	4454	4758	5061	5364	5667
43		5336	5640	5943	6246	6549	6852	7154	7457	7759	8061
44		8302	8604	8905	9206	9507					
	16							0168	0469	0769	1068
145		1368	1667	1967	2266	2564	2863	3161	3460	3757	4055
46		4353	4650	4947	5244	5541	5838	6134	6430	6726	7022
47		7317	7613	7908	8203	8498	8792	9086	9381	9674	9968
48	17	0268	0555	0842	1121	1404	1786	2169	2551	2933	3315
N.	0	1	2	3	4	5	6	7	8	9	

TABLE I.—LOG. OF NOS.

N.	0	1	2	3	4	5	6	7	8	9
140	17 3186	3478	3760	4060	4351	4641	4932	5222	5512	5802
150	6091	6381	6670	6959	7248	7536	7825	8113	8401	8689
51	8977	9265	9552	9839						
52	18 1844	2129	2415	2700	2985	3270	3555	3839	4123	4408
53	4611	4895	5179	5462	5745	6028	6311	6594	6876	7159
54	7521	7803	8084	8366	8647	8928	9210	9490	9771	0051
155	0332	0612	0892	1172	1451	1730	2010	2289	2568	2846
56	3125	3403	3681	3959	4237	4514	4791	5069	5346	5623
57	5000	6178	6452	6727	7001	7274	7547	7820	8092	8364
58	8657	8932	9206	9481	9755					
59	1397	1670	1943	2216	2488	2761	3033	3305	3577	3848
160	4190	4391	4682	4934	5204	5475	5746	6016	6286	6556
61	6522	7036	7365	7634	7904	8173	8441	8710	8979	9247
62	9515	9783								
63	2188	2454	2720	2986	3252	3518	3783	4049	4314	4579
64	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221
165	7484	7747	8010	8273	8536	8798	9060	9322	9585	9846
66	0108	0370	0631	0892	1153	1414	1675	1936	2196	2456
67	2717	2978	3238	3498	3758	4018	4278	4538	4798	5051
68	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630
69	7867	8144	8400	8657	8913	9170	9426	9682	9938	0193
170	0449	0704	0960	1215	1470	1724	1979	2233	2488	2742
71	2396	3250	3504	3757	4011	4264	4517	4770	5023	5275
72	5523	5781	6033	6285	6537	6789	7041	7292	7544	7795
73	8046	8297	8548	8799	9049	9300	9550	9800	0050	0300
74	0549	0799	1048	1297	1547	1795	2044	2293	2541	2790
175	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266
76	5513	5753	6006	6252	6499	6745	6991	7236	7482	7728
77	7973	8219	8464	8709	8954	9198	9443	9687	9932	0178
78	0420	0664	0908	1151	1395	1638	1882	2125	2368	2610
79	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031
180	5273	5514	5755	5996	6237	6477	6718	6959	7198	7437
81	7679	7919	8158	8398	8637	8877	9116	9355	9594	9833
82	0071	0310	0548	0787	1025	1263	1501	1738	1976	2214
83	2451	2688	2926	3162	3399	3636	3873	4109	4346	4582
84	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937
185	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279
86	9513	9746	9980							
87	1842	2074	2305	2537	2770	3001	3233	3464	3696	3927
88	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232
89	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525
190	8754	8982	9210	9439	9667	9895				
91	1073	1291	1498	1715	1942	2169	2396	2622	2849	3075
92	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332
93	5557	5782	6007	6232	6457	6681	6905	7130	7354	7578
94	7802	8025	8249	8473	8696	8920	9143	9366	9589	9812
195	0075	0257	0420	0702	0925	1147	1369	1591	1813	2034
96	2256	2478	2699	2920	3142	3363	3584	3804	4025	4246
97	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446
98	6675	6894	7104	7323	7542	7761	7979	8198	8416	8635
99	8853	9071	9289	9507	9725	9943				
200	1090	1247	1444	1641	1838	2114	2331	2547	2764	2980
01	3196	3412	3628	3844	4060	4275	4490	4706	4921	5136
N.	0	1	2	3	4	5	6	7	8	9

N.	0	1	2	3	4	5	6	7	8	9
202	30 5351	5596	5781	5996	6211	6425	6639	6854	7068	7282
03	74.6	7710	7924	8137	8351	8564	8778	8991	9204	9417
04	9630	9843								
	31		0056	0268	0481	0693	0905	1118	1330	1542
205	1754	1966	2177	2389	2600	2819	3023	3224	3445	3636
06	3867	4078	4289	4499	4710	4920	5130	5340	5551	5761
07	5370	6180	6390	6599	6801	7018	7227	7437	7646	7854
08	8063	8272	8481	8689	8898	9106	9314	9523	9730	9936
09	32 0146	0354	0562	0769	0977	1184	1391	1598	1806	2013
210	2219	2426	2633	2839	3046	3252	3458	3665	3871	4077
11	4283	4489	4694	4900	5105	5310	5516	5721	5926	6131
12	6336	6541	6745	6950	7154	7359	7563	7767	7972	8176
13	8380	8583	8787	8991	9194	9398	9601	9805		
14	33 0414	0617	0819	1022	1225	1427	1630	1832	2034	2236
215	2439	2642	2842	3044	3246	3447	3649	3850	4051	4253
16	4454	4655	4856	5056	5257	5458	5657	5859	6059	6260
17	6460	6660	6860	7060	7260	7453	7659	7856	8058	8257
18	8457	8656	8855	9054	9253	9451	9650	9849		
19	34 0444	0642	0841	1039	1237	1435	1632	1830	2028	2225
220	2423	2620	2817	3014	3212	3409	3606	3803	3999	4196
21	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157
22	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110
23	8305	8500	8694	8889	9083	9277	9472	9666	9860	
24	35 0248	0442	0636	0829	1023	1216	1410	1603	1796	1989
225	2182	2376	2568	2761	2954	3147	3339	3532	3724	3916
26	4108	4301	4493	4685	4876	5068	5260	5452	5643	5835
27	6026	6217	6408	6599	6791	6981	7173	7363	7554	7744
28	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646
29	9836									
30		0025	0215	0404	0593	0783	0973	1161	1350	1539
230	1738	1917	2105	2274	2483	2671	2859	3048	3236	3424
31	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301
32	5488	5675	5862	6049	6236	6423	6610	6796	6983	7170
33	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030
34	9216	9401	9587	9773	9958					
35	37					0143	0328	0513	0698	0883
235	1058	1233	1437	1622	1807	1991	2175	2360	2544	2729
36	2112	3036	3390	3464	3647	3831	4015	4198	4382	4565
37	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394
38	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216
39	8398	8580	8761	8943	9124	9306	9487	9668	9849	0030
240	0211	0392	0573	0754	0935	1115	1296	1476	1656	1837
41	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636
42	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428
43	5108	5287	5466	6142	6321	6499	6677	6856	7034	7212
44	7390	7568	7746	7923	8101	8279	8457	8634	8811	8989
245	9166	9343	9521	9698	9875					
46	30 0035	1112	1289	1464	1641	0052	0228	0405	0582	0759
47	2897	2673	3048	3224	3400	1217	1393	2169	2345	2521
48	4452	4627	4802	4977	5152	3575	3751	3926	4101	4277
49	6199	6374	6548	6723	6896	5326	5501	5676	5850	6025
250	7940	8114	8287	8461	8634	7071	7245	7419	7592	7766
51	9674	9847								
52	1400	1573	1745	1917	2089	0536	0711	0883	1056	1228
53	3121	3292	3464	3635	3807	2261	2433	2605	2777	2949
54	4834	5005	5176	5346	5517	3977	4149	4321	4492	4663
255	6540	6711	6881	7051	7221	5688	5858	6029	6199	6370
55										
N.	0	1	2	3	4	5	6	7	8	9

TABLE I.—LOG. OF NOS.

N.	0	1	2	3	4	5	6	7	8	9	
856	40	8940	8410	8579	8749	8918	9087	9257	9426	9595	9764
57		9933									
58	41	1090	0109	0271	0440	0608	0777	0946	1114	1283	1451
59		3300	1788	1956	2124	2293	2461	2629	2796	2964	3132
			3467	3635	3803	3970	4137	4305	4472	4639	4806
200		4973	5140	5307	5474	5641	5808	5974	6141	6308	6474
61		0640	6607	6773	7139	7306	7472	7638	7804	7970	8135
62		8301	8467	8633	8798	8964	9129	9295	9460	9625	9791
63		9956									
64	42	1604	0121	0286	0451	0616	0781	0945	1110	1275	1439
			1768	1933	2097	2262	2426	2590	2754	2918	3082
265		3946	3410	3573	3737	3901	4064	4228	4392	4555	4718
66		4999	3045	3208	3371	3534	3697	3860	4023	4186	4349
67		6511	6674	6837	6999	7161	7324	7486	7648	7811	7973
68		8135	8297	8459	8621	8783	8944	9106	9268	9429	9591
69		9752									
	43		0075	0236	0398	0559	0720	0881	1042	1203	
270		1364	1525	1685	1846	2007	2167	2328	2488	2649	2809
71		2269	2130	2290	2450	2610	2770	2930	3090	3249	3409
72		4569	4728	4888	5048	5207	5367	5526	5685	5844	6004
73		6163	6322	6481	6640	6799	6957	7116	7275	7433	7592
74		7751	7900	8057	8216	8374	8532	8690	8849	9007	9165
275		9333	9491	9648	9806	9964					
	44						0122	0279	0437	0594	0752
76		0909	1066	1224	1381	1538	1695	1852	2009	2166	2323
77		2440	2637	2793	2950	3107	3263	3420	3576	3732	3889
78		4045	4201	4357	4513	4669	4825	4981	5137	5293	5449
79		5604	5760	5915	6071	6226	6382	6537	6693	6848	7003
280		7158	7313	7468	7623	7778	7933	8088	8242	8397	8552
81		8706	8861	9015	9170	9324	9478	9633	9787	9941	
	45										0095
82		0949	0403	0557	0711	0865	1018	1172	1326	1479	1633
83		1786	1940	2093	2247	2400	2553	2706	2859	3012	3165
84		3328	3471	3624	3777	3930	4082	4235	4387	4540	4692
285		4945	4997	5150	5302	5454	5606	5758	5910	6062	6214
86		6366	6518	6670	6821	6973	7125	7276	7428	7579	7731
87		7892	8033	8184	8336	8487	8638	8789	8940	9091	9242
88		9392	9543	9694	9845	9995					
	46						0146	0296	0447	0597	0747
89		0896	1048	1198	1348	1499	1649	1799	1949	2098	2248
290		2396	2548	2697	2847	2997	3146	3296	3445	3594	3744
91		3623	4042	4191	4341	4490	4639	4787	4936	5085	5234
92		5383	5532	5680	5829	5977	6126	6274	6423	6571	6719
93		6968	7016	7164	7312	7460	7608	7756	7904	8052	8200
94		8347	8495	8643	8790	8938	9085	9233	9380	9528	9675
295		9922	9969								
	47										1145
96		1292	1436	1585	1732	1878	2025	2171	2317	2464	2610
97		2756	2903	3049	3195	3341	3487	3633	3779	3925	4070
98		4216	4362	4508	4653	4799	4944	5090	5235	5381	5526
99		5671	5816	5962	6107	6252	6397	6542	6687	6832	6977
300		7121	7266	7411	7555	7700	7845	7989	8133	8278	8422
01		8267	8411	8555	8699	9143	9287	9431	9575	9719	9863
02		0007	0151	0295	0438	0582	0725	0869	1012	1156	1300
03	48	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731
04		2874	3016	3159	3302	3445	3587	3730	3873	4015	4157
305		4300	4442	4585	4727	4869	5011	5153	5295	5438	5579
05		5721	5863	6005	6147	6289	6431	6572	6714	6855	6997
06		7129	7270	7411	7553	7704	7845	7986	8128	8269	8410
07		8521	8662	8803	8944	9114	9255	9396	9537	9677	9818
08		9458									
	49		0090	0240	0390	0540	0661	0801	0941	1081	1222
310		1398	1598	1648	1798	1998	2068	2208	2341	2481	2621
N.	0	1	2	3	4	5	6	7	8	9	

TABLE I.—LOG. OF NOS.

N.	0	1	2	3	4	5	6	7	8	9	
311	49	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015
12		4155	4294	4433	4572	4711	4850	4989	5128	5267	5406
13		5544	5683	5822	5960	6099	6237	6376	6515	6653	6791
14		6930	7068	7206	7344	7482	7621	7759	7897	8035	8173
315		8311	8448	8586	8724	8862	8999	9137	9275	9412	9550
16		9687	9825	9962							
	50			0090	0236	0374	0511	0648	0785	0922	
17		1059	1196	1333	1470	1607	1744	1881	2017	2154	2291
18		2427	2564	2700	2837	2973	3109	3246	3382	3518	3654
19		3791	3927	4063	4199	4335	4471	4607	4743	4878	5014
320		5150	5286	5421	5557	5692	5828	5964	6099	6234	6370
21		6505	6640	6776	6911	7046	7181	7316	7451	7586	7721
22		7836	7971	8106	8240	8375	8510	8644	8779	8914	9048
23		9203	9337	9471	9606						
	51			0009	0143	0277	0411	0545	0679	0812	0946
24		0545	0679	0812	0946	1081	1215	1349	1482	1616	1750
325		1893	2027	2160	2294	2428	2561	2694	2828	2961	3094
26		3218	3351	3484	3617	3750	3883	4016	4149	4282	4415
27		4548	4681	4813	4946	5079	5211	5344	5476	5609	5741
28		5874	6006	6138	6271	6403	6535	6668	6800	6932	7064
29		7196	7328	7460	7592	7724	7855	7987	8119	8251	8382
330		8514	8646	8777	8909	9040	9172	9303	9434	9566	9697
31		9826	9958								
	52			0090	0221	0353	0483	0615	0746	0878	1007
32		1138	1269	1400	1530	1661	1792	1922	2053	2183	2314
33		2444	2575	2705	2835	2966	3096	3226	3356	3486	3616
34		3747	3877	4006	4136	4266	4396	4526	4656	4785	4915
335		5045	5174	5304	5434	5563	5693	5822	5951	6081	6210
36		6339	6469	6598	6727	6856	6985	7114	7243	7372	7501
37		7630	7759	7888	8016	8145	8274	8402	8531	8660	8788
38		8917	9045	9174	9302	9430	9559	9687	9815	9943	
	53			0072	0201	0330	0459	0588	0717	0845	0973
39		0900	0328	0456	0584	0712	0840	0968	1096	1223	1351
340		1479	1607	1734	1862	1990	2117	2245	2372	2500	2627
41		2754	2882	3009	3136	3263	3391	3518	3645	3772	3899
42		4026	4153	4280	4407	4534	4661	4787	4914	5041	5168
43		5394	5521	5647	5774	5900	6027	6153	6280	6406	6532
44		6558	6685	6811	6937	7063	7189	7315	7441	7567	7693
345		7819	7945	8071	8197	8322	8448	8574	8699	8825	8951
46		9076	9202	9327	9453	9578	9703	9829	9954		
	54			0079	0204	0329	0454	0579	0704	0829	0954
47		0330	0455	0580	0705	0830	0955	1080	1205	1330	1454
48		1579	1704	1829	1953	2078	2203	2327	2452	2577	2701
49		2825	2950	3074	3199	3323	3447	3571	3696	3820	3944
350		4068	4192	4316	4440	4564	4688	4812	4936	5060	5183
51		5307	5431	5555	5678	5802	5925	6049	6172	6296	6419
52		6543	6666	6789	6913	7036	7159	7282	7406	7529	7652
53		7775	7898	8021	8144	8266	8389	8512	8635	8758	8881
54		9003	9126	9249	9371	9494	9616	9739	9861	9984	
	55			0106	0229	0352	0475	0598	0721	0844	0967
355		0926	0351	0473	0595	0717	0840	0963	1084	1206	1328
56		1450	1572	1694	1816	1938	2060	2181	2303	2425	2547
57		2668	2790	2912	3033	3155	3276	3398	3519	3640	3762
58		3883	4004	4126	4247	4368	4489	4610	4731	4852	4974
59		5094	5215	5336	5457	5578	5699	5820	5940	6061	6182
360		6302	6423	6544	6664	6785	6905	7026	7146	7267	7387
61		7507	7628	7748	7868	7988	8108	8228	8348	8468	8588
62		8709	8829	8949	9069	9189	9308	9428	9548	9667	9787
63		9907									
	56			0026	0148	0265	0385	0504	0624	0743	0863
64		1101	1221	1340	1459	1578	1697	1817	1936	2055	2174
365		2293	2412	2531	2650	2769	2887	3006	3125	3244	3363
N.	0	1	2	3	4	5	6	7	8	9	

TABLE I.—LOG. OF NOS.

N.	0	1	2	3	4	5	6	7	8	9	
366	56	3481	3600	3718	3837	3956	4074	4192	4311	4429	4548
67		4608	4724	4803	4921	5139	5257	5376	5494	5612	5730
68		5848	5966	6084	6202	6320	6438	6555	6673	6791	6909
69		7026	7144	7262	7379	7497	7614	7732	7850	7967	8084
370		8202	8310	8436	8554	8671	8788	8905	9023	9140	9257
71		9374	9491	9608	9725	9842	9959				
	57							0076	0193	0309	0426
72		0543	0660	0776	0893	1010	1126	1243	1359	1476	1592
73		1709	1825	1942	2058	2174	2291	2407	2523	2639	2756
74		2872	2988	3104	3220	3336	3452	3568	3684	3800	3916
375		4031	4147	4263	4379	4494	4610	4726	4841	4957	5072
76		5188	5303	5419	5534	5650	5765	5880	5996	6111	6226
77		6341	6456	6572	6687	6802	6917	7032	7147	7262	7377
78		7492	7607	7722	7836	7951	8066	8181	8295	8410	8525
79		8639	8754	8868	8983	9097	9212	9326	9441	9555	9669
380		9784	9898								
	58			0012	0126	0240	0355	0469	0583	0697	0811
81		0925	1039	1153	1267	1381	1495	1608	1722	1836	1950
82		2063	2177	2291	2404	2518	2631	2745	2859	2972	3085
83		3199	3312	3426	3539	3652	3765	3879	3992	4105	4218
84		4331	4444	4557	4670	4783	4896	5009	5122	5235	5348
385		5461	5574	5686	5799	5912	6024	6137	6250	6362	6475
86		6587	6700	6812	6925	7037	7150	7262	7374	7487	7599
87		7711	7823	7935	8047	8160	8272	8384	8496	8608	8720
88		8832	8944	9055	9167	9279	9391	9503	9615	9726	9838
89		9950									
	59		0061	0173	0284	0396	0508	0619	0730	0842	0953
390		1065	1176	1287	1399	1510	1621	1732	1843	1955	2066
91		2177	2288	2399	2510	2621	2732	2843	2954	3064	3175
92		3286	3397	3508	3618	3729	3840	3950	4061	4172	4282
93		4393	4503	4614	4724	4834	4945	5055	5165	5275	5386
94		5496	5606	5717	5827	5937	6047	6157	6267	6377	6487
395		6587	6707	6817	6927	7037	7146	7256	7366	7476	7586
96		7695	7805	7914	8024	8134	8243	8353	8462	8572	8681
97		8791	8900	9009	9119	9228	9337	9446	9556	9665	9774
98		9883	9992								
	60		0101	0210	0319	0428	0537	0646	0755	0864	0974
99		0973	1082	1191	1299	1408	1517	1625	1734	1843	1951
400		2060	2169	2277	2386	2494	2602	2711	2819	2928	3036
01		3144	3253	3361	3469	3577	3686	3794	3902	4010	4118
02		4226	4334	4442	4550	4658	4766	4874	4982	5090	5197
03		5305	5413	5520	5628	5736	5843	5951	6059	6166	6274
04		6361	6469	6576	6684	6791	6899	7006	7113	7221	7328
405		7455	7562	7670	7777	7884	7991	8098	8205	8312	8419
06		8526	8633	8740	8847	8954	9060	9167	9274	9381	9488
07		9584	9701	9808	9914						
	61		0021	0129	0237	0345	0453	0561	0669	0777	0884
08		0660	0767	0873	0979	1086	1192	1300	1407	1514	1621
09		1723	1830	1936	2042	2148	2254	2360	2466	2572	2678
410		2784	2890	2996	3102	3207	3313	3419	3525	3630	3736
11		3843	3948	4053	4159	4264	4370	4475	4581	4686	4791
12		4897	5003	5108	5213	5319	5424	5529	5635	5740	5845
13		5950	6055	6160	6265	6370	6476	6581	6686	6791	6896
14		7000	7105	7210	7315	7420	7525	7630	7734	7839	7943
415		8048	8153	8257	8362	8467	8571	8676	8780	8884	8989
16		9093	9198	9302	9406	9511	9615	9719	9823	9928	
	62										0032
17		0136	0240	0344	0448	0552	0657	0760	0864	0968	1072
18		1176	1280	1384	1488	1592	1695	1799	1903	2007	2110
19		2214	2318	2421	2525	2628	2732	2836	2939	3042	3146
420		3249	3353	3456	3559	3663	3766	3869	3973	4076	4179
21		4292	4395	4498	4591	4695	4798	4901	5004	5107	5210
22		5313	5415	5518	5621	5724	5827	5930	6033	6135	6238
N.	0	1	2	3	4	5	6	7	8	9	

N.	0	1	2	3	4	5	6	7	8	9	
482	68	6340	6443	6546	6648	6751	6853	6956	7059	7161	7263
94		7368	7468	7571	7673	7775	7878	7980	8082	8184	8287
485		8389	8491	8593	8695	8797	8899	9002	9104	9206	9308
96		9410	9512	9613	9715	9817	9919				
	68							0081	0183	0284	0386
97		0488	0590	0691	0793	0895	0996	1098	1199	1301	1402
98		1444	1545	1647	1748	1849	1951	2052	2153	2255	2356
99		2457	2559	2660	2761	2862	2963	3064	3165	3266	3367
430		3469	3569	3670	3771	3872	3973	4074	4175	4276	4377
31		4477	4578	4679	4780	4880	4981	5081	5182	5283	5383
32		5484	5584	5685	5785	5886	5986	6086	6187	6287	6388
- 33		6488	6588	6688	6789	6889	6989	7089	7189	7290	7390
34		7490	7590	7690	7790	7890	7990	8090	8190	8290	8390
435		8490	8590	8689	8789	8889	8989	9089	9189	9287	9387
36		9487	9586	9686	9785	9885	9984				
	64							0084	0183	0283	0388
37		0481	0581	0680	0780	0879	0978	1077	1176	1276	1375
38		1474	1573	1672	1771	1871	1970	2069	2168	2267	2366
39		2465	2563	2662	2761	2860	2959	3058	3157	3255	3354
440		3453	3551	3650	3749	3847	3946	4044	4143	4242	4340
41		4439	4537	4636	4734	4832	4931	5029	5127	5226	5324
42		5428	5525	5623	5721	5819	5917	6015	6113	6211	6309
43		6404	6502	6600	6698	6796	6894	6992	7089	7187	7285
44		7383	7481	7579	7676	7774	7872	7969	8067	8165	8262
445		8360	8458	8555	8653	8750	8848	8945	9043	9140	9237
46		9335	9432	9530	9627	9724	9821	9919			
	65								0016	0113	0210
47		0307	0405	0502	0599	0696	0793	0890	0987	1084	1181
48		1278	1375	1472	1569	1666	1762	1859	1956	2053	2150
49		2246	2343	2440	2536	2633	2730	2826	2923	3019	3116
480		3219	3309	3406	3502	3598	3695	3791	3888	3984	4080
51		4177	4273	4369	4465	4562	4658	4754	4850	4946	5042
52		5136	5231	5326	5422	5517	5613	5715	5811	5906	6002
53		6006	6104	6200	6296	6392	6488	6577	6673	6768	6864
54		7056	7152	7247	7343	7438	7534	7629	7725	7821	7916
455		8011	8107	8202	8298	8393	8489	8584	8679	8774	8870
56		8965	9060	9155	9250	9346	9441	9536	9631	9726	9821
57		9916									
	66		0011	0106	0201	0296	0391	0486	0581	0676	0771
58		0866	0960	1055	1150	1245	1339	1434	1529	1623	1718
59		1813	1907	2002	2096	2191	2286	2380	2475	2569	2663
490		2758	2852	2947	3041	3135	3230	3324	3418	3513	3607
61		3701	3795	3889	3983	4078	4172	4266	4360	4454	4548
62		4642	4736	4830	4924	5018	5112	5206	5300	5393	5487
63		5581	5675	5769	5862	5956	6050	6143	6237	6331	6424
64		6518	6612	6705	6799	6892	6986	7079	7173	7266	7359
465		7453	7546	7640	7733	7826	7920	8013	8106	8200	8293
66		8386	8479	8572	8665	8759	8852	8945	9038	9131	9224
67		9317	9410	9503	9596	9689	9782	9875	9967		
	67									0099	0193
68		0246	0339	0431	0524	0617	0710	0802	0895	0988	1080
69		1173	1265	1358	1451	1543	1636	1728	1821	1913	2005
470		2098	2190	2283	2375	2467	2559	2652	2744	2837	2929
71		3021	3113	3205	3297	3389	3482	3574	3666	3758	3850
72		3942	4034	4126	4218	4310	4402	4494	4586	4677	4769
73		4861	4953	5045	5137	5229	5320	5411	5503	5595	5687
74		5778	5870	5962	6053	6145	6236	6328	6419	6511	6602
475		6604	6705	6806	6908	7009	7110	7211	7312	7414	7516
76		7607	7708	7799	7891	7973	8065	8156	8248	8339	8431
77		8518	8609	8700	8791	8882	8973	9064	9155	9246	9337
78		9498	9519	9610	9700	9791	9882	9973			
	68								0063	0154	0245
79		0336	0428	0517	0607	0698	0789	0879	0970	1060	1151
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TABLE I.—LOG. OF NOS.

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480	68	1941	1332	1452	1513	1603	1693	1784	1874	1965	2065
81		2145	2235	2326	2416	2506	2596	2686	2777	2867	2957
82		3047	3137	3227	3317	3407	3497	3587	3677	3767	3857
83		3947	4037	4127	4217	4307	4396	4486	4576	4666	4756
84		4845	4935	5025	5115	5205	5294	5383	5473	5563	5652
485		5748	5831	5921	6010	6100	6189	6279	6368	6458	6547
86		6636	6726	6815	6904	6994	7083	7173	7261	7351	7440
87		7330	7418	7507	7596	7686	7775	7864	7953	8042	8131
88		8230	8318	8407	8496	8585	8673	8762	8851	8940	9029
89		9130	9218	9307	9396	9485	9573	9662	9751	9840	9929
	69									0019	0107
490		0126	0215	0303	0392	0481	0570	0658	0747	0836	0925
91		1025	1114	1203	1292	1381	1470	1559	1648	1737	1826
92		1925	2014	2103	2192	2281	2370	2459	2548	2637	2726
93		2825	2914	3003	3092	3181	3270	3359	3448	3537	3626
94		3725	3814	3903	3992	4081	4170	4259	4348	4437	4526
495		4625	4714	4803	4892	4981	5070	5159	5248	5337	5426
96		5525	5614	5703	5792	5881	5970	6059	6148	6237	6326
97		6425	6514	6603	6692	6781	6870	6959	7048	7137	7226
98		7325	7414	7503	7592	7681	7770	7859	7948	8037	8126
99		8225	8314	8403	8492	8581	8670	8759	8848	8937	9026
500		9125	9214	9303	9392	9481	9570	9659	9748	9837	9926
01	70	9938	0027	0116	0205	0294	0383	0472	0561	0650	0739
02		0739	0828	0917	1006	1095	1184	1273	1362	1451	1540
03		1639	1728	1817	1906	1995	2084	2173	2262	2351	2440
04		2539	2628	2717	2806	2895	2984	3073	3162	3251	3340
505		3439	3528	3617	3706	3795	3884	3973	4062	4151	4240
06		4339	4428	4517	4606	4695	4784	4873	4962	5051	5140
07		5239	5328	5417	5506	5595	5684	5773	5862	5951	6040
08		6139	6228	6317	6406	6495	6584	6673	6762	6851	6940
09		7039	7128	7217	7306	7395	7484	7573	7662	7751	7840
510		7939	8028	8117	8206	8295	8384	8473	8562	8651	8740
11		8839	8928	9017	9106	9195	9284	9373	9462	9551	9640
12		9739	9828	9917	0006	0095	0184	0273	0362	0451	0540
13	71	0639	0728	0817	0906	0995	1084	1173	1262	1351	1440
14		1539	1628	1717	1806	1895	1984	2073	2162	2251	2340
515		2439	2528	2617	2706	2795	2884	2973	3062	3151	3240
16		3339	3428	3517	3606	3695	3784	3873	3962	4051	4140
17		4239	4328	4417	4506	4595	4684	4773	4862	4951	5040
18		5139	5228	5317	5406	5495	5584	5673	5762	5851	5940
19		6039	6128	6217	6306	6395	6484	6573	6662	6751	6840
520		6939	7028	7117	7206	7295	7384	7473	7562	7651	7740
21		7839	7928	8017	8106	8195	8284	8373	8462	8551	8640
22		8739	8828	8917	9006	9095	9184	9273	9362	9451	9540
23		9639	9728	9817	9906	9995	0084	0173	0262	0351	0440
24	72	0539	0628	0717	0806	0895	0984	1073	1162	1251	1340
525		1439	1528	1617	1706	1795	1884	1973	2062	2151	2240
26		2339	2428	2517	2606	2695	2784	2873	2962	3051	3140
27		3239	3328	3417	3506	3595	3684	3773	3862	3951	4040
28		4139	4228	4317	4406	4495	4584	4673	4762	4851	4940
29		5039	5128	5217	5306	5395	5484	5573	5662	5751	5840
530		5939	6028	6117	6206	6295	6384	6473	6562	6651	6740
31		6839	6928	7017	7106	7195	7284	7373	7462	7551	7640
32		7739	7828	7917	8006	8095	8184	8273	8362	8451	8540
33		8639	8728	8817	8906	8995	9084	9173	9262	9351	9440
34		9539	9628	9717	9806	9895	9984	0073	0162	0251	0340
535		0439	0528	0617	0706	0795	0884	0973	1062	1151	1240
35		1339	1428	1517	1606	1695	1784	1873	1962	2051	2140
36		2239	2328	2417	2506	2595	2684	2773	2862	2951	3040
37	73	3139	3228	3317	3406	3495	3584	3673	3762	3851	3940
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TABLE I.—LOG. OF NOS.

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538	73 07:2	0863	0444	1094	1105	1186	1266	1347	1428	1508
39	15:9	1669	1750	1830	1911	1991	2072	2152	2233	2313
540	2394	2474	2555	2635	2715	2796	2876	2956	3037	3117
41	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919
42	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720
43	4800	48:0	4900	5040	5190	5300	5390	5480	5570	5660
44	5999	5679	5758	5838	5918	5998	6078	6157	6237	6317
545	6397	6476	6556	6636	6715	6795	6874	6954	7034	7113
46	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908
47	7887	80:7	8146	8225	8305	8384	8463	8543	8622	8701
48	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493
49	9579	9651	9731	9810	9889	9968				
	74						0047	0126	0205	0284
550	0363	0442	0521	0600	0678	0757	0836	0915	0994	1073
51	1153	1230	1309	1388	1467	1546	1624	1703	1782	1860
52	1930	2018	2096	2175	2254	2332	2411	2490	2568	2647
53	2725	2804	2882	2961	3039	3118	3196	3274	3353	3431
54	3510	3588	3667	3745	3823	3902	3980	4058	4137	4215
555	4393	4371	4450	4528	4606	4684	4762	4840	4919	4997
56	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777
57	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556
58	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334
59	7413	7490	7567	7645	7723	7800	7878	7955	8033	8111
500	8198	8266	8343	8421	8498	8576	8653	8731	8808	8885
61	8963	9040	9118	9195	9272	9350	9427	9504	9581	9658
62	9736	9814	9891	9968						
	75				0045	0122	0200	0277	0354	0431
63	0508	0585	0663	0740	0817	0894	0971	1048	1125	1202
64	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972
565	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740
66	2816	2893	2970	3047	3123	3200	3277	3353	3430	3507
67	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272
68	4348	4425	4501	4578	4654	4731	4807	4883	4960	5036
69	5112	5189	5265	5341	5417	5494	5570	5646	5722	5798
570	5875	5951	6027	6103	6180	6256	6332	6408	6484	6560
71	6636	6712	6789	6864	6940	7016	7092	7168	7244	7320
72	7396	7472	7548	7624	7700	7776	7851	7927	8003	8078
73	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836
74	8919	8988	9063	9139	9214	9290	9366	9441	9517	9592
575	9668	9743	9819	9894	9970					
	76					0045	0121	0196	0272	0347
76	0428	0498	0573	0649	0724	0799	0875	0950	1025	1100
77	1176	1251	1326	1402	1477	1552	1627	1702	1777	1852
78	1928	2003	2078	2153	2228	2303	2378	2453	2528	2603
79	2679	2754	2829	2903	2979	3053	3128	3203	3278	3353
580	3428	3503	3578	3653	3727	3802	3877	3952	4027	4101
81	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848
82	4893	4968	5042	5117	5191	5266	5340	5415	5489	5563
83	5689	5763	5837	5911	5985	6059	6133	6207	6281	6355
84	6413	6487	6561	6635	6709	6783	6857	6931	7005	7079
585	7156	7230	7304	7378	7452	7526	7600	7674	7748	7822
86	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564
87	86:36	8712	8786	8860	8934	9008	9082	9156	9230	9304
88	9377	9451	9525	9599	9673	9747	9820	9894	9968	
	77									0042
89	0115	0189	0263	0336	0410	0484	0558	0631	0705	0778
590	0850	0924	0998	1072	1146	1220	1293	1367	1441	1514
91	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248
92	2329	2402	2475	2548	2621	2694	2767	2840	2913	2986
93	3055	3128	3201	3274	3347	3420	3493	3566	3639	3712
94	3786	3859	3932	4005	4078	4151	4224	4297	4370	4443
595	4517	4590	4663	4736	4809	4882	4955	5028	5101	5174
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506	77 5946	5319	5399	5465	5539	5610	5683	5756	5829	5902
97	5974	6047	6120	6192	6265	6338	6411	6483	6556	6629
98	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354
99	7437	7499	7572	7644	7717	7789	7862	7934	8007	8079
600	8151	8224	8296	8369	8441	8513	8585	8658	8730	8802
01	8475	8547	8619	8691	8763	8835	8907	8979	9051	9123
02	9597	9669	9741	9813	9885	9957	0029	0101	0173	0245
03	0317	0389	0461	0533	0605	0677	0749	0821	0893	0965
04	1037	1109	1181	1253	1325	1396	1468	1540	1612	1684
605	1755	1827	1899	1971	2043	2114	2186	2258	2329	2401
06	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117
07	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832
08	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546
09	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259
610	5330	5401	5472	5543	5614	5685	5756	5827	5898	5970
11	6041	6112	6183	6254	6325	6396	6468	6539	6610	6681
12	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390
13	7461	7531	7602	7673	7744	7815	7885	7956	8027	8098
14	8168	8239	8310	8381	8451	8522	8593	8663	8734	8805
615	8975	9046	9117	9188	9259	9329	9400	9471	9542	9613
16	9581	9651	9722	9792	9863	9933	0004	0074	0144	0215
17	0285	0356	0426	0496	0567	0637	0707	0777	0848	0918
18	0988	1059	1129	1199	1270	1340	1410	1480	1550	1620
19	1691	1761	1831	1901	1971	2041	2111	2182	2252	2322
620	2339	2409	2479	2549	2619	2689	2759	2829	2899	2969
21	3039	3109	3179	3249	3319	3389	3459	3529	3599	3669
22	3770	3840	3910	3980	4050	4120	4190	4260	4330	4400
23	4480	4550	4620	4690	4760	4830	4900	4970	5040	5110
24	5185	5255	5325	5395	5465	5535	5605	5675	5745	5815
625	5880	5950	6020	6090	6160	6230	6300	6370	6440	6510
26	6574	6644	6714	6784	6854	6924	6994	7064	7134	7204
27	7217	7287	7357	7427	7497	7567	7637	7707	7777	7847
28	7960	8030	8100	8170	8240	8310	8380	8450	8520	8590
29	8651	8721	8791	8861	8931	9001	9071	9141	9211	9281
630	9341	9411	9481	9551	9621	9691	9761	9831	9901	9971
31	0022	0092	0162	0232	0302	0372	0442	0511	0580	0649
32	0717	0787	0857	0927	0997	1067	1137	1206	1275	1344
33	1404	1474	1544	1614	1684	1754	1823	1893	1962	2031
34	2089	2159	2229	2299	2369	2439	2509	2578	2648	2717
635	2774	2844	2914	2984	3054	3124	3194	3264	3334	3404
36	3457	3527	3597	3667	3737	3807	3877	3947	4017	4087
37	4130	4200	4270	4340	4410	4480	4550	4620	4690	4760
38	4821	4891	4961	5031	5101	5171	5241	5311	5381	5451
39	5501	5571	5641	5711	5781	5851	5921	5991	6061	6131
640	6180	6250	6320	6390	6460	6530	6600	6670	6740	6810
41	6858	6928	6998	7068	7138	7208	7278	7348	7418	7488
42	7535	7605	7675	7745	7815	7885	7955	8025	8095	8165
43	8211	8281	8351	8421	8491	8561	8631	8701	8771	8841
44	8906	8976	9046	9116	9186	9256	9326	9396	9466	9536
645	9500	9570	9640	9710	9780	9850	9920	9990	0060	0130
46	0233	0303	0373	0443	0513	0583	0653	0723	0793	0863
47	0904	0974	1044	1114	1184	1254	1324	1394	1464	1534
48	1575	1645	1715	1785	1855	1925	1995	2065	2135	2205
49	2245	2315	2385	2455	2525	2595	2665	2735	2805	2875
650	2913	2983	3053	3123	3193	3263	3333	3403	3473	3543
51	3581	3651	3721	3791	3861	3931	4001	4071	4141	4211
52	4248	4318	4388	4458	4528	4598	4668	4738	4808	4878
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N.	0	1	2	3	4	5	6	7	8	9
653	81 4913	4960	5046	5113	5179	5246	5312	5378	5445	5511
54	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175
655	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838
56	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499
57	7565	7632	7698	7764	7830	7896	7962	8028	8094	8160
58	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820
59	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478
660	9544	9610	9676	9741	9807	9873	9939			
61	82 0202	0267	0333	0399	0464	0530	0596	0661	0727	0792
62	0858	0924	0989	1055	1120	1186	1251	1317	1383	1449
63	1513	1579	1645	1710	1776	1841	1906	1972	2037	2103
64	2168	2234	2299	2364	2430	2495	2560	2626	2691	2757
665	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409
66	3474	3539	3605	3670	3735	3800	3865	3931	3996	4061
67	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711
68	4777	4841	4907	4972	5036	5101	5166	5231	5296	5361
69	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010
670	6075	6140	6204	6269	6334	6399	6464	6528	6593	6658
71	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305
72	7369	7434	7499	7563	7628	7692	7757	7821	7886	7950
73	8015	8080	8144	8209	8273	8338	8402	8467	8531	8596
74	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239
675	9304	9368	9432	9497	9561	9625	9690	9754	9818	9882
76	9947									
77	83 0589	0653	0717	0781	0845	0909	0973	1037	1102	1166
78	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806
79	1870	1934	1998	2062	2126	2190	2253	2317	2381	2445
680	2509	2573	2637	2701	2764	2828	2892	2956	3020	3083
81	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721
82	3784	3848	3912	3975	4039	4103	4166	4230	4293	4357
83	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993
84	5056	5120	5183	5246	5310	5373	5437	5500	5564	5627
685	5691	5754	5817	5881	5944	6007	6071	6134	6198	6261
86	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894
87	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525
88	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156
89	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786
690	8849	8912	8975	9038	9101	9164	9227	9290	9353	9415
91	9478	9541	9604	9667	9730	9792	9855	9918	9981	
92	84 0108	0169	0232	0294	0357	0420	0482	0545	0608	0671
93	0732	0795	0859	0921	0984	1047	1109	1172	1234	1297
94	1360	1422	1485	1547	1610	1672	1735	1797	1860	1922
695	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547
96	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170
97	3233	3295	3357	3420	3482	3544	3607	3669	3731	3793
98	3855	3918	3980	4042	4104	4166	4228	4291	4353	4415
99	4477	4539	4601	4663	4725	4788	4850	4912	4974	5036
700	5098	5160	5222	5284	5346	5408	5470	5532	5594	5656
01	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275
02	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894
03	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511
04	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128
705	8189	8251	8313	8374	8436	8497	8559	8620	8682	8743
06	8805	8866	8928	8989	9051	9112	9174	9235	9296	9358
07	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972
08	0033	0095	0156	0217	0279	0340	0401	0462	0524	0585
09	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197
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11		1670	1721	1772	1823	1874	1925	1976	2027	2078
12		2480	2531	2582	2633	2684	2735	2786	2837	2888
13		3090	3141	3192	3243	3294	3345	3396	3447	3498
14		3698	3749	3800	3851	3902	3953	4004	4055	4106
715		4306	4357	4408	4459	4510	4561	4612	4663	4714
16		4913	4964	5015	5066	5117	5168	5219	5270	5321
17		5519	5570	5621	5672	5723	5774	5825	5876	5927
18		6124	6175	6226	6277	6328	6379	6430	6481	6532
19		6729	6780	6831	6882	6933	6984	7035	7086	7137
720		7332	7383	7434	7485	7536	7587	7638	7689	7740
21		7935	7986	8037	8088	8139	8190	8241	8292	8343
22		8537	8588	8639	8690	8741	8792	8843	8894	8945
23		9136	9187	9238	9289	9340	9391	9442	9493	9544
24		9739	9790	9841	9892	9943	9994			
	86					0038	0089	0140	0191	0242
725		0338	0389	0440	0491	0542	0593	0644	0695	0746
26		0937	0988	1039	1090	1141	1192	1243	1294	1345
27		1534	1585	1636	1687	1738	1789	1840	1891	1942
28		2131	2182	2233	2284	2335	2386	2437	2488	2539
29		2737	2788	2839	2890	2941	2992	3043	3094	3145
730		3332	3383	3434	3485	3536	3587	3638	3689	3740
31		3917	3968	4019	4070	4121	4172	4223	4274	4325
32		4511	4562	4613	4664	4715	4766	4817	4868	4919
33		5104	5155	5206	5257	5308	5359	5410	5461	5512
34		5696	5747	5798	5849	5900	5951	6002	6053	6104
735		6297	6348	6399	6450	6501	6552	6603	6654	6705
36		6978	7029	7080	7131	7182	7233	7284	7335	7386
37		7468	7519	7570	7621	7672	7723	7774	7825	7876
38		8056	8107	8158	8209	8260	8311	8362	8413	8464
39		8644	8695	8746	8797	8848	8899	8950	9001	9052
740		9232	9283	9334	9385	9436	9487	9538	9589	9640
41		9818	9869	9920	9971					
	87					0053	0104	0155	0206	0257
42		0404	0455	0506	0557	0608	0659	0710	0761	0812
43		0960	1011	1062	1113	1164	1215	1266	1317	1368
44		1573	1624	1675	1726	1777	1828	1879	1930	1981
745		2156	2207	2258	2309	2360	2411	2462	2513	2564
46		2739	2790	2841	2892	2943	2994	3045	3096	3147
47		3321	3372	3423	3474	3525	3576	3627	3678	3729
48		3902	3953	4004	4055	4106	4157	4208	4259	4310
49		4482	4533	4584	4635	4686	4737	4788	4839	4890
750		5061	5112	5163	5214	5265	5316	5367	5418	5469
51		5640	5691	5742	5793	5844	5895	5946	5997	6048
52		6218	6269	6320	6371	6422	6473	6524	6575	6626
53		6795	6846	6897	6948	6999	7050	7101	7152	7203
54		7371	7422	7473	7524	7575	7626	7677	7728	7779
755		7947	8008	8069	8120	8171	8222	8273	8324	8375
56		8522	8573	8624	8675	8726	8777	8828	8879	8930
57		9096	9147	9198	9249	9300	9351	9402	9453	9504
58		9669	9720	9771	9822	9873	9924			
	88							0013	0064	0115
59		0249	0300	0351	0402	0453	0504	0555	0606	0657
760		0814	0865	0916	0967	1018	1069	1120	1171	1222
61		1385	1436	1487	1538	1589	1640	1691	1742	1793
62		1955	2006	2057	2108	2159	2210	2261	2312	2363
63		2523	2574	2625	2676	2727	2778	2829	2880	2931
64		3093	3144	3195	3246	3297	3348	3399	3450	3501
765		3661	3712	3763	3814	3865	3916	3967	4018	4069
66		4239	4290	4341	4392	4443	4494	4545	4596	4647
67		4725	4776	4827	4878	4929	4980	5031	5082	5133
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768	88 5361	5418	5474	5531	5587	5644	5700	5757	5813	5870
69	5396	5453	6039	6096	6152	6209	6265	6322	6378	6434
770	6491	6547	6604	6660	6716	6773	6829	6885	6942	6998
71	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561
72	7617	7674	7730	7786	7842	7899	7955	8011	8067	8123
73	8180	8236	8292	8348	8404	8460	8517	8573	8629	8685
74	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246
775	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806
76	9869	9918	9974							
77	89 0421	0477	0533	0589	0645	0700	0756	0812	0868	0924
78	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482
79	1538	1593	1649	1705	1760	1816	1872	1927	1983	2039
780	9035	9100	9166	9231	9297	9362	9427	9492	9557	9622
81	9687	9752	9817	9882	9947	10012	10077	10142	10207	10272
82	3207	3272	3337	3402	3467	3532	3597	3662	3727	3792
83	3768	3833	3898	3963	4028	4093	4158	4223	4288	4353
84	4316	4381	4446	4511	4576	4641	4706	4771	4836	4901
785	4870	4935	4999	5064	5129	5194	5259	5324	5389	5454
86	5423	5488	5553	5618	5683	5748	5813	5878	5943	6008
87	5775	6030	6085	6140	6195	6250	6305	6360	6415	6470
88	6525	6580	6635	6690	6745	6800	6855	6910	6965	7020
89	7076	7131	7186	7241	7296	7351	7406	7461	7516	7571
790	7627	7682	7737	7792	7847	7902	7957	8012	8067	8122
91	8176	8231	8286	8341	8396	8451	8506	8561	8616	8671
92	8725	8780	8835	8890	8945	8999	9054	9109	9164	9219
93	9273	9328	9383	9438	9492	9547	9602	9657	9712	9767
94	9821	9875	9930	9985						
795	90 0357	0422	0476	0531	0586	0640	0695	0749	0804	0858
96	0913	0968	1023	1077	1131	1186	1240	1295	1349	1404
97	1458	1513	1567	1622	1676	1731	1785	1840	1894	1949
98	2003	2057	2112	2166	2221	2275	2329	2384	2438	2493
99	2547	2601	2655	2710	2764	2818	2873	2927	2981	3035
800	3070	3144	3199	3253	3307	3361	3415	3470	3524	3578
01	3632	3687	3741	3795	3849	3903	3957	4011	4065	4119
02	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661
03	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202
04	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742
805	5796	5850	5904	5958	6012	6066	6120	6173	6227	6281
06	6335	6389	6443	6497	6550	6604	6658	6712	6766	6820
07	6873	6927	6981	7035	7089	7143	7197	7250	7304	7358
08	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895
09	7949	8003	8056	8110	8163	8217	8271	8324	8378	8431
810	8485	8539	8592	8646	8699	8753	8807	8860	8914	8967
11	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503
12	9556	9609	9663	9717	9770	9823	9877	9930	9984	
13	0030	0144	0197	0251	0304	0358	0411	0464	0518	0571
14	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104
815	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637
16	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169
17	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700
18	2753	2806	2860	2913	2966	3019	3073	3125	3178	3231
19	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761
820	3814	3867	3920	3973	4026	4079	4132	4184	4237	4290
21	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819
22	4872	4925	4978	5030	5083	5136	5189	5242	5294	5347
23	5400	5453	5505	5558	5611	5664	5717	5769	5822	5875
24	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401
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TABLE I.—LOG. OF NOS.

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26		6960	7033	7025	7136	7190	7243	7295	7348	7400	7453
27		7506	7558	7611	7663	7716	7768	7821	7873	7925	7978
28		8030	8083	8135	8188	8240	8293	8345	8397	8450	8502
29		8555	8607	8659	8713	8764	8816	8869	8921	8973	9026
830		9078	9130	9183	9235	9287	9340	9392	9444	9497	9549
31		9601	9653	9706	9758	9810	9862	9914	9967		
	92									0019	0071
32		0123	0176	0228	0280	0332	0384	0436	0489	0541	0593
33		0645	0697	0749	0801	0853	0906	0958	1010	1062	1114
34		1166	1218	1270	1323	1374	1426	1478	1530	1582	1634
835		1687	1739	1790	1842	1894	1946	1998	2050	2102	2154
36		2206	2258	2310	2362	2414	2466	2518	2570	2622	2674
37		2725	2777	2829	2881	2933	2985	3037	3089	3140	3192
38		3244	3296	3348	3400	3451	3503	3555	3607	3658	3710
39		3762	3814	3866	3917	3969	4021	4072	4124	4176	4228
840		4279	4331	4383	4434	4486	4538	4589	4641	4693	4744
41		4796	4848	4899	4951	5003	5054	5106	5157	5209	5261
42		5313	5364	5415	5467	5518	5570	5621	5673	5725	5776
43		5828	5879	5931	5982	6034	6085	6137	6188	6240	6291
44		6342	6394	6445	6497	6548	6600	6651	6703	6754	6805
845		6857	6908	6960	7011	7062	7114	7165	7216	7268	7319
46		7370	7422	7473	7524	7576	7627	7678	7730	7781	7832
47		7883	7935	7986	8037	8089	8140	8192	8243	8295	8346
48		8398	8449	8498	8550	8601	8652	8703	8754	8805	8857
49		8908	8959	9010	9061	9112	9163	9214	9266	9317	9368
850		9419	9470	9521	9572	9623	9674	9725	9776	9827	9879
51		9930	9981								
	93										
52		0040	0091	0142	0193	0244	0295	0346	0397	0448	0499
53		0549	0600	0651	0702	0753	0804	0855	0906	0957	1008
54		1058	1109	1160	1211	1262	1313	1364	1415	1466	1517
855		1568	1619	1670	1721	1772	1823	1874	1925	1976	2027
56		2078	2129	2180	2231	2282	2333	2384	2435	2486	2537
57		2588	2639	2690	2741	2792	2843	2894	2945	2996	3047
58		3098	3149	3200	3251	3302	3353	3404	3455	3506	3557
59		3608	3659	3710	3761	3812	3863	3914	3965	4016	4067
860		4118	4169	4220	4271	4322	4373	4424	4475	4526	4577
61		4628	4679	4730	4781	4832	4883	4934	4985	5036	5087
62		5138	5189	5240	5291	5342	5393	5444	5495	5546	5597
63		5648	5699	5750	5801	5852	5903	5954	6005	6056	6107
64		6158	6209	6260	6311	6362	6413	6464	6515	6566	6617
865		6668	6719	6770	6821	6872	6923	6974	7025	7076	7127
66		7178	7229	7280	7331	7382	7433	7484	7535	7586	7637
67		7688	7739	7790	7841	7892	7943	7994	8045	8096	8147
68		8198	8249	8300	8351	8402	8453	8504	8555	8606	8657
69		8708	8759	8810	8861	8912	8963	9014	9065	9116	9167
870		9218	9269	9320	9371	9422	9473	9524	9575	9626	9677
71	94	0018	0069	0120	0171	0222	0273	0324	0375	0426	0477
72		0528	0579	0630	0681	0732	0783	0834	0885	0936	0987
73		1038	1089	1140	1191	1242	1293	1344	1395	1446	1497
74		1548	1599	1650	1701	1752	1803	1854	1905	1956	2007
875		2058	2109	2160	2211	2262	2313	2364	2415	2466	2517
76		2568	2619	2670	2721	2772	2823	2874	2925	2976	3027
77		3078	3129	3180	3231	3282	3333	3384	3435	3486	3537
78		3588	3639	3690	3741	3792	3843	3894	3945	3996	4047
79		4098	4149	4200	4251	4302	4353	4404	4455	4506	4557
880		4608	4659	4710	4761	4812	4863	4914	4965	5016	5067
81		4978	5029	5080	5131	5182	5233	5284	5335	5386	5437
82		5488	5539	5590	5641	5692	5743	5794	5845	5896	5947
83		5998	6049	6100	6151	6202	6253	6304	6355	6406	6457
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884	94 6432	6501	6551	6600	6649	6698	6747	6796	6845	6894
885	6943	6992	7041	7091	7140	7189	7238	7287	7336	7385
886	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875
887	7974	7973	8022	8071	8119	8168	8217	8266	8315	8364
888	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853
889	8902	8951	8999	9048	9097	9146	9195	9244	9293	9341
890	9390	9439	9488	9536	9585	9634	9683	9732	9780	9829
891	9878	9926	9975							
892	95	0065	0414	0462	0511	0560	0608	0657	0706	0754
893	0652	0900	0949	0997	1046	1095	1143	1192	1240	1289
894	1337	1386	1435	1483	1532	1580	1629	1677	1726	1775
895	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260
896	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744
897	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228
898	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711
899	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194
900	4242	4291	4339	4387	4436	4484	4532	4580	4628	4677
901	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158
902	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640
903	5682	5730	5778	5826	5874	5922	5970	6018	6066	6114
904	6162	6210	6258	6306	6354	6402	6450	6498	6546	6594
905	6642	6690	6738	6786	6834	6882	6930	6978	7026	7074
906	7122	7170	7218	7266	7314	7362	7410	7458	7506	7554
907	7602	7650	7698	7746	7794	7842	7890	7938	7986	8034
908	8082	8130	8178	8226	8274	8322	8370	8418	8466	8514
909	8564	8612	8660	8708	8756	8804	8852	8900	8948	8996
910	9041	9089	9137	9185	9233	9281	9329	9377	9425	9473
911	9512	9560	9608	9656	9704	9752	9800	9848	9896	9944
912	9995									
913	96	0042	0090	0138	0185	0233	0280	0328	0376	0423
914	0471	0518	0566	0614	0661	0709	0756	0804	0851	0899
915	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374
916	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848
917	1896	1943	1990	2038	2085	2132	2180	2227	2275	2322
918	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795
919	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268
920	3316	3363	3410	3457	3505	3552	3599	3646	3693	3741
921	3788	3835	3882	3929	3977	4024	4071	4118	4165	4213
922	4280	4327	4374	4421	4468	4515	4562	4609	4656	4704
923	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155
924	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625
925	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095
926	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564
927	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033
928	7070	7117	7163	7210	7257	7304	7351	7398	7445	7491
929	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969
930	8016	8063	8109	8156	8203	8249	8296	8343	8390	8436
931	8483	8530	8576	8623	8670	8716	8763	8810	8856	8903
932	8950	8996	9043	9090	9136	9183	9230	9276	9323	9369
933	9416	9462	9509	9556	9602	9649	9695	9742	9789	9835
934	9882	9928	9975							
935	97	0347	0393	0440	0486	0533	0579	0626	0672	0719
936	0612	0658	0705	0751	0797	0844	0890	0937	0983	1030
937	1076	1122	1169	1215	1261	1308	1354	1401	1447	1493
938	1539	1585	1632	1678	1724	1771	1817	1864	1910	1957
939	2003	2049	2095	2142	2188	2234	2281	2327	2373	2419
940	2466	2512	2558	2604	2651	2697	2743	2789	2835	2882
941	3198	3174	3290	3266	3313	3359	3405	3451	3497	3544
N.	0	1	2	3	4	5	6	7	8	9

N.	0	1	2	3	4	5	6	7	8	9	
941	97	3690	3636	3689	3738	3774	3800	3866	3913	3959	4005
42		4051	4097	4143	4189	4237	4281	4327	4374	4420	4466
43		4519	4558	4604	4650	4696	4742	4788	4834	4880	4926
44		4973	5018	5064	5110	5156	5202	5248	5294	5340	5386
945		5432	5478	5524	5570	5616	5662	5707	5753	5799	5845
45		5891	5937	5983	6029	6075	6121	6167	6213	6258	6304
47		6359	6396	6442	6488	6533	6579	6625	6671	6717	6763
48		6808	6854	6900	6946	6992	7037	7083	7129	7175	7220
49		7266	7313	7358	7404	7449	7495	7541	7586	7632	7678
950		7724	7769	7815	7861	7906	7952	7998	8043	8089	8135
51		8180	8226	8272	8317	8363	8409	8454	8500	8546	8591
52		8637	8683	8728	8774	8819	8865	8911	8956	9002	9047
53		9093	9138	9184	9230	9275	9321	9366	9412	9457	9503
54		9548	9594	9639	9685	9730	9776	9821	9867	9912	9958
955	98	0003	0049	0094	0140	0185	0231	0276	0322	0367	0412
56		0458	0503	0549	0594	0640	0685	0730	0776	0821	0867
57		0912	0957	1003	1048	1093	1139	1184	1230	1275	1320
58		1366	1411	1456	1502	1547	1592	1637	1683	1728	1773
59		1819	1864	1909	1954	2000	2045	2090	2135	2181	2226
960		2271	2317	2362	2407	2452	2497	2543	2588	2633	2679
61		2723	2769	2814	2859	2904	2949	2995	3040	3085	3130
62		3175	3220	3265	3311	3356	3401	3446	3491	3536	3581
63		3626	3671	3716	3762	3807	3852	3897	3942	3987	4032
64		4077	4122	4167	4212	4257	4302	4347	4392	4437	4482
965		4527	4572	4617	4662	4707	4752	4797	4842	4887	4932
66		4977	5022	5067	5112	5157	5202	5247	5292	5337	5382
67		5426	5471	5516	5561	5606	5651	5696	5741	5786	5831
68		5875	5920	5965	6010	6055	6100	6145	6190	6234	6279
69		6324	6369	6413	6458	6503	6548	6593	6637	6682	6727
970		6772	6817	6861	6906	6951	6996	7040	7085	7130	7175
71		7219	7264	7309	7353	7398	7443	7488	7532	7577	7622
72		7666	7711	7756	7800	7845	7890	7934	7979	8024	8068
73		8113	8158	8202	8247	8291	8336	8381	8425	8470	8514
74		8559	8603	8648	8693	8737	8782	8826	8871	8916	8960
975		9005	9049	9094	9138	9183	9227	9272	9316	9361	9405
76		9450	9494	9539	9583	9628	9672	9717	9761	9806	9850
77		9895	9939	9984							
980	99	0039	0083	0128	0172	0217	0261	0305	0350	0394	0438
78		0783	0827	0871	0916	0960	1004	1049	1093	1137	1182
985		1226	1270	1315	1359	1403	1448	1492	1536	1581	1625
81		1669	1713	1758	1802	1846	1890	1935	1979	2023	2067
82		2111	2156	2200	2244	2288	2333	2377	2421	2465	2509
83		2554	2598	2642	2686	2730	2774	2819	2863	2907	2951
84		2995	3039	3083	3127	3171	3215	3260	3304	3348	3392
985		3436	3480	3524	3568	3612	3657	3701	3745	3789	3833
86		3877	3921	3965	4009	4053	4097	4141	4185	4229	4273
87		4317	4361	4405	4449	4493	4537	4581	4625	4669	4713
88		4757	4801	4845	4889	4933	4977	5021	5065	5109	5152
89		5196	5240	5284	5328	5372	5416	5460	5504	5547	5591
990		5635	5679	5723	5767	5811	5854	5898	5942	5986	6030
91		6074	6118	6161	6205	6249	6293	6337	6380	6424	6468
93		6512	6555	6599	6643	6687	6731	6774	6818	6862	6906
95		6949	6993	7037	7080	7124	7168	7211	7255	7299	7343
94		7386	7430	7474	7517	7561	7605	7649	7692	7736	7779
985		7823	7867	7910	7954	7998	8041	8085	8128	8172	8216
96		8259	8303	8347	8390	8434	8477	8521	8565	8608	8652
97		8696	8739	8783	8826	8869	8912	8956	9000	9043	9087
98		9131	9174	9218	9261	9305	9348	9392	9435	9479	9522
99		9566	9609	9652	9696	9739	9782	9826	9870	9913	9957
1000	00	0000	0043	0087	0130	0174	0217	0261	0304	0347	0391
N.	0	1	2	3	4	5	6	7	8	9	

TABLE II.

LOGARITHMS OF SINES, COSEC'S, TANGENTS, &c.

TABLE II.—LOG. SINES, TANG'S, &C.

0°		1° - 15' 1° - 15' 1° - 15°						179°		180°	
Deg.	L. Sin.	Diff. for 5'	L. Cossec.	L. Cos.	L. Sec.	L. Tang.	Diff. for 5'	L. Cot.	Deg.	Hours.	
15	8.110926	802	1.889074	9.999903	0.000037	8.110903	802	1.889037	15	56	
15	.119332	798	.890668	.02	.38	.119370	798	.890630	45	59	
30	.121725	793	.878275	.02	.38	.121763	793	.878237	30	58	
45	.124104	789	.875896	.02	.38	.124142	789	.875857	15	57	
46	.126471	785	.873529	.01	.39	.126510	785	.873490	14	56	
15	.128825	780	.871175	.01	.40	.128864	781	.871136	45	55	
30	.131166	776	.868834	.00	.40	.131206	776	.868794	30	54	
45	.133494	772	.866506	.00	.40	.133534	772	.866466	15	53	
47	.135810	768	.864190	.59	.41	.135851	768	.864149	13	52	
15	.138114	764	.861886	.59	.41	.138155	764	.861845	45	51	
30	.140406	760	.859594	.59	.41	.140447	760	.859553	30	50	
45	.142685	756	.857315	.58	.42	.142727	756	.857273	15	49	
18	.144953	752	.855047	.57	.43	.144994	752	.855004	12	48	
15	.147209	748	.852791	.57	.43	.147250	748	.852748	45	47	
30	.149453	744	.850547	.56	.44	.149497	744	.850503	30	46	
45	.151688	741	.848314	.56	.44	.151730	741	.848270	15	45	
19	.153908	737	.846092	.56	.44	.153952	737	.846048	11	44	
15	.156118	733	.843882	.55	.45	.156162	733	.843838	45	43	
30	.158316	729	.841684	.55	.45	.158361	729	.841639	30	42	
45	.160504	726	.839496	.54	.46	.160550	726	.839450	15	41	
20	.162681	722	.837319	.54	.46	.162727	722	.837273	10	40	
15	.164847	719	.835153	.54	.46	.164893	719	.835107	45	39	
30	.167002	715	.832998	.53	.47	.167049	715	.832951	30	38	
45	.169146	711	.830854	.52	.48	.169194	711	.830806	15	37	
51	.171280	708	.828720	.52	.48	.171339	708	.828672	9	36	
15	.173404	705	.826596	.52	.48	.173452	705	.826548	45	35	
30	.175517	701	.824483	.51	.49	.175566	701	.824434	30	34	
45	.177620	698	.822380	.51	.49	.177670	698	.822331	15	33	
22	.179713	694	.820287	.50	.50	.179773	694	.820237	8	32	
15	.181796	691	.818204	.50	.50	.181846	691	.818154	45	31	
30	.183868	688	.816132	.49	.51	.183919	688	.816081	30	30	
45	.185931	684	.814069	.48	.52	.185982	684	.814017	15	29	
23	.187984	682	.812016	.48	.52	.188036	682	.811964	7	28	
15	.190028	678	.809972	.48	.52	.190081	678	.809919	45	27	
30	.192062	675	.807938	.47	.53	.192115	675	.807885	30	26	
45	.194087	672	.805913	.47	.53	.194140	672	.805860	15	25	
24	.196102	669	.803898	.46	.54	.196156	669	.803844	6	24	
15	.198108	665	.801892	.46	.54	.198162	666	.801838	45	23	
30	.200104	663	.799896	.45	.55	.200159	663	.799841	30	22	
45	.202092	659	.797908	.45	.55	.202147	660	.797853	15	21	
25	.204070	657	.795930	.44	.56	.204126	657	.795874	5	20	
15	.206040	653	.793960	.44	.56	.206096	654	.793904	45	19	
30	.208000	651	.792000	.43	.57	.208057	651	.791943	30	18	
45	.209952	648	.790048	.43	.57	.210009	648	.789991	15	17	
26	.211895	645	.788103	.42	.58	.211953	645	.788047	4	16	
15	.213829	642	.786171	.42	.58	.213887	642	.786133	45	15	
30	.215755	639	.784245	.41	.59	.215814	639	.784186	30	14	
45	.217672	636	.782328	.41	.59	.217731	637	.782269	15	13	
27	.219581	634	.780419	.40	.60	.219641	634	.780359	3	12	
15	.221482	631	.778518	.40	.60	.221542	631	.778458	45	11	
30	.223374	628	.776626	.39	.61	.223435	628	.776565	30	10	
45	.225258	625	.774742	.39	.61	.225319	625	.774681	15	9	
28	.227133	623	.772867	.39	.62	.227195	623	.772805	2	8	
15	.229001	623	.770999	.37	.63	.229064	623	.770926	45	7	
30	.230861	620	.769130	.37	.63	.230924	620	.769076	30	6	
45	.232713	617	.767267	.37	.63	.232770	617	.767234	15	5	
29	.234557	615	.765413	.36	.64	.234621	615	.765379	1	4	
15	.236393	612	.763560	.35	.65	.236469	612	.763522	45	3	
30	.238221	609	.761719	.35	.65	.238316	609	.761714	30	2	
45	.240042	607	.759885	.34	.66	.240160	607	.759892	15	1	
30	8.241855	604	1.758145	9.999933	0.000067	8.241922	605	1.758075	0	50	
Deg.	L. Cos.	Diff. for 5'	L. Sec.	L. Sin.	L. Cossec.	L. Cot.	Diff. for 5'	L. Tang.	Deg.	Hours.	

— 00°

1° = 3' 10" = 4"

89° 54'

TABLE II.—LOG. SINES, TANG'S, &c.

1°		1' - 15' 1" - 15' 1" - 15°						178°		11'	
Deg.	L. Sin.	Diff. for 15" or 1'	L. Cossec.	L. Cos.	L. Sec.	L. Tang.	Diff. for 15" or 1'	L. Cot.	Deg.	Hours.	
10	8.417919	1203	1.529081	9.999851	0.000149	8.418068	1904	1.581932	50	53	
30	.420325	1194	.579675	.50	.50	.420475	1197	.579325	30	55	
11	.422717	1189	.577283	.48	.52	.422869	1190	.577131	29	56	
30	.425006	1183	.574804	.46	.54	.425250	1184	.574750	30	54	
12	.427402	1176	.572333	.44	.56	.427618	1177	.572322	28	52	
30	.429816	1170	.570184	.43	.57	.429973	1171	.570027	30	50	
13	.432158	1164	.567844	.41	.59	.432315	1165	.567653	27	48	
30	.434484	1158	.565516	.39	.61	.434645	1159	.565355	30	46	
14	.436800	1151	.563200	.38	.62	.436962	1152	.563038	26	44	
30	.439103	1145	.560897	.36	.64	.439367	1146	.560733	30	42	
15	.441394	1140	.558606	.34	.66	.441560	1141	.558440	25	40	
30	.443674	1133	.556326	.33	.67	.443841	1134	.556152	30	38	
16	.445941	1127	.554059	.31	.69	.446110	1128	.553881	24	36	
30	.448196	1122	.551804	.30	.71	.448367	1123	.551633	30	34	
17	.450440	1116	.549560	.27	.73	.450613	1117	.549387	23	32	
30	.452672	1110	.547328	.25	.75	.452847	1111	.547153	30	30	
18	.454893	1105	.545107	.22	.77	.455070	1106	.544930	22	28	
30	.457103	1099	.542897	.22	.78	.457281	1100	.542711	30	26	
19	.459301	1094	.540699	.20	.80	.459481	1095	.540511	21	24	
30	.461489	1088	.538511	.18	.82	.461671	1089	.538322	30	22	
20	.463665	1082	.536335	.16	.84	.463849	1083	.536151	30	20	
30	.465830	1077	.534170	.14	.86	.466016	1078	.533984	19	18	
1	.467985	1072	.532015	.13	.87	.468172	1072	.531826	19	16	
30	.470129	1067	.529871	.11	.89	.470318	1068	.529662	30	14	
2	.472263	1062	.527737	.09	.91	.472454	1063	.527504	18	12	
30	.474386	1056	.525614	.07	.93	.474579	1057	.525481	30	10	
3	.476496	1051	.523502	.05	.95	.476693	1052	.523397	17	8	
30	.478601	1046	.521394	.03	.97	.478798	1047	.521302	30	6	
4	.480693	1041	.519307	.01	.99	.480892	1042	.519106	16	4	
30	.482775	1036	.517225	.999799	.000201	.482976	1037	.517024	30	2	
5	.484848	1031	.515152	.97	.03	.485051	1032	.514949	15	0	
30	.486910	1026	.513090	.95	.05	.487115	1027	.512883	30	52	
6	.488963	1021	.511037	.93	.07	.489170	1022	.510830	14	56	
30	.491008	1017	.508994	.92	.08	.491214	1018	.508786	30	54	
17	.493040	1012	.506960	.90	.10	.493250	1013	.506750	13	52	
30	.495064	1007	.504936	.88	.12	.495276	1008	.504724	30	50	
18	.497079	1003	.502921	.86	.14	.497293	1004	.502700	12	48	
30	.499084	998	.500916	.84	.16	.499300	999	.500700	30	46	
19	.501080	993	.498920	.82	.18	.501298	994	.498702	11	44	
30	.503067	989	.496933	.80	.20	.503287	990	.496713	30	42	
20	.505045	984	.494955	.78	.22	.505267	985	.494733	10	40	
30	.507014	980	.492988	.76	.24	.507238	981	.492752	30	38	
11	.508974	975	.491029	.74	.26	.509200	976	.490800	9	36	
30	.510925	971	.489075	.72	.28	.511153	972	.488847	30	34	
12	.512867	967	.487133	.69	.31	.513098	968	.486902	8	32	
30	.514801	962	.485199	.67	.33	.515034	963	.484966	30	30	
13	.516726	958	.483274	.65	.35	.516961	959	.483033	7	28	
30	.518643	954	.481357	.63	.37	.518880	955	.481130	30	26	
14	.520551	950	.479449	.61	.39	.520790	951	.479210	6	24	
30	.522451	946	.477549	.59	.41	.522692	947	.477308	30	22	
15	.524343	941	.475657	.57	.43	.524586	943	.475414	5	20	
30	.526228	938	.473774	.55	.45	.526471	939	.473529	30	18	
16	.528109	933	.471896	.53	.47	.528349	934	.471651	4	16	
30	.529986	929	.470031	.51	.49	.530218	930	.469782	30	14	
17	.531858	925	.468173	.48	.52	.532080	926	.467920	3	12	
30	.533725	922	.466321	.46	.54	.533933	923	.466067	30	10	
18	.535583	918	.464477	.44	.56	.535779	919	.464221	2	8	
30	.537435	913	.462644	.42	.58	.537616	914	.462384	30	6	
19	.539281	910	.460814	.40	.60	.539447	911	.460554	1	4	
30	.541120	906	.458993	.38	.62	.541289	907	.458731	30	2	
20	8.542910	900	1.457181	9.999735	0.000265	8.543084	900	1.456916	0	0	

TABLE II.—LOG. SINES, TANG'S, &c.

0° 20'

1° - 15' 1" - 15' 1" - 15°

177°

Hours.	Deg.	L. Sin.	Diff. for 15" or 1'	L. Cossec.	L. Cos.	L. Sec.	L. Tang.	Diff. for 15" or 1'	L. Cot.	Diff.	Deg.
8	0	8.502811	902	1.437181	9.999735	0.000265	8.543084	603	1.456916		60
	2	.544624	896	.433776		.31	.344851	900	.455109		58
	4	.546422	895	.433778		.31	.346691	896	.453300		56
	6	.548212	891	.431788		.29	.348483	892	.451517		54
	8	.549995	887	.430005		.27	.350268	888	.449732		52
	10	.551770	884	.448230		.24	.352046	885	.447954		50
	12	.553539	880	.446461		.22	.353817	881	.446183		48
	14	.555300	877	.444700		.20	.355580	878	.444430		46
	16	.557054	873	.442946		.18	.357336	874	.442684		44
	18	.558800	870	.441200		.15	.359095	871	.440945		42
	20	.560541	866	.439459		.13	.360852	867	.439212		40
	22	.562273	863	.437727		.10	.362603	864	.437437		38
	24	.564009	860	.436001		.08	.364351	861	.435709		36
	26	.565749	856	.434281		.06	.366093	857	.433987		34
	28	.567481	853	.432569		.04	.367827	854	.432273		32
	30	.569217	849	.430863		.01	.369564	850	.430564		30
	32	.570959	846	.429164	9.999601	.000391	.371317	847	.428863		28
	34	.572698	843	.427472		.06	.373062	844	.427168		26
	36	.574434	839	.425786		.04	.374800	840	.425480		24
	38	.576169	836	.424107		.02	.376541	838	.423799		22
	40	.577900	833	.422434		.00	.378284	835	.422123		20
	42	.579628	830	.420768		.00	.380029	832	.420455		18
	44	.581352	827	.419108		.01	.381776	829	.418792		16
	46	.583074	823	.417454		.02	.383524	826	.417136		14
	48	.584793	820	.415807		.03	.385273	823	.415487		12
	50	.586509	817	.414160		.05	.387023	820	.413843		10
	52	.588224	814	.412531		.07	.388774	817	.412206		8
	54	.589938	811	.410902		.09	.390525	814	.410574		6
	56	.591651	808	.409279		.12	.392276	811	.408949		4
	58	.593363	805	.407662		.15	.394027	808	.407330		2
9	0	.595074	802	.406052		.18	.395778	805	.405717		0
	2	.596783	799	.404447		.21	.397529	802	.404110		358
	4	.598491	796	.402848		.24	.399280	799	.402508		356
	6	.599999	793	.401255		.27	.401031	796	.400913		354
	8	.601506	790	.399668		.30	.402782	793	.399323		352
	10	.603013	787	.398087		.33	.404533	790	.397739		350
	12	.604519	784	.396511		.36	.406284	787	.396161		348
	14	.606026	781	.394942		.39	.408035	784	.394589		346
	16	.607532	778	.393377		.42	.409786	781	.393023		344
	18	.609039	775	.391819		.45	.411537	778	.391461		342
	20	.610545	772	.390269		.48	.413288	775	.389906		340
	22	.612051	769	.388719		.51	.415039	772	.388356		338
	24	.613557	766	.387170		.54	.416790	769	.386811		336
	26	.615063	763	.385620		.57	.418541	766	.385272		334
	28	.616569	760	.384071		.60	.420292	763	.383738		332
	30	.618075	757	.382523		.63	.422043	760	.382210		330
	32	.619581	754	.380974		.66	.423794	757	.380687		328
	34	.621087	751	.379426		.69	.425545	754	.379173		326
	36	.622593	748	.377877		.72	.427296	751	.377667		324
	38	.624099	745	.376328		.75	.429047	748	.376169		322
	40	.625605	742	.374780		.78	.430798	745	.374674		320
	42	.627111	739	.373231		.81	.432549	742	.373183		318
	44	.628617	736	.371683		.84	.434300	739	.371694		316
	46	.630123	733	.370134		.87	.436051	736	.370207		314
	48	.631629	730	.368586		.90	.437802	733	.368723		312
	50	.633135	727	.367037		.93	.439553	730	.367240		310
	52	.634641	724	.365488		.96	.441304	727	.365759		308
	54	.636147	721	.363939		.99	.443055	724	.364279		306
	56	.637653	718	.362390		.02	.444806	721	.362800		304
	58	.639159	715	.360841		.05	.446557	718	.361323		302
	60	.640665	712	.359292		.08	.448308	715	.359848		300
9	00	8.639680		1.390620	9.999587	0.000413	8.640693		1.359607		00

0° 20'

1° - 4' 10" - 4°

87°

1° = 15' 1° = 15' 1° = 15° 177° 11°

Hours.	Deg.	L. Sin.	Diff. for 15' or 1"	L. Cossec.	L. Cos.	L. Sec.	L. Tang.	Diff. for 15' or 1"	L. Cot.	Deg.	Hours.
10	0	8.630680	732	1.360320	9.999587	0.000413	8.640093	723	1.359607	30	49
	2	.641134	719	.358876		84	.641540	721	.359460	30	55
	4	.642563	717	.357437		81	.642982	719	.357918	29	54
	6	.643998	715	.356002		78	.644420	716	.356580	30	54
	8	.645428	713	.354572		75	.645853	714	.355147	28	52
	10	.646853	710	.353147		72	.647281	711	.353719	30	50
	12	.648274	708	.351726		70	.648704	709	.352286	27	48
	14	.649690	706	.350310		67	.650123	707	.350857	27	46
	16	.651102	703	.348898		64	.651538	704	.349422	26	44
	18	.652508	701	.347492		61	.652947	702	.347983	30	42
	20	.653911	698	.346089		59	.654352	700	.346548	25	40
	22	.655308	697	.344692		55	.655753	698	.345107	24	38
	24	.656702	694	.343298		53	.657149	696	.343658	24	36
	26	.658091	692	.341909		50	.658541	693	.342209	23	34
	28	.659475	690	.340525		47	.659928	691	.340762	23	32
	30	.660855	687	.339145		44	.661311	689	.339309	30	30
	32	.662230	685	.337770		41	.662689	687	.337851	22	30
	34	.663601	684	.336399		38	.664063	685	.336387	30	28
	36	.664969	684	.335031		35	.665433	683	.334927	21	26
	38	.666331	681	.333669		32	.666799	680	.333469	30	24
	40	.667689	677	.332311		29	.668160	678	.332014	30	22
	42	.669043	675	.330957		26	.669517	676	.330563	30	20
	44	.670393	673	.329607		23	.670870	674	.329116	19	30
	46	.671737	673	.328261		21	.672218	672	.327672	30	18
	48	.673081	671	.326919		18	.673563	670	.326230	18	12
	50	.674418	669	.325582		15	.674903	668	.324797	30	10
	52	.675751	666	.324249		12	.676239	666	.323371	17	30
	54	.677080	664	.322920		09	.677571	664	.321950	6	4
	56	.678405	662	.321595		06	.678899	662	.320534	16	30
	58	.679727	661	.320273		03	.680224	660	.319122	30	28
11	0	.681044	658	.318956	0.000500	00	.681544	658	.318456	15	49
	2	.682356	656	.317644	.999496	04	.682860	656	.317140	30	50
	4	.683665	654	.316335		07	.684172	654	.315828	14	30
	6	.684971	653	.315029		09	.685480	652	.314520	30	56
	8	.686272	648	.313728		88	.686784	650	.313216	13	52
	10	.687569	647	.312431		84	.688085	648	.311915	30	50
	12	.688862	645	.311138		81	.689381	646	.310619	12	48
	14	.690152	643	.309848		78	.690674	644	.309326	30	46
	16	.691438	641	.308562		75	.691963	642	.308037	11	30
	18	.692720	639	.307280		72	.693248	640	.306752	44	42
	20	.693998	637	.306002		69	.694529	639	.305471	10	40
	22	.695273	635	.304727		66	.695807	637	.304193	30	38
	24	.696543	633	.303457		62	.697081	635	.302919	9	36
	26	.697810	631	.302190		59	.698351	633	.301649	30	34
	28	.699073	630	.300927		56	.699617	631	.300383	8	32
	30	.700333	628	.299667		53	.700880	629	.299120	30	30
	32	.701589	626	.298411		50	.702133	628	.297861	7	30
	34	.702841	624	.297159		47	.703384	626	.296606	30	28
	36	.704090	622	.295910		44	.704646	624	.295354	6	24
	38	.705335	621	.294665		40	.705895	622	.294105	30	22
	40	.706577	619	.293423		37	.707140	620	.292859	5	30
	42	.707815	617	.292185		34	.708381	619	.291619	30	18
	44	.709049	615	.290951		31	.709618	617	.290382	4	16
	46	.710280	614	.289720		27	.710853	615	.289147	30	14
	48	.711507	612	.288493		24	.712083	614	.287917	3	12
	50	.712732	610	.287268		21	.713311	612	.286689	30	10
	52	.713952	608	.286046		18	.714534	610	.285466	2	8
	54	.715169	607	.284831		14	.715753	608	.284246	30	6
	56	.716383	607	.283617		11	.716972	608	.283028	1	4
	58	.717593	605	.282407		08	.718185	607	.281815	30	2
	60	.718800	603	.281200	9.999404	0.000596	.719396	605	.280604	0	0

TABLE II.—LOG. SINES, TANG'S, &C.

30		1° - 15' 1° - 15' 1° - 15'					170°		11°	
Deg.	L. Sin.	Diff. for 15' or 1"	L. Cossec.	L. Cos.	L. Sec.	L. Tang.	Diff. for 15' or 1"	L. Cot.	Deg.	Hours
30	8.785075	510	1.214325	9.999189	0.000811	8.786486	518	1.213514	30	45
30	786707	514	.213243	.85	.15	.787522	516	.212478	30	46
31	787739	513	.212264	.82	.18	.788554	515	.211446	29	50
31	788763	512	.211237	.78	.22	.789585	514	.210415	30	54
32	789787	511	.210213	.74	.26	.790613	513	.209387	28	58
30	790809	509	.209191	.70	.30	.791639	512	.208361	30	52
33	791822	508	.208172	.66	.34	.792662	510	.207338	27	56
30	792845	507	.207155	.62	.38	.793683	509	.206317	30	48
34	793859	506	.206141	.58	.42	.794701	508	.205299	25	52
30	794872	505	.205128	.54	.46	.795718	507	.204282	30	46
35	795881	504	.204119	.50	.50	.796731	506	.203269	25	48
30	796889	502	.203111	.46	.54	.797743	504	.202257	30	42
36	797894	502	.202106	.42	.58	.798752	503	.201248	24	46
30	798897	500	.201103	.38	.62	.799759	502	.200241	30	40
37	799897	499	.200103	.34	.66	.800762	501	.199237	23	44
30	800896	498	.199104	.30	.70	.801766	500	.198234	30	38
38	801891	497	.198103	.26	.74	.802765	499	.197235	22	42
30	802885	495	.197115	.22	.78	.803763	497	.196237	30	36
39	803876	494	.196124	.18	.82	.804758	496	.195242	21	46
30	804865	493	.195135	.14	.86	.805751	495	.194243	30	40
40	805852	492	.194148	.10	.90	.806742	494	.193258	20	44
41	806837	491	.193163	.06	.94	.807731	493	.192269	30	38
42	807819	490	.192181	.02	.98	.808717	492	.191283	19	42
30	808799	488	.191191	.999098	-0.000902	.809701	491	.190293	30	36
42	809777	485	.190223	.94	.06	.810683	490	.189317	18	46
30	810753	487	.189247	.90	.10	.811663	489	.188337	30	40
43	811727	485	.188273	.86	.14	.812641	487	.187359	17	44
30	812698	484	.187302	.82	.18	.813616	486	.186384	30	38
44	813667	483	.186333	.78	.22	.814589	486	.185411	16	48
30	814634	482	.185366	.73	.27	.815561	484	.184439	30	42
45	815598	481	.184402	.69	.31	.816529	483	.183471	15	46
30	816561	480	.183439	.65	.35	.817496	482	.182504	30	40
46	817522	479	.182477	.61	.39	.818461	481	.181539	14	44
30	818480	478	.181520	.57	.43	.819423	480	.180577	30	38
47	819435	477	.180564	.52	.48	.820384	479	.179616	13	42
30	820390	476	.179610	.48	.52	.821342	478	.178658	30	36
48	821342	475	.178657	.44	.56	.822298	477	.177703	12	46
30	822293	474	.177707	.40	.60	.823253	476	.176747	30	40
49	823241	473	.176759	.36	.64	.824205	475	.175791	11	44
30	824186	472	.175814	.31	.69	.825155	474	.174835	30	38
50	825130	471	.174870	.27	.73	.826103	473	.173880	10	42
30	826072	470	.173924	.23	.77	.827049	471	.172925	30	36
51	827011	469	.172980	.19	.81	.827992	471	.172008	9	46
30	827949	467	.172035	.15	.85	.828934	470	.171096	30	40
52	828884	467	.171110	.10	.90	.829874	469	.170186	8	44
30	829812	466	.170182	.06	.94	.830812	468	.169288	30	38
53	830750	464	.169250	.02	.98	.831748	467	.168392	7	42
30	831674	464	.168321	.998907	-0.001093	.832682	466	.167511	30	36
54	832607	463	.167393	.93	.07	.833614	465	.166636	6	46
30	833532	462	.166468	.89	.11	.834543	464	.165767	30	40
55	834456	460	.165544	.85	.15	.835471	463	.164902	5	44
30	835377	460	.164623	.80	.20	.836407	462	.164043	30	38
56	836297	459	.163703	.76	.24	.837321	461	.163189	4	42
30	837215	457	.162785	.72	.28	.838243	460	.162341	30	36
57	838130	457	.161870	.67	.33	.839163	459	.161498	3	46
30	839044	456	.160955	.63	.37	.840081	458	.160659	30	40
58	839955	455	.160044	.59	.42	.841008	457	.159823	2	44
30	840866	454	.159134	.54	.46	.841932	456	.158991	30	38
59	841774	454	.158222	.50	.50	.842854	455	.158176	1	42
30	842680	453	.157309	.45	.55	.843775	454	.157365	30	36
60	843585	452	1.156415	9.998941	0.001059	8.444644	454	1.155356	0	44

TABLE II.—LOG. SINES, TANG'S, &c.

0° 80		1° — 15' 1° — 15' 1° — 15°										1710 1	
Hours.	Deg.	L. Sin.	Diff. for 15' or 1'	L. Cos.ec.	L. Cos.	Diff. for 15' or 1'	L. Sec.	L. Tang.	Diff. for 15' or 1'	L. Cot.	Deg.		
32	0	0.143555	224	0.856445	0.995733	4	0.004247	0.147702	229	0.85218	60		
	4	1	4453	224	5547	4	42.5	8718	228	1322	59		
	8	2	5340	223	4651	4	4283	9032	228	0366	58		
	12	3	6243	223	3757	4	4301	150544	227	049450	57		
	16	4	7136	222	2864	4	4318	1454	227	8546	56		
	20	5	8036	222	1974	4	4337	2363	226	7637	55		
	24	6	8915	222	1085	4	4354	3269	226	6731	54		
	28	7	9e02	221	0196	4	4372	4174	226	5826	53		
	32	8	15068	221	049314	4	4391	5077	226	4923	52		
	36	9	1569	220	8431	4	4409	5978	225	4022	51		
	40	10	2451	220	7549	5			225				
	44	11	3330	219	6670	4	4426	6877	224	3123	50		
	48	12	4206	219	5792	4	4445	7775	224	2225	49		
	52	13	5084	219	4916	4	4463	8671	223	1321	48		
	56	14	5957	218	4043	4	4481	9565	223	0435	47		
				218		5	4500	160457	222	839543	46		
33	0	15	6820	218	3171	5	4518	1347	222	8653	45		
	4	16	7700	217	2300	4	4536	2230	221	7764	44		
	8	17	8569	217	1431	4	4554	3123	221	6877	43		
	12	18	9435	216	0565	5	4573	4008	221	5992	42		
	16	19	16030	216	839899	4	4591	4892	220	5108	41		
	20	20	1164	215	8836	5	4610	5774	220	4226	40		
	24	21	2026	215	7974	4	4628	6654	219	3346	39		
	28	22	2885	214	7115	4	4647	7532	219	2468	38		
	32	23	3743	214	6257	5	4666	8409	219	1591	37		
	36	24	4600	213	5400	5	4684	9284	218	0716	36		
	40	25	5454	213	4546	5	4703	170157	218	899643	35		
	44	26	6307	213	3693	4	4722	1029	217	8071	34		
	48	27	7159	212	2841	5	4740	1899	217	8101	33		
	52	28	8006	212	1992	5	4757	2767	217	7273	32		
	56	29	8856	211	1144	5	4772	3634	216	6366	31		
	34	0	30	9702	211	0298	5	4797	4490	216	5501	30	
4		31	170546	211	899454	5	4811	5362	215	4638	29		
8		32	1389	210	8611	5	4825	6224	215	3776	28		
12		33	2230	210	7770	5	4845	7084	215	2916	27		
16		34	3070	209	6930	5	4873	7943	214	2057	26		
20		35	3908	209	6092	5	4892	8800	214	1200	25		
24		36	4744	208	5256	5	4911	9655	213	0345	24		
28		37	5578	208	4422	5	4930	180508	213	819492	23		
32		38	6411	208	3590	5	4949	1300	213	8440	22		
36		39	7243	207	2757	5	4968	2211	212	7789	21		
40		40	8072	207	1928	5	4987	3059	212	6941	20		
44		41	8900	207	1100	5	5007	3907	211	6093	19		
48		42	9727	206	0273	5	5026	4757	211	5247	18		
52		43	180551	206	819449	5	5046	5597	210	4403	17		
56		44	1374	205	862	5	5065	6430	210	3561	16		
35		0	45	2196	205	7804	5	5084	7280	210	2720	15	
	4	46	3016	204	6984	5	5104	8120	209	1880	14		
	8	47	3854	204	6166	5	5123	8957	209	1043	13		
	12	48	4681	204	5349	5	5143	9794	209	0206	12		
	16	49	5467	203	4533	5	5162	190029	208	803371	11		
	20	50	6290	203	3720	5	5182	1462	208	8538	10		
	24	51	7062	203	2908	5	5202	2274	207	7706	9		
	28	52	7903	202	2097	5	5221	3124	207	6876	8		
	32	53	8712	202	1288	5	5241	3953	207	6047	7		
	36	54	9519	201	0481	5	5261	4780	206	5220	6		
	40	55	190325	201	909675	5	5281	5606	206	4394	5		
	44	56	1130	201	8270	5	5300	6430	206	3570	4		
	48	57	1933	200	8067	5	5320	7253	205	2747	3		
	52	58	2734	200	7266	5	5340	8074	205	1926	2		
	56	59	3574	199	6466	5	5360	8894	204	1106	1		
	60	60	9194332	199	0805668	5	0.005320	0.197712		0.800284	0		
Hours.	Deg.	L. Cos.	Diff. for 15' or 1'	L. Sec.	L. Sin.	Diff. for 15' or 1'	L. Cos.ec.	L. Cot.	Diff. for 15' or 1'	L. Tang.	Deg.		

TABLE II.—LOG. SINES, TANG'S, &c.

90°		1° - 15' 1" - 15'				170°			11°		
deg.	L. Sin.	Diff. for 15' or 1"	L. Cosc.	L. Cos.	Diff. for 15' or 1"	L. Sec.	L. Tang.	Diff. for 15' or 1"	L. Cot.	Deg.	Hours.
0	9.194338		0.805868	9.994620		0.005320	9.193719		0.800288	80	23 0
1	5139	199	4871	4800	5	5400	900523	204	79471	59	3
2	5925	198	4075	4580	5	5420	1345	203	8555	58	5
3	6719	198	3291	4500	5	5440	2151	203	7641	57	5
4	7511	198	2489	4540	5	5460	2971	203	7022	56	4
5	8302	197	1698	4519	5	5481	3783	202	6217	55	4
6	9091	197	9099	4499	5	5501	4592	202	5408	54	3
7	9879	197	0121	4479	5	5521	5400	202	4600	53	3
8	900686	196	79334	4459	5	5541	6207	201	3793	52	2
9	1451	196	8549	4438	5	5562	7013	201	2367	51	2
10	2235	195	7765	4418	5	5582	7817	200	2183	50	1
11	3017	195	6983	4398	5	5602	8619	200	1321	49	1
12	3797	195	6203	4377	5	5623	9420	200	0540	48	1
13	4577	194	5423	4357	5	5643	91920	199	7976	47	1
14	5354	194	4646	4336	5	5664	1018	199	802	46	0
15	6131	194	3869	4316	5	5684	1815	199	7185	45	23 0
16	6906	193	3094	4295	5	5705	2611	198	6329	44	5
17	7679	193	2321	4274	5	5726	3405	198	5485	43	5
18	8452	192	1548	4254	5	5746	4198	198	4642	42	5
19	9222	192	0778	4233	5	5767	4989	197	3801	41	4
20	9991	192	0009	4212	5	5788	5779	197	2931	40	4
21	910760	191	789240	4192	5	5808	6568	197	2123	39	3
22	1528	191	8474	4170	5	5830	7356	196	1314	38	3
23	2302	191	7708	4150	5	5850	8142	196	0503	37	2
24	3055	191	6945	4129	5	5871	8926	196	1074	36	2
25	3818	190	6189	4108	5	5892	9710	195	0290	35	1
26	4579	190	5429	4087	5	5913	90492	195	77250	34	1
27	5338	190	4662	4066	5	5934	1879	195	8723	33	1
28	6097	189	3903	4045	5	5955	2652	194	7948	32	1
29	6854	189	3146	4024	5	5976	3430	194	7170	31	0
30	7609	189	2391	4003	5	5997	3606	194	6394	30	23 0
31	8364	188	1636	3982	5	6018	4329	193	5618	29	0
32	9118	188	0884	3960	5	6040	5156	193	4844	28	5
33	9868	188	0132	3939	5	6061	5929	193	4071	27	4
34	90618	187	77932	3918	5	6082	6700	193	3300	26	4
35	1367	187	8633	3896	5	6104	7471	192	2529	25	4
36	2115	186	7885	3875	5	6125	8240	192	1760	24	3
37	2861	186	7139	3854	5	6146	9007	192	0993	23	3
38	3606	186	6394	3832	5	6168	9774	191	0236	22	2
39	4350	185	5650	3811	5	6189	90539	191	79461	21	2
40	5092	185	4908	3789	5	6211	1303	190	8697	20	1
41	5833	185	4167	3768	5	6232	2065	190	7935	19	1
42	6572	185	3429	3746	5	6254	2826	190	7174	18	1
43	7311	184	2689	3725	5	6275	3586	190	6414	17	0
44	8048	184	1952	3703	5	6297	4345	189	5655	16	0
45	8784	184	1216	3681	5	6319	5103	189	4907	15	21 0
46	9519	183	0481	3660	5	6340	5859	189	4141	14	20
47	90252	183	769748	3638	5	6362	6614	188	3386	13	20
48	0984	182	9016	3616	5	6384	7368	189	2639	12	19
49	1714	182	8286	3594	5	6406	8120	188	1880	11	18
50	2444	182	7556	3572	5	6428	8872	187	1129	10	18
51	3172	182	6828	3550	5	6450	9623	187	0376	9	18
52	3900	181	6101	3528	5	6472	90371	187	79689	8	17
53	4625	181	5375	3506	6	6494	1119	186	8891	7	17
54	5349	181	4651	3484	5	6516	1865	186	8135	6	17
55	6072	180	3928	3462	5	6538	2610	186	7390	5	17
56	6794	180	3206	3440	5	6560	3354	186	6646	4	16
57	7515	180	2485	3418	5	6582	4077	185	5903	3	16
58	8235	179	1765	3396	5	6604	4839	185	5161	2	16
59	8953	179	1047	3374	6	6626	5579	185	4421	1	16
60	939670	179	0760330	9.993351		0.006649	9.946319		0.753681	0	20 0

TABLE II.—LOG. SINES, TANG'S, &c.

0° 100		1° - 15' 1° - 15' 1° - 15'								160° 11°		
Hours.	Deg.	L. Sin.	Diff. for 15' or 1'	L. Cossec.	L. Cos.	Diff. for 15' or 1'	L. Sec.	L. Tang.	Diff. for 15' or 1'	L. Cot.	Deg.	Hours.
40	0	0.239670	179	0.760330	0.993351	6	0.006649	0.246319	184	0.753681	60	19 00
	4	240386	179	753614	3323	6	6671	7057	184	2343	59	56
	8	1101	179	8839	3307	5	6693	7794	184	2206	58	52
	12	1814	178	8186	3284	6	6716	8530	183	1470	57	48
	16	2538	178	7474	3262	5	6738	9264	183	0736	56	44
	20	3238	177	6769	3240	6	6760	9998	183	0002	55	40
	24	3947	177	6053	3217	6	6783	10730	183	74270	54	36
	28	4656	177	5344	3195	5	6805	1461	182	8539	53	32
	32	5363	177	4637	3172	6	6828	2191	182	780	52	28
	36	6069	176	3931	3149	5	6851	2920	182	7080	51	24
	40	6775	176	3225	3127	6	6873	3648	181	6352	50	20
	44	7472	176	2523	3104	6	6896	4374	181	5626	49	16
	48	8161	175	1819	3081	6	6919	5100	181	4900	48	12
	52	8853	175	1117	3059	5	6941	5824	181	4176	47	8
	56	9533	175	0417	3036	6	6964	6547	180	3453	46	4
41	0	102092	174	749718	3013	6	6987	7269	180	2731	45	19 00
	4	0730	174	9020	2990	6	7010	7990	180	2010	44	56
	8	1677	174	8323	2967	6	7033	8710	180	1290	43	52
	12	2373	173	7627	2944	6	7056	9429	179	0571	42	48
	16	3067	173	6933	2921	6	7079	10146	179	73954	41	44
	20	3761	173	6239	2898	6	7102	10863	179	9137	40	40
	24	4453	173	5547	2875	6	7125	11578	178	8492	39	36
	28	5144	173	4856	2852	6	7148	12292	178	7708	38	32
	32	5834	173	4166	2829	6	7171	13005	178	6985	37	28
	36	6523	172	3477	2806	6	7194	13717	178	6283	36	24
	40	7211	172	2789	2783	6	7217	14428	177	5572	35	20
	44	7898	171	2102	2760	6	7240	15138	177	4862	34	16
	48	8583	171	1417	2736	6	7264	15847	177	4153	33	12
	52	9268	171	0732	2713	6	7287	16555	176	3445	32	8
	56	9951	170	0049	2690	6	7310	17261	176	2739	31	4
42	0	106033	170	739267	2666	6	7334	17967	176	2033	30	18 00
	4	1314	170	8686	2643	6	7357	18671	176	1329	29	56
	8	1994	170	8006	2619	6	7381	19375	175	0625	28	52
	12	2673	169	7327	2596	6	7404	20077	175	73923	27	48
	16	3351	169	6649	2572	6	7428	20779	175	9221	26	44
	20	4027	169	5973	2548	6	7452	21479	175	8521	25	40
	24	4703	168	5277	2525	6	7475	22178	174	7822	24	36
	28	5377	168	4573	2501	6	7499	22875	174	7124	23	32
	32	6051	168	3849	2477	6	7522	23573	174	6427	22	28
	36	6723	168	3277	2454	6	7546	24269	174	5731	21	24
	40	7394	168	2607	2430	6	7570	24964	174	5036	20	20
	44	8035	167	1935	2406	6	7594	25658	173	4341	19	16
	48	8731	167	1233	2382	6	7618	26352	173	3648	18	12
	52	9402	167	0538	2359	6	7641	27047	173	2957	17	8
	56	10069	166	73331	2335	6	7665	27741	172	2266	16	4
43	0	10735	166	9265	2311	6	7689	28434	172	1576	15	17 00
	4	1400	166	8600	2287	6	7713	29128	172	0887	14	56
	8	1964	165	7936	2263	6	7737	29821	172	0199	13	52
	12	2528	165	7274	2239	6	7762	30514	172	71919	12	48
	16	3092	165	6612	2214	6	7786	1174	171	6526	11	44
	20	3656	165	5951	2190	6	7810	1859	171	8141	10	40
	24	4220	165	5292	2166	6	7834	2542	171	7454	9	36
	28	4784	164	4633	2142	6	7858	3225	170	6775	8	32
	32	5348	164	3975	2118	6	7882	3907	170	6093	7	28
	36	5912	164	3319	2093	6	7907	4588	170	5412	6	24
	40	6476	163	2663	2069	6	7931	5268	170	4732	5	20
	44	7040	163	2007	2044	6	7956	5947	169	4053	4	16
	48	7604	163	1353	2020	6	7980	6625	169	3375	3	12
	52	8168	163	0703	1996	6	8004	7301	169	2699	2	8
	56	8732	163	0052	1971	6	8029	7977	169	2023	1	4
43	60	929599	163	0719401	0.991947	6	0.008053	0.288652	169	0.711346	0	16 00

0° 100°

1° - 4' 10° - 4'

170° 54

110°		1° — 15' 1" — 15' 1" — 15°					160°		11		
Hours.	Deg.	L. Sin.	Diff. for 15' or 1"	L. Cos.	L. Cot.	L. Sec.	L. Tang.	Diff. for 15' or 1"	L. Cot.	Deg.	Hours.
44	0	9.260399	162	0.719401	9.991947	0.008053	9.260699	166	0.711349	60	15
	1	1948	162	8732	1992	8	9386	166	0974	59	56
	2	1897	162	8163	1898	6	9192	166	0061	58	52
	3	2544	162	7456	1873	6	8127	166	09329	57	48
	4	3191	161	6889	1848	6	8151	166	1342	56	44
	5	3636	161	6164	1823	6	8177	167	2012	55	40
	6	4480	161	5360	1798	6	8202	167	2682	54	36
	7	5194	161	4678	1774	6	8226	167	3350	53	32
	8	5766	160	4234	1749	6	8251	167	4017	52	28
	9	6400	160	3582	1724	6	8276	167	4684	51	24
	10	7048	160	2982	1699	6	8301	166	5349	50	20
	11	7687	160	2313	1674	6	8326	166	6013	49	16
	12	8328	159	1674	1648	6	8351	166	6677	48	12
	13	8984	159	1086	1624	6	8376	165	7340	47	8
	14	9660	159	0480	1599	6	8401	165	8001	46	4
45	0	9.290236	159	.709784	1574	6	8426	165	8662	45	15
	1	0671	158	9129	1549	6	8451	165	9322	44	56
	2	1504	158	8496	1524	6	8476	164	9980	43	52
	3	2137	158	7863	1499	6	8501	164	10628	42	48
	4	2768	158	7232	1473	6	8527	164	11455	41	44
	5	3399	157	6601	1448	6	8552	164	12281	40	40
	6	4029	157	5971	1422	6	8578	163	13107	39	36
	7	4658	157	5342	1397	6	8603	163	13932	38	32
	8	5286	157	4714	1372	6	8629	163	14757	37	28
	9	5913	156	4087	1346	6	8654	163	15581	36	24
	10	6539	156	3461	1321	6	8679	163	16405	35	20
	11	7164	156	2836	1295	6	8705	162	17228	34	16
	12	7786	156	2212	1269	6	8731	162	18051	33	12
	13	8412	155	1588	1244	6	8756	162	18873	32	8
	14	9034	155	0966	1218	6	8782	162	19694	31	4
46	0	9655	155	0345	1193	6	8807	162	20516	30	15
	1	966276	155	99734	1167	6	8833	161	9199	29	56
	2	0885	155	9105	1141	6	8858	161	9846	28	52
	3	1514	155	8486	1115	6	8883	161	10499	27	48
	4	2133	154	7868	1090	6	8908	161	11148	26	44
	5	2749	154	7251	1064	6	8936	160	11792	25	40
	6	3365	153	6635	1038	6	8962	160	12431	24	36
	7	3979	153	6021	1012	6	8989	160	13065	23	32
	8	4593	153	5407	986	6	9014	160	13694	22	28
	9	5207	153	4792	960	6	9040	159	14318	21	24
	10	5819	153	4171	934	6	9066	159	14936	20	20
	11	6430	153	3576	908	6	9092	159	15549	19	16
	12	7041	152	2959	882	6	9118	159	16157	18	12
	13	7650	152	2359	856	6	9145	159	16760	17	8
	14	8259	152	1741	829	6	9171	158	17358	16	4
47	0	8867	152	1133	803	6	9197	158	17952	15	15
	1	9474	151	0536	777	6	9223	158	18541	14	56
	2	10090	151	0939	750	6	9250	158	19125	13	52
	3	0685	151	0315	724	6	9276	158	19704	12	48
	4	1309	151	0711	697	6	9303	157	20278	11	44
	5	1893	150	0107	671	6	9330	157	20847	10	40
	6	2495	150	7305	644	6	9356	157	21411	9	36
	7	3097	150	6503	618	6	9383	157	21970	8	32
	8	3698	150	5702	592	6	9408	157	22524	7	28
	9	4298	150	4903	565	6	9433	156	23073	6	24
	10	4896	149	4104	538	6	9458	156	23617	5	20
	11	5495	149	3305	512	6	9483	156	24156	4	16
	12	6092	149	2506	485	6	9515	156	24690	3	12
	13	6689	149	1707	458	6	9542	155	25219	2	8
	14	7284	149	0908	431	6	9569	155	25743	1	4
47	0	9.217879	149	0.682121	9.990404	0.009596	9.227475	155	0.672585	0	15
Hours.	Deg.	L. Cos.	Diff. for 15' or 1"	L. Sec.	L. Sin.	Diff. for 15' or 1"	L. Cot.	Diff. for 15' or 1"	L. Tang.	Deg.	Hours.

0°		10°	1° = 15' 1" = 15' 2" = 15"				167°				11°	
Hours.	Deg.	L. Sin.	Diff. for 15' or 1"	L. Cossec.	L. Cot.	Diff. for 15' or 1"	L. Sec.	L. Tang.	Diff. for 15' or 1"	L. Cot.	Deg.	Hours.
18	0	9.217579	148	0.622121	9.999994	6	0.009996	9.237475	153	0.672525	60	11 00
	4	8473	148	1527	0378	7	0022	8093	153	1905	58	
	8	9066	148	0934	0351	7	9049	8715	153	2285	59	
	12	9658	148	0348	0324	7	0076	9334	153	0666	57	48
	16	990250	147	97750	0297	7	9703	9933	154	0047	56	44
	20	0940	147	9160	0270	7	9730	99570	154	669430	55	40
	24	1430	147	8570	0243	7	9757	1187	154	8813	54	36
	28	9019	147	7961	0216	7	9784	1603	154	8197	53	32
	32	9806	147	7304	0188	7	9812	9418	154	7589	52	28
	36	3194	146	6608	0161	7	9839	3033	153	6967	51	24
	40	3780	146	6930	0136	7	9866	2646	153	6354	50	20
	44	4366	146	5634	0107	7	9893	4259	153	5741	49	16
	48	4950	146	5050	0079	7	9921	4871	153	5139	48	12
	52	5534	146	4466	0052	7	9948	5489	153	4518	47	8
	56	6118	145	3882	0025	7	9975	6093	153	3907	46	4
19	0	6700	145	3300	999997	7	.010003	6703	153	2297	45	11 00
	4	7281	145	2719	9970	7	0020	7311	152	9699	44	56
	8	7863	145	2138	9942	7	0058	7920	152	9098	43	52
	12	8443	144	1558	9915	7	0083	8527	152	1473	42	48
	16	9020	144	9930	9887	7	0113	9133	151	8367	41	44
	20	9599	144	0401	9860	7	0140	9739	151	0261	40	40
	24	230176	144	66984	9833	7	0168	93044	151	659656	39	36
	28	0753	144	9947	9806	7	0196	0949	151	9051	38	32
	32	1339	144	8671	9777	7	0223	1552	151	8448	37	28
	36	1904	143	8006	9749	7	0251	2155	151	7845	36	24
	40	2478	143	7322	9721	7	0279	2757	150	7243	35	20
	44	3051	143	6649	9693	7	0307	3358	150	6649	34	16
	48	3624	143	6376	9666	7	0334	3958	150	6043	33	12
	52	4195	143	5705	9637	7	0363	4558	150	5449	32	8
	56	4767	142	5233	9610	7	0390	5157	150	4843	31	4
50	0	5337	142	4643	9582	7	0418	5755	149	4245	30	10 00
	4	5906	142	4004	9553	7	0447	6353	149	3647	29	56
	8	6475	142	3525	9525	7	0475	6950	149	3050	28	52
	12	7043	142	2957	9497	7	0503	7546	149	2454	27	48
	16	7610	141	2390	9469	7	0531	8141	148	1859	26	44
	20	8178	141	1824	9441	7	0559	8735	148	1265	25	40
	24	8742	141	1258	9413	7	0587	9329	148	6671	24	36
	28	9307	141	693	9385	7	0615	9922	148	0078	23	32
	32	9870	141	0130	9356	7	0644	9514	148	648430	22	28
	36	9434	140	65956	9328	7	0672	1108	148	8894	21	24
	40	0996	140	9004	9299	7	0701	1697	147	8303	20	20
	44	1558	140	8443	9271	7	0729	2287	147	7713	19	16
	48	2119	140	7881	9243	7	0757	2876	147	7124	18	12
	52	2679	140	7321	9214	7	0786	3465	147	6535	17	8
	56	3239	139	6761	9186	7	0814	4053	147	5947	16	4
51	0	3797	139	6203	9157	7	0843	4640	147	5360	15	9 00
	4	4355	139	5645	9128	7	0872	5227	146	4773	14	56
	8	4913	139	5087	9100	7	0900	5813	146	4187	13	52
	12	5469	139	4531	9071	7	0929	6398	146	3602	12	48
	16	6024	139	3976	9042	7	0958	6982	146	3018	11	44
	20	6580	138	3420	9014	7	0986	7566	146	2434	10	40
	24	7134	138	2866	8985	7	1015	8149	145	1851	9	36
	28	7687	138	2313	8956	7	1044	8731	145	1269	8	32
	32	8240	138	1760	8927	7	1073	9313	145	6877	7	28
	36	8792	138	1208	8898	7	1102	9894	145	0106	6	24
	40	9343	137	657	8869	7	1131	9474	145	639230	5	20
	44	9893	137	0107	8840	7	1160	1053	145	8947	4	16
	48	9443	137	448557	8811	7	1189	1632	144	8368	3	12
	52	0992	137	9006	8782	7	1218	2210	144	7790	2	8
	56	1540	137	8490	8753	7	1247	2787	144	7213	1	4
	50	035008	137	0547912	998794	7	0.011276	9.363364	144	0.638636	0	0 00

60 1670

1' = 4' 10" = 4"

770 50

TABLE II.—LOG. SINES, TANG'S, &C.

0° 13°			1° = 15' 1" = 15' 1" = 15°						166° 11°			
Hours.	Deg.	L. Sin.	Diff. for 15' or 1"	L. Cos. c.	L. Cos.	Diff. for 15' or 1"	L. Sec.	L. Tang.	Diff. for 15' or 1"	L. Cot.	Deg.	Hours.
52	0	9.352068	137	0.647912	9.988794	7	0.011976	9.363364	144	0.636636	66	7
	4	9635	136	7365	8695	7	1305	3040	144	6000	39	56
	8	3181	136	6819	8665	7	1335	4516	143	5484	58	52
	12	3726	136	6274	8636	7	1364	5000	143	4910	57	47
	16	4271	136	5739	8607	7	1393	5664	143	4336	56	44
	20	4815	136	5185	8578	7	1422	6237	143	3763	55	40
	24	5358	136	4642	8548	7	1452	6810	143	3190	54	36
	28	5901	136	4099	8519	7	1481	7382	143	2618	53	32
	32	6442	135	3558	8489	7	1511	7953	143	2047	52	27
	36	6984	135	3016	8460	7	1540	8524	142	1476	51	24
	40	7524	135	2476	8430	7	1570	9094	142	906	50	20
	44	8064	135	1936	8401	7	1599	9663	142	327	49	16
	48	8603	135	1397	8371	7	1629	370233	142	620769	48	12
	52	9141	134	859	8342	7	1658	9799	142	991	47	8
	56	9679	134	321	8312	7	1688	1367	141	8833	46	4
53	0	360215	134	639785	8282	7	1718	1933	141	8067	45	7
	4	0751	134	9249	8252	7	1748	2499	141	7501	44	3
	8	1387	134	8713	8222	7	1778	3065	141	6935	43	52
	12	1822	133	8178	8193	7	1807	3629	141	6371	42	48
	16	2250	133	7644	8163	7	1837	4193	141	5807	41	44
	20	2689	133	7111	8133	7	1867	4756	141	5244	40	40
	24	3122	133	6578	8103	7	1897	5319	140	4681	39	36
	28	3554	133	6046	8073	7	1927	5881	140	4119	38	32
	32	3983	133	5515	8043	7	1957	6442	140	3558	37	28
	36	4416	132	4984	8013	7	1987	7003	140	2997	36	24
	40	4846	132	4454	7983	7	2017	7563	140	2437	35	20
	44	5275	132	3925	7953	8	2047	8123	140	1878	34	16
	48	5703	132	3397	7922	8	2078	8681	139	1319	33	12
	52	6131	132	2869	7892	7	2108	9239	139	761	32	8
	56	6559	131	2341	7862	7	2138	9797	139	2003	31	4
54	0	8185	131	1815	7832	8	2168	380254	139	619646	30	0
	4	8711	131	1289	7801	7	2199	0010	139	9090	29	56
	8	9236	131	0764	7771	7	2229	1465	139	8535	28	52
	12	9761	131	0239	7740	7	2260	2921	138	7979	27	48
	16	370885	131	639715	7710	7	2290	2575	138	7425	26	44
	20	0608	130	9192	7679	7	2321	3129	138	6871	25	40
	24	1330	130	8670	7649	8	2351	3681	138	6319	24	36
	28	1852	130	8148	7618	8	2382	4234	138	5766	23	32
	32	2374	130	7626	7588	8	2412	4786	138	5214	22	28
	36	2894	130	7106	7557	8	2443	5337	138	4663	21	24
	40	3414	130	6586	7526	8	2474	5888	137	4112	20	20
	44	3933	130	6067	7495	8	2505	6438	137	3562	19	16
	48	4452	130	5548	7465	8	2535	6987	137	3013	18	12
	52	4970	129	5030	7434	8	2566	7536	137	2464	17	8
	56	5487	129	4513	7403	8	2597	8084	137	1916	16	4
55	0	6003	129	3997	7372	8	2628	8631	137	1369	15	0
	4	6519	129	3481	7341	8	2659	9178	137	822	14	56
	8	7035	129	2965	7310	8	2690	9725	136	275	13	52
	12	7549	128	2451	7279	8	2721	30270	136	609730	12	48
	16	8063	128	1937	7248	8	2752	6815	136	9185	11	44
	20	8577	128	1423	7217	8	2783	1260	136	8240	10	40
	24	9091	128	9011	7186	8	2814	1803	136	7297	9	36
	28	9605	128	3896	7155	8	2845	2347	135	6353	8	32
	32	53	128	619887	7124	8	2876	2890	135	5409	7	28
	36	0684	127	9376	7093	8	2907	3531	135	4466	6	24
	40	1134	127	8866	7061	8	2938	4073	135	3522	5	20
	44	1644	127	8356	7030	8	2970	4614	135	2578	4	16
	48	2152	127	7846	6998	8	3002	5154	135	1634	3	12
	52	2661	127	7336	6967	8	3033	5694	135	789	2	8
	56	3168	127	6826	6935	8	3065	6233	134	3767	1	4
55	60	3675	127	6316325	9.986904	8	0.013086	9.396771	134	0.603889	0	0

TABLE II.—LOG. SINES, TANG'S, &c.

140		1° - 15' 1° - 15' 1° - 15'							165° - 11°			
Hours.	Deg.	L. Sin.	Diff. for 15' or 1"	L. Cossec.	L. Cos.	Diff. for 15' or 1"	L. Sec.	L. Tang.	Diff. for 15' or 1"	L. Cot.	Deg.	Hours.
56	0	9.383675	127	0.616325	9.986004	8	0.013096	9.396771	134	0.603229	60	3
	1	418.2	126	5818	6873	8	3127	7309	134	2939	59	56
	2	464.7	126	5313	6841	8	3159	7846	134	2154	58	52
	3	5192	126	4808	6809	8	3191	8383	134	1617	57	48
	4	5697	126	4303	6778	8	3223	8919	134	1081	56	44
	5	6201	126	3799	6746	8	3254	9453	134	545	55	40
	6	6704	126	3296	6714	8	3286	9990	133	6010	54	36
	7	7207	125	2793	6683	8	3317	10524	133	59476	53	32
	8	7709	125	2291	6651	8	3349	1058	133	8942	52	28
	9	8210	125	1790	6619	8	3381	1501	133	8409	51	24
	10	8711	125	1289	6587	8	3413	2124	133	7876	50	20
	11	9211	125	789	6555	8	3445	2656	133	7344	49	16
	12	9710	125	290	6523	8	3477	3187	133	6813	48	12
	13	390309	125	60791	6491	8	3509	3718	133	6282	47	8
	14	0708	124	9292	6459	8	3541	4249	132	5751	46	4
57	0	15	1203	124	8794	8	3573	4779	132	5221	45	3
	16	1703	124	8297	8305	8	3605	5308	132	4692	44	56
	17	9199	124	7801	7813	8	3637	5836	132	4164	43	52
	18	8685	124	7305	7317	8	3669	6364	132	3636	42	48
	19	8191	123	6809	6821	8	3701	6892	132	3108	41	44
	20	7675	123	6315	6286	8	3734	7419	131	2581	40	40
	21	7151	123	5821	6254	8	3766	7945	131	2055	39	36
	22	6623	123	5327	6222	8	3798	8471	131	1529	38	32
	23	6098	123	4833	6190	8	3831	8997	131	1003	37	28
	24	5568	123	4339	6157	8	3863	9521	131	479	36	24
	25	6150	123	3850	6125	8	3895	10045	131	58955	35	20
	26	6641	123	3359	6093	8	3928	10569	131	8431	34	16
	27	7131	123	2869	6061	8	3961	1092	131	8008	33	12
	28	7622	122	2378	6029	8	3993	1615	130	5385	32	8
	29	8111	122	1889	5997	8	4026	2137	130	7863	31	4
58	0	30	8606	122	1400	8	4058	2658	130	7342	30	0
	1	31	9088	122	8912	8	4091	3179	130	6821	29	56
	2	9575	122	4825	5876	8	4124	3699	130	6301	28	52
	3	400692	122	599936	5843	8	4157	4219	130	5781	27	48
	4	0549	121	9451	5811	8	4189	4738	130	5262	26	44
	5	1035	121	8965	5778	8	4222	5257	129	4743	25	40
	6	1520	121	8480	5745	8	4255	5775	129	4225	24	36
	7	2005	121	7995	5712	8	4288	6293	129	3707	23	32
	8	2489	121	7511	5679	8	4321	6810	129	3190	22	28
	9	2972	121	7028	5646	8	4354	7329	129	2674	21	24
	10	3455	121	6545	5613	8	4387	7842	129	2158	20	20
	11	3938	120	6062	5580	8	4420	8358	129	1642	19	16
	12	4420	120	5580	5547	8	4453	8873	128	1127	18	12
	13	4901	120	5099	5514	8	4486	9387	128	613	17	8
	14	5381	120	4619	5480	8	4520	9901	128	1009	16	4
59	0	45	5882	120	4138	8	4553	10415	128	579585	15	1
	1	6341	120	3659	5414	8	4586	0927	128	9073	14	56
	2	6820	120	3180	5380	8	4620	1440	128	8560	13	52
	3	7299	119	2701	5347	8	4653	1952	128	8048	12	48
	4	7777	119	2223	5314	8	4686	2463	128	7537	11	44
	5	8254	119	1746	5280	8	4720	2974	127	7026	10	40
	6	8731	119	1269	5247	8	4753	3484	127	6516	9	36
	7	9207	119	793	5213	8	4787	3994	127	6006	8	32
	8	9682	119	318	5180	8	4820	4503	127	5496	7	28
	9	10157	119	589843	5146	8	4854	5011	127	4989	6	24
	10	0632	118	9368	5113	8	4887	5519	127	4481	5	20
	11	1106	118	8894	5079	8	4921	6027	127	3973	4	16
	12	1579	118	8421	5045	8	4955	6534	127	3466	3	12
	13	2052	118	7948	5011	8	4989	7041	126	2959	2	8
	14	2525	118	7475	4978	8	5022	7547	126	2453	1	4
59	0	30	9.412996	0.587004	9.984944	8	0.015006	9.428652	126	0.571948	0	0

140

1° - 15' 1° - 15'

165° - 11°

TABLE II.—LOG. SINES, TANG'S, &c.

1° - 15'			1° - 15'			1° - 15'			164°			10°	
Hours.	Deg.	L. Sin.	Diff. for 15" or 1'	L. Cos. c.	L. Cos.	Diff. for 15" or 1'	L. Sec.	L. Tang.	Diff. for 15" or 1'	L. Cot.	Deg.	Hours.	
0	0	9.412396	118	0.587004	0.984944	8	0.015056	9.428053	196	0.571948	60	59	
	4	3467	118	6533	4910	8	5030	9032	196	8557	59	60	
	8	3938	117	6062	4876	8	5124	9032	196	0938	58	52	
	12	4403	117	5592	4842	8	5158	9586	196	0434	57	46	
	16	4878	117	5122	4808	8	5192	4.30070	196	5.80030	56	44	
	20	5347	117	4653	4774	8	5226	6573	195	9427	55	40	
	24	5815	117	4185	4740	8	5260	1075	195	8925	54	36	
	28	6283	117	3717	4706	8	5294	1577	195	8423	53	32	
	32	6751	117	3249	4672	8	5328	2079	195	7921	52	28	
	36	7218	116	2782	4638	8	5362	2580	195	7420	51	24	
	40	7684	116	2316	4601	8	5397	3081	195	6919	50	20	
	44	8150	116	1850	4569	8	5431	3581	195	6419	49	16	
	48	8615	116	1385	4535	8	5465	4080	195	5920	48	12	
	52	9079	116	921	4500	8	5500	4579	195	5421	47	8	
	56	9544	116	456	4466	8	5534	5078	194	4922	46	4	
1	0	4.20007	116	5.79993	4431	8	5569	5576	194	4424	45	59	
	4	0470	116	8530	4397	8	5603	6073	194	3927	44	56	
	8	0933	115	8067	4363	9	5637	6570	194	3430	43	52	
	12	1395	115	8103	4328	8	5672	7067	194	2933	42	48	
	16	1857	115	8143	4294	9	5706	7563	194	2437	41	44	
	20	2319	115	7682	4259	9	5741	8059	194	1941	40	40	
	24	2779	115	7222	4224	9	5776	8554	194	1445	39	36	
	28	3238	115	6762	4189	8	5811	9049	193	0951	38	32	
	32	3697	115	6303	4153	9	5845	9545	193	0458	37	28	
	36	4156	115	5844	4120	9	5880	4.46036	193	5.59964	36	24	
	40	4615	114	5385	4085	9	5915	6530	193	9470	35	20	
	44	5072	114	4928	4050	9	5950	1092	193	8977	34	16	
	48	5530	114	4470	4015	8	5985	1515	193	8483	33	12	
	52	5987	114	4013	3981	9	6019	2006	193	7994	32	8	
	56	6443	114	3557	3946	9	6054	2497	193	7503	31	4	
2	0	6899	114	3101	3911	9	6089	2988	193	7012	30	58	
	4	7354	114	2646	3875	9	6125	3479	193	6521	29	56	
	8	7807	114	2191	3840	9	6160	3969	193	6031	28	52	
	12	8259	113	1737	3805	9	6195	4458	193	5542	27	48	
	16	8717	113	1283	3770	9	6230	4947	193	5053	26	44	
	20	9170	113	830	3735	9	6265	5435	193	4565	25	40	
	24	9623	113	377	3700	9	6300	5923	193	4077	24	36	
	28	4.30975	113	5.69925	3664	9	6336	6411	193	3589	23	32	
	32	0527	113	0473	3629	9	6371	6899	191	3102	22	28	
	36	0072	113	9023	3594	9	6406	7384	191	2616	21	24	
	40	1492	113	8572	3559	9	6442	7870	191	2130	20	20	
	44	1979	113	8121	3523	9	6477	8356	191	1644	19	16	
	48	2472	113	7672	3487	9	6513	8841	191	1159	18	12	
	52	2972	113	7223	3451	9	6548	9326	191	674	17	8	
	56	3473	113	6774	3416	9	6584	9810	191	0190	16	4	
3	0	3875	113	6325	3381	9	6619	4.50294	191	5.49706	15	57	
	4	4322	113	5878	3345	9	6655	0.777	191	9223	14	56	
	8	4779	113	5431	3309	9	6691	1980	191	8740	13	52	
	12	5015	113	4984	3273	9	6727	1743	190	8257	12	48	
	16	5462	111	4538	3238	9	6762	2294	190	7776	11	44	
	20	5904	111	4092	3202	9	6798	2706	190	7294	10	40	
	24	6353	111	3647	3166	9	6834	3187	190	6813	9	36	
	28	6799	111	3202	3130	9	6870	3668	190	6332	8	32	
	32	7248	111	2758	3094	9	6906	4148	190	5852	7	28	
	36	7688	111	2314	3058	9	6942	4628	190	5372	6	24	
	40	8127	111	1871	3022	9	6978	5107	190	4893	5	20	
	44	8572	110	1429	2986	9	7014	5586	119	4414	4	16	
	48	9014	110	0986	2950	9	7050	6064	119	3936	3	12	
	52	9458	110	0544	2914	9	7086	6542	119	3458	2	8	
	56	9877	110	0103	2878	9	7122	7019	119	2981	1	4	
3	00	9.440336	110	0.559662	0.989849	9	0.017158	9.457498	190	0.522304	6	56	

1° - 105°

1° - 10° - 4°

74° - 4°

1° 10°

1° — 15' 1° — 15'

163° 10°

Hours.	Deg.	L. Sin.	Diff. for 15'' or 1'	L. Cos. ec.	L. Cos.	Diff. for 15'' or 1'	L. Sec.	L. Tang.	Diff. for 15'' or 1'	L. Cot.	Deg.	Hours.
4	0	9.440338	110	0.559662	9.982842	0	0.017158	9.457496	110	0.542504	60	53
	4	0778	110	09222	2805	0	7145	7973	110	2017	59	56
	8	1218	110	8782	2739	0	7231	8449	110	1551	58	52
	12	1654	110	8342	2733	0	7207	8925	110	1075	57	48
	16	2090	109	7904	2696	0	7304	9400	110	0900	56	44
	20	2535	109	7465	2600	0	7399	9875	112	0125	55	40
	24	2973	109	7027	2624	0	7370	103049	112	53051	54	36
	28	3410	109	6590	2587	0	7413	0622	112	0177	53	32
	32	3847	109	6153	2550	0	7450	1247	112	3702	52	28
	36	4284	109	5716	2514	0	7488	1770	112	8230	51	24
	40	4720	109	5280	2477	0	7523	2243	112	7757	50	20
	44	5155	109	4845	2441	0	7557	2714	112	7290	49	16
	48	55 40	109	4419	2404	0	7596	3180	112	6814	48	12
	52	5925	108	3975	2367	0	76 33	3655	112	6342	47	8
	56	6459	108	3541	2331	0	7669	4128	112	5872	46	4
	0	6893	108	3107	2294	0	7706	4599	117	5401	45	53
	4	7326	108	2674	2257	0	7743	5040	117	4931	44	50
	8	7759	108	2241	2220	0	7780	553	117	4461	43	52
	12	8191	108	1809	2183	0	7817	6008	117	3992	42	48
	16	8623	108	1377	2146	0	7854	6477	117	3523	41	44
	20	9054	108	0946	2109	0	7891	6945	117	3053	40	40
	24	9485	107	0515	2072	0	7928	7413	117	2587	39	36
	28	9915	107	0085	2035	0	7965	7880	117	2120	38	32
	32	10345	107	549555	1998	0	8002	8347	117	1653	37	28
	36	0775	107	9225	1961	0	8039	8814	116	1186	36	24
	40	1204	107	8796	1924	0	8076	9280	116	0720	35	20
	44	1632	107	8368	1889	0	8114	9746	116	0254	34	16
	48	2060	107	7940	1849	0	8151	10201	116	529790	33	12
	52	2488	107	7512	1812	0	8188	0670	116	0324	32	8
	56	2915	107	7085	1774	0	8226	1141	116	8839	31	4
	0	3342	106	6658	1737	0	8263	1605	116	8395	30	52
	4	3768	106	6232	1699	0	8301	2069	116	7931	29	56
	8	4194	106	5806	1662	0	8338	2532	116	7468	28	52
	12	4619	106	5381	1624	0	8376	2995	116	7005	27	48
	16	5044	106	4956	1587	0	8413	3457	115	6543	26	44
	20	5469	106	4531	1550	0	8450	3919	115	6081	25	40
	24	5893	106	4107	1512	0	8488	4374	115	5619	24	36
	28	6316	106	3684	1474	0	8526	4842	115	5158	23	32
	32	6739	106	3261	1436	0	8564	5302	115	4697	22	28
	36	7162	105	2838	1399	0	8601	5763	115	4237	21	24
	40	7584	105	2416	1361	0	8639	6223	115	3777	20	20
	44	8006	105	1994	1323	0	8677	6683	115	3317	19	16
	48	8427	105	1573	1285	0	8715	7142	115	2858	18	12
	52	8848	105	1152	1247	0	8753	7601	115	2399	17	8
	56	9268	105	0732	1209	0	8791	8059	114	1941	16	4
	0	9688	105	0312	1171	0	8829	8517	114	1483	15	53
	4	10108	105	53862	1133	0	8867	8975	114	1025	14	56
	8	0327	105	9473	1095	0	8905	9432	114	0568	13	52
	12	0746	104	9054	1057	0	8943	9889	114	0111	12	48
	16	1164	104	8636	1019	0	8981	10345	114	519055	11	44
	20	1782	104	8218	0981	0	9019	0801	114	0109	10	40
	24	2199	104	7801	0942	0	9058	1257	114	8743	9	36
	28	2616	104	7384	0904	0	9096	1712	114	8288	8	32
	32	3032	104	6968	0866	0	9134	2166	114	7834	7	28
	36	3448	104	6552	0827	0	9173	2621	114	7379	6	24
	40	3864	104	6136	0789	0	9211	3075	113	6925	5	20
	44	4279	104	5721	0751	0	9249	3529	113	6471	4	16
	48	4694	103	5306	0713	0	9288	3982	113	6018	3	12
	52	5108	103	4892	0674	0	9326	4434	113	5566	2	8
	56	5522	103	4478	0635	0	9365	4887	113	5113	1	4
	0	9.165935	103	0.534065	9.980596	0	0.019404	9.485339	113	0.514661	0	52

7° 100°

1° — 4' 10° — 4'

73° 4'

1°		17°		1' - 15' 1" - 15' 1" - 15°				16°		18°		
Hours.	Deg.	L. Sin.	Diff. for 15' or 1"	L. Cosec.	L. Cos.	Diff. for 15' or 1"	L. Sec.	L. Tang.	Diff. for 15' or 1"	L. Cot.	Deg.	Hours.
8	0	9.465935	103	0.534065	9.980530	9	0.019404	9.465935	113	0.514061	00	51 00
	1	6348	103	3652	0558	10	0442	5701	113	4920	00	51 01
	2	6761	103	3529	0519	10	0481	6948	113	3750	00	51 02
	3	7173	103	3387	0480	9	0520	8693	112	2577	00	51 03
	4	7585	103	3219	0442	10	0558	7143	112	1407	00	51 04
	5	7996	103	3094	0403	10	0597	7593	112	2497	00	51 05
	6	8407	102	2993	0364	10	0636	8043	112	1857	00	51 06
	7	8817	102	2913	0325	10	0675	8492	112	1308	00	51 07
	8	9227	102	2853	0286	10	0714	8941	112	859	00	51 08
	9	9637	102	2803	0247	10	0753	9390	112	600	00	51 09
	10	470046	102	520054	0208	10	0792	9838	112	0102	00	51 10
	11	0455	109	9545	0169	10	0831	4902914	112	509714	00	51 11
	12	0863	102	9137	0130	10	0870	0733	112	9967	00	51 12
	13	1271	102	8729	0091	10	0909	1180	112	9280	00	51 13
	14	1679	102	8321	0052	10	0948	1627	111	8373	00	51 14
9	0	9065	102	7915	0013	10	0988	9073	111	7927	00	51 15
	1	9492	102	7508	979973	10	020027	9519	111	7481	00	51 16
	2	9899	101	7101	9634	10	0066	9965	111	7035	00	51 17
	3	3304	101	6694	9694	10	0108	3410	111	6590	00	51 18
	4	3719	101	6287	9655	10	0145	2855	111	6145	00	51 19
	5											
	6	4115	101	5885	9616	10	0184	4900	111	5701	00	51 20
	7	4519	101	5481	9778	10	0221	4745	111	5257	00	51 21
	8	4923	101	5077	9737	10	0263	5186	111	4814	00	51 22
	9	5327	101	4673	9697	10	0303	5630	111	4370	00	51 23
	10	5731	101	4269	9658	10	0342	6072	110	3927	00	51 24
	11											
	12	6133	101	3867	9618	10	0382	6515	110	3485	00	51 25
	13	6538	100	3464	9579	10	0421	6957	110	3043	00	51 26
	14	6939	100	3062	9539	10	0461	7399	110	2601	00	51 27
	15	7340	100	2660	9499	10	0501	7841	110	2159	00	51 28
	16	7741	100	2258	9459	10	0541	8282	110	1718	00	51 29
	17											
10	0	8142	100	1858	9420	10	0580	8722	110	1276	00	51 30
	1	8542	100	1458	9380	10	0620	9168	110	832	00	51 31
	2	8943	100	1059	9340	10	0660	9615	110	626	00	51 32
	3	9343	100	658	9300	10	0700	500042	110	49053	27	51 33
	4	9741	100	259	9260	10	0740	0481	110	9519	26	51 34
	5											
	6	490140	100	519000	9220	10	0780	0920	110	9090	25	51 35
	7	0639	99	9461	9180	10	0820	1354	109	8641	24	51 36
	8	0937	99	9063	9140	10	0860	1797	109	8203	23	51 37
	9	1334	99	8666	9099	10	0901	2235	109	7765	22	51 38
	10	1731	99	8269	9059	10	0941	2672	109	7328	21	51 39
	11											
	12	2129	99	7872	9019	10	0981	3109	109	6891	20	51 40
	13	2525	99	7475	8979	10	1021	3546	109	6454	19	51 41
	14	2921	99	7079	8939	10	1061	3982	109	6017	18	51 42
	15	3316	99	6684	8898	10	1102	4418	109	5582	17	51 43
	16	3712	99	6288	8858	10	1142	4854	109	5146	16	51 44
	17											
11	0	4107	99	5893	8818	10	1182	5290	109	4711	15	51 45
	1	4501	99	5499	8777	10	1223	5724	107	4276	14	51 46
	2	4895	98	5105	8736	10	1264	6158	108	3841	13	51 47
	3	5289	98	4711	8694	10	1304	6592	108	3407	12	51 48
	4	5682	98	4318	8653	10	1345	7027	108	2973	11	51 49
	5											
	6	6075	98	3925	8613	10	1385	7460	108	2540	10	51 50
	7	6467	98	3533	8574	10	1426	7893	108	2107	9	51 51
	8	6859	98	3141	8533	10	1467	8326	108	1674	8	51 52
	9	7251	98	2749	8493	10	1507	8758	108	1242	7	51 53
	10	7643	98	2357	8452	10	1548	9191	108	809	6	51 54
	11											
	12	8033	98	1967	8411	10	1589	9622	108	376	5	51 55
	13	8424	97	1578	8370	10	1630	510054	108	489046	4	51 56
	14	8814	97	1189	8329	10	1671	0485	108	9515	3	51 57
	15	9204	97	7796	8288	10	1712	0816	108	9094	2	51 58
	16	9593	97	0407	8247	10	1753	1346	107	8534	1	51 59
	17	9982	97	0018	8206	10	0201794	9.511776	107	0.663294	0	51 60
	18											
Hours.	Deg.	L. Cos.	Diff. for 15' or 1"	L. Sec.	L. Sin.	Diff. for 15' or 1"	L. Cosec.	L. Tang.	Diff. for 15' or 1"	L. Cot.	Deg.	Hours.

1° 10'

1' - 15' 1" - 15°

18°

TABLE II.—LOC. SINES, TANG'S, &c.

180		1' - 15' 1" - 15' 1" - 150				160		10			
Deg.	L. Sin.	Diff. for 15' or 1"	L. Cosec.	L. Cos.	Diff. for 15' or 1"	L. Sec.	L. Tang.	Diff. for 15' or 1"	L. Cot.	Deg.	Hours.
0	0.513648	99	0.487358	0.973670	11	0.024330	0.536978	102	0.463088	60	43 00
1	3009	91	6991	5627	11	4373	7322	102	2618	59	58
2	3375	91	6625	5523	11	4417	7792	102	2808	58	52
3	3741	91	6259	5539	11	4461	8262	102	2798	57	46
4	4107	91	5893	5496	11	4504	8611	102	2789	56	41
5	4473	91	5528	5452	11	4548	9090	102	2780	55	36
6	4837	91	5163	5408	11	4592	9429	102	2771	54	30
7	5203	91	4798	5365	11	4635	9637	102	2762	53	25
8	5568	91	4434	5321	11	4679	540945	102	452755	52	20
9	5930	91	4070	5277	11	4723	0653	102	9347	51	14
10	6294	91	3706	5233	11	4767	1061	102	8939	50	10
11	6657	91	3343	5189	11	4811	1468	102	8532	49	06
12	7020	91	2980	5145	11	4855	1875	102	8125	48	02
13	7382	91	2618	5101	11	4899	2281	102	7719	47	58
14	7745	90	2255	5057	11	4943	2688	101	7312	46	54
15	8107	90	1893	5013	11	4987	3094	101	6906	45	50
16	8468	90	1532	4969	11	5031	3490	101	6501	44	46
17	8830	90	1170	4925	11	5075	3895	101	6095	43	42
18	9190	90	0810	4880	11	5150	4310	101	5690	42	38
19	9551	90	0449	4836	11	5164	4715	101	5285	41	34
20	9911	90	0089	4792	11	5208	5119	101	4881	40	30
21	590271	90	470792	4747	11	5253	5524	101	4478	39	26
22	0631	90	9369	4703	11	5297	5928	101	4073	38	22
23	0990	90	9010	4659	11	5341	6331	101	3669	37	18
24	1349	90	8651	4614	11	5386	6735	101	3265	36	14
25	1708	89	8292	4570	11	5430	7138	101	2862	35	10
26	2066	89	7934	4525	11	5475	7541	100	2459	34	06
27	2424	89	7576	4481	11	5519	7943	100	2057	33	02
28	2781	89	7219	4436	11	5564	8345	100	1655	32	58
29	3138	89	6862	4391	11	5609	8747	100	1253	31	54
30	3495	89	6505	4346	11	5654	9149	100	0851	30	50
31	3852	89	6148	4302	11	5698	9550	100	0450	29	46
32	4208	89	5792	4257	11	5743	9951	100	0049	28	42
33	4564	89	5436	4213	11	5788	550322	100	449049	27	38
34	4919	89	5081	4167	11	5833	0782	100	9048	26	34
35	5275	89	4725	4122	11	5878	1153	100	8647	25	30
36	5630	89	4370	4077	11	5923	1523	100	8247	24	26
37	5984	89	4016	4032	11	5968	1892	100	7848	23	22
38	6339	89	3661	3987	11	6013	2252	100	7448	22	18
39	6693	89	3307	3942	11	6058	2731	100	7049	21	14
40	7046	88	2954	3897	11	6103	3149	100	6651	20	10
41	7400	88	2600	3852	11	6148	3548	99	6252	19	06
42	7753	88	2247	3807	11	6193	3948	99	5854	18	02
43	8105	88	1895	3761	11	6238	4344	99	5456	17	58
44	8458	88	1542	3716	11	6284	4742	99	5058	16	54
45	8810	88	1190	3671	11	6329	5139	99	4661	15	50
46	9161	88	0838	3625	11	6375	5536	99	4264	14	46
47	9513	88	0487	3580	11	6420	5933	99	3867	13	42
48	9864	88	0136	3535	11	6465	6329	99	3471	12	38
49	530215	88	469785	3489	11	6511	6726	99	3074	11	34
50	0585	87	9435	3444	11	6556	7121	99	2679	10	30
51	0915	87	9085	3398	11	6602	7517	99	2283	9	26
52	1265	87	8735	3352	11	6648	7913	99	1887	8	22
53	1615	87	8385	3307	11	6693	8308	99	1492	7	18
54	1964	87	8036	3261	11	6739	8703	99	1097	6	14
55	2312	87	7686	3215	11	6785	9097	99	6993	5	10
56	2661	87	7339	3169	11	6831	9492	99	6598	4	06
57	3009	87	6991	3124	11	6878	9885	99	6202	3	02
58	3357	87	6643	3079	11	6924	560979	99	439731	2	58
59	3705	87	6295	3032	11	6968	0673	99	9287	1	54
60	9.534022	87	0.463946	0.973670	11	0.027914	0.546166	99	0.439834	0	40
Deg.	L. Cos.	Diff. for 15' or 1"	L. Sec.	L. Sin.	Diff. for 15' or 1"	L. Cossec.	L. Cot.	Diff. for 15' or 1"	L. Tang.	Deg.	Hours.

100

1' - 15' 1" - 15'

160

10

TABLE II.—LOG. SINES, TANG'S, &c.

100		$P = 15'' \quad P' = 15' \quad P'' = 15''$				1500				10		
Hours.	Deg.	L. Sin.	Diff. for 15'' or P	L. Coscc.	L. Cos.	Diff. for 15'' or P	L. Sec.	L. Tang.	Diff. for 15'' or P	L. Cot.	Deg.	Hours.
0	0	0.531072	87	0.465098	0.972986	11	0.027014	0.561966	98	0.438934	60	30
1	1	4709	87	5601	2940	11	7090	1459	98	8541	59	56
2	2	4745	87	5255	2994	11	7196	1851	98	8149	58	52
3	3	5092	88	4908	2948	11	7192	2244	98	7756	57	48
4	4	5438	88	4562	2892	12	7199	2636	99	7364	56	44
5	5	5789	88	4217	2755	11	7245	3028	98	6972	55	40
6	6	6128	88	3872	2709	11	7291	3419	98	6581	54	36
7	7	6474	88	3526	2663	11	7337	3811	98	6190	53	32
8	8	6819	88	3181	2617	11	7383	4202	98	5798	52	28
9	9	7163	88	2837	2570	11	7430	4593	97	5407	51	24
10	10	7507	88	2492	2524	11	7476	4983	97	5017	50	20
11	11	7851	88	2147	2478	11	7522	5373	97	4627	49	16
12	12	8194	88	1806	2431	12	7569	5763	97	4237	48	12
13	13	8538	88	1462	2385	11	7615	6153	97	3847	47	8
14	14	8880	88	1120	2338	12	7662	6542	97	3457	46	4
15	15	9223	88	777	2291	11	7709	6932	97	3067	45	0
16	16	9565	88	435	2245	12	7755	7320	97	2676	44	56
17	17	9907	88	93	2198	12	7802	7707	97	2285	43	52
18	18	540149	88	450751	2151	12	7849	8093	97	1894	42	48
19	19	6380	88	9410	2105	12	7895	8485	97	1503	41	44
20	20	6721	88	9069	2058	12	7942	8873	97	1113	40	40
21	21	7072	88	8728	2011	12	7989	9261	97	7230	39	36
22	22	7423	88	8387	1964	12	8036	9649	97	6839	38	32
23	23	7774	88	8047	1917	12	8082	570096	97	420964	37	28
24	24	8125	88	7708	1870	12	8129	0422	97	9578	36	24
25	25	8476	88	7368	1823	12	8177	0809	96	9191	35	20
26	26	8827	88	7029	1776	12	8224	1195	96	8805	34	16
27	27	9178	88	6690	1729	12	8271	1581	96	8419	33	12
28	28	9529	88	6351	1682	12	8318	1967	96	8033	32	8
29	29	9880	88	6013	1635	12	8365	2352	96	7648	31	4
30	30	10231	88	5675	1587	12	8413	2739	96	7262	30	0
31	31	10582	88	5337	1540	12	8460	3123	96	6877	29	56
32	32	10933	88	5000	1493	12	8507	3507	96	6493	28	52
33	33	11284	88	4662	1446	12	8554	3892	96	6108	27	48
34	34	11635	88	4326	1398	12	8602	4276	96	5724	26	44
35	35	11986	88	3989	1351	12	8649	4660	96	5340	25	40
36	36	12337	88	3653	1303	12	8697	5044	96	4956	24	36
37	37	12688	88	3317	1256	12	8744	5427	96	4571	23	32
38	38	13039	88	2981	1209	12	8791	5810	96	4187	22	28
39	39	13390	88	2646	1161	12	8839	6193	96	3803	21	24
40	40	13741	88	2311	1113	12	8887	6576	96	3420	20	20
41	41	14092	88	1976	1066	12	8934	6959	96	3037	19	16
42	42	14443	88	1641	1018	12	8982	7341	95	2654	18	12
43	43	14794	88	1307	970	12	9030	7723	95	2271	17	8
44	44	15145	88	974	922	12	9078	8104	95	1886	16	4
45	45	15496	88	640	874	12	9126	8486	95	1504	15	0
46	46	15847	88	306	827	12	9173	8867	95	1123	14	56
47	47	16198	88	41973	779	12	9221	9248	95	6739	13	52
48	48	16549	88	6641	731	12	9269	9629	95	6357	12	48
49	49	16900	88	9086	683	12	9317	580000	95	41991	11	44
50	50	17251	88	8970	635	12	9365	0389	95	9611	10	40
51	51	17602	88	8644	587	12	9413	0769	95	9231	9	36
52	52	17953	88	8317	539	12	9460	1149	95	8851	8	32
53	53	18304	88	7990	490	12	9508	1529	95	8472	7	28
54	54	18655	88	7654	442	12	9556	1907	95	8093	6	24
55	55	19006	88	7320	394	12	9604	2286	95	7714	5	20
56	56	19357	88	6990	345	12	9652	2665	95	7335	4	16
57	57	19708	88	6659	297	12	9700	3044	95	6956	3	12
58	58	20059	88	6329	249	12	9748	3422	94	6577	2	8
59	59	20410	88	6000	200	12	9796	3800	94	6200	1	4
60	60	20761	88	5671	152	12	9844	4177	94	5823	0	0

TABLE II.—LOG. SINES, TANG'S, &C.

1°			22°	1° - 15' 1" = 15' 1" - 15°				187°	10°			
Hours.	Deg.	L. Sin.	Diff. for 15'' or 1'	L. Cossec.	L. Cos.	Diff. for 15'' or 1'	L. Sec.	L. Tang.	Diff. for 15'' or 1'	L. Cot.	Deg.	Hours.
29	0	9.573578	78	0.425424	9.987186	13	0.032834	9.606410	91	0.394596	60	31
	1	3 398	78	6112	7115	13	2885	6773	91	3227	59	56
	2	4230	78	5890	7094	13	2936	7136	91	2954	58	52
	3	4512	78	5488	7042	13	2988	7509	91	2500	57	48
	4	4824	78	5176	6961	13	3039	7863	91	2137	56	44
29	5	5135	78	4865	6900	13	3090	8225	91	1775	55	40
	6	5447	78	4553	6850	13	3141	8588	90	1412	54	36
	7	5758	78	4244	6808	13	3192	8950	90	1050	53	32
	8	6068	78	3932	6756	13	3244	9312	90	688	52	28
	9	6379	77	3621	6705	13	3295	9674	90	326	51	24
29	10	6689	77	3311	6653	13	3347	610039	90	389964	50	20
	11	6999	77	3001	6602	13	3398	6037	90	9603	49	16
	12	7309	77	2691	6550	13	3450	6753	90	924	48	12
	13	7618	77	2382	6499	13	3501	7119	90	889	47	8
	14	7927	77	2073	6447	13	3553	7480	90	850	46	4
29	0	8236	77	1764	6395	13	3605	1841	90	819	45	31
	1	8545	77	1455	6344	13	3656	2201	90	779	44	27
	2	8854	77	1146	6292	13	3708	2562	90	748	43	23
	3	9162	77	838	6240	13	3760	2922	90	708	42	19
	4	9469	77	531	6188	13	3812	3281	90	679	41	15
29	0	9777	77	223	6136	13	3864	3641	90	639	40	11
	1	580085	77	119915	6085	13	3915	4000	90	600	39	7
	2	932	77	968	6033	13	3967	4359	90	561	38	3
	3	939	76	930	5981	13	4019	4718	90	522	37	29
	4	1005	76	895	5929	13	4072	5077	89	482	36	25
29	0	1311	76	860	5876	13	4124	5435	90	455	35	20
	1	1618	76	829	5824	13	4176	5794	90	426	34	16
	2	1924	76	807	5772	13	4229	6152	89	396	33	12
	3	2229	76	777	5720	13	4280	6509	89	367	32	8
	4	2535	76	7465	5668	13	4332	6867	89	337	31	4
30	0	2840	76	7160	5616	13	4384	7224	89	307	30	30
	1	3145	76	6855	5563	13	4437	7582	89	278	29	26
	2	3449	76	6551	5511	13	4489	7939	89	248	28	22
	3	3753	76	6247	5458	13	4542	8295	89	219	27	18
	4	4058	76	5942	5406	13	4594	8652	89	189	26	14
30	0	4361	76	5639	5353	13	4647	9008	89	160	25	10
	1	4665	76	5335	5301	13	4699	9364	89	130	24	6
	2	4969	76	5031	5248	13	4752	9721	89	101	23	2
	3	5272	76	4729	5196	13	4804	620076	89	37924	22	29
	4	5575	75	4425	5143	13	4857	6432	89	956	21	24
30	0	5877	75	4123	5090	13	4910	6787	89	621	20	19
	1	6179	75	3821	5037	13	4963	7142	89	885	19	15
	2	6481	75	3519	4984	13	5015	7497	89	850	18	11
	3	6783	75	3217	4931	13	5068	7852	89	814	17	8
	4	7085	75	2915	4878	13	5121	8207	88	779	16	4
31	0	7387	75	2613	4825	13	5174	8561	88	743	15	29
	1	7689	75	2312	4772	13	5227	8915	88	708	14	25
	2	7991	75	2011	4720	13	5280	9269	88	673	13	21
	3	8293	75	1711	4668	13	5334	9623	88	637	12	17
	4	8595	75	1411	4615	13	5387	9976	88	602	11	13
31	0	8900	75	1110	4560	13	5440	4380	88	567	10	10
	1	9190	75	810	4507	13	5493	4683	88	531	9	6
	2	9480	75	511	4454	13	5546	5033	88	496	8	2
	3	9770	75	211	4401	13	5599	5384	88	461	7	28
	4	590088	75	40012	4347	13	5653	5741	88	425	6	24
31	0	9087	75	9613	4294	13	5706	6093	88	390	5	20
	1	9389	74	9314	4240	13	5759	6446	88	354	4	16
	2	9691	74	9016	4187	13	5813	6797	88	320	3	12
	3	1242	74	8719	4133	13	5867	7149	88	285	2	8
	4	1590	74	8420	4080	13	5920	7500	88	250	1	4
31	0	9591878	74	0.407122	9.961026	13	0.035774	9.627852	88	0.372148	0	29
Hours.	Deg.	L. Cos.	Diff. for 15'' or 1'	L. Sec.	L. Sin.	Diff. for 15'' or 1'	L. Cossec.	L. Cot.	Diff. for 15'' or 1'	L. Tang.	Deg.	Hours.

7° 113°

1° - 4' 10" - 4°

87° 34'

TABLE II.—LOG. SINES, TANG'S, &c

1° 30'		1° - 15' 1" - 15' 1" -- 15°										156° 10'		
Hours.	Deg.	L. Sin	Diff. for 15'' or 1'	L. Cosc.	L. Cos.	Diff. for 15'' or 1'	L. Sec.	L. Tang.	Diff. for 15'' or 1'	L. Cot.	Deg.	Hours.		
32	0 0	9.591878		0.408129	9.964096		0.035974	9.627852		0.272148	60	27		
	4 1	2175	74	7825	3972	13	6028	8203	88	1797	59			
	8 2	2473	74	7527	3919	13	6081	8554	88	1446	58			
	12 3	2770	74	7230	3865	13	6135	8905	88	1095	57			
	16 4	3066	74	6934	3811	13	6189	9255	88	0745	56			
	20 5	3363	74	6637	3757	13	6243	9606	87	0394	55			
	24 6	3659	74	6341	3703	13	6297	9956	87	0044	54			
	28 7	3956	74	6044	3650	13	6350	1.0306	87	3.6094	53			
	32 8	4251	74	5749	3596	13	6404	0.655	87	8345	52			
	36 9	4547	74	5453	3542	13	6458	1005	87	8895	51			
	40 10	4842	74	5158	3488	13	6512	1354	87	8646	50			
	44 11	5137	74	4863	3433	14	6567	1704	87	8896	49			
	48 12	5432	74	4568	3379	13	6621	2053	87	7947	48			
	52 13	5727	74	4273	3325	14	6675	2402	87	7398	47			
	56 14	6021	73	3978	3271	13	6729	2750	87	7350	46			
	33	0 15	6315	73	3683	3217	13	6783	3098	87	6902		45	27
		4 16	6609	73	3391	3162	14	6838	3447	87	6553		44	
8 17		6903	73	3097	3108	13	6892	3795	87	6205	43			
12 18		7197	73	2803	3054	13	6946	4143	87	5857	42			
16 19		7490	73	2510	3000	14	7000	4490	87	5510	41			
20 20		7783	73	2217	2945	13	7055	4838	87	5162	40			
24 21		8075	73	1925	2890	13	7110	5185	87	4815	39			
28 22		8368	73	1632	2836	14	7164	5532	87	4468	38			
32 23		8660	73	1340	2781	14	7219	5879	87	4121	37			
36 24		8952	73	1048	2726	14	7274	6226	87	3774	36			
40 25	9244	73	756	2672	14	7328	6572	87	3428	35				
44 26	9536	73	464	2617	14	7383	6919	86	3081	34				
48 27	9827	73	0173	2562	14	7438	7265	86	2735	33				
52 28	1.0118	73	3.0689	2507	14	7493	7611	86	2389	32				
56 29	0409	73	9591	2453	14	7547	7956	86	2044	31				
34	0 30	0700	72	9300	2398	14	7602	8302	86	1696	30	26		
	4 31	0990	72	9010	2343	14	7657	8647	86	1353	29			
	8 32	1280	72	8720	2288	14	7712	8992	86	1008	28			
	12 33	1570	72	8430	2233	14	7767	9337	86	0663	27			
	16 34	1860	72	8140	2178	14	7822	9682	86	0318	26			
	20 35	2150	72	7850	2123	14	7877	1.0027	86	3.5973	25			
	24 36	2439	72	7561	2067	14	7933	0373	86	6698	24			
	28 37	2728	72	7272	2012	14	7988	0718	86	9284	23			
	32 38	3017	72	6983	1957	14	8043	1060	86	8940	22			
	36 39	3305	72	6695	1902	14	8098	1403	86	8597	21			
40 40	3593	72	6407	1846	14	8154	1747	86	8253	20				
44 41	3882	72	6118	1791	14	8209	2091	86	7909	19				
48 42	4170	72	5830	1736	14	8264	2434	86	7566	18				
52 43	4457	72	5543	1680	14	8320	2777	86	7223	17				
56 44	4745	72	5255	1625	14	8375	3120	86	6880	16				
35	0 45	5038	72	4968	1569	14	8431	3463	86	6537	15	25		
	4 46	5319	72	4681	1513	14	8487	3806	85	6194	14			
	8 47	5606	72	4394	1458	14	8542	4148	85	5852	13			
	12 48	5892	72	4108	1402	14	8598	4490	85	5510	12			
	16 49	6179	71	3821	1347	14	8653	4832	85	5168	11			
	20 50	6465	71	3535	1291	14	8709	5174	85	4826	10			
	24 51	6751	71	3249	1235	14	8765	5516	85	4484	9			
	28 52	7036	71	2964	1179	14	8821	5857	85	4143	8			
	32 53	7322	71	2678	1123	14	8877	6199	85	3801	7			
	36 54	7607	71	2393	1067	14	8933	6540	85	3460	6			
40 55	7892	71	2108	1011	14	8989	6881	85	3119	5				
44 56	8177	71	1823	955	14	9045	7222	85	2778	4				
48 57	8461	71	1539	899	14	9101	7562	85	2438	3				
52 58	8745	71	1255	843	14	9158	7903	85	2097	2				
56 59	9029	71	0971	786	14	9214	8243	85	1757	1				
60 60	9.609313		0.390687	9.960730		0.039270	9.648583		0.351417	0	24			
Hours.	Deg.	L. Cos.	Diff. for 15'' or 1'	L. Sec.	L. Sin.	Diff. for 15'' or 1'	L. Cosc.	L. Cot.	Diff. for 15'' or 1'	L. Tang.	Deg.	Hours.		

TABLE II.—LOG. SINES, TANG'S, &c.

1° 24°			1° - 15' 1" - 15' 1" - 15°						155° 10°			
Hours.	Ueg.	L. Sin.	Diff. for 15'' or 1'	L. Cossec.	L. Cos.	Diff. for 15'' or 1'	L. Sec.	L. Tang.	Diff. for 15'' or 1'	L. Cot.	Ueg.	Hours.
36	0	0	9.609313	71	0.390187	9.980730	14	0.039270	9.648583	85	0.351417	60
	4	1	8597	71	0103	0674	14	9386	8923	85	1077	59
	8	2	9690	71	0120	0617	14	9383	9248	85	0737	58
	12	3	610163	71	389637	0561	14	9439	9602	85	0398	57
	16	4	0447	71	9553	0505	14	9485	9942	85	0058	56
	20	5	0799	71	9271	0448	14	9552	650981	85	349719	55
	24	6	1019	70	8988	0392	14	9602	0690	85	9380	54
	28	7	1294	70	8706	0335	14	9685	0959	85	9041	53
	32	8	1576	70	8424	0279	14	9721	1297	85	8703	52
	36	9	1858	70	8142	0222	14	9778	1636	84	8364	51
	40	10	2140	70	7860	0166	14	9834	1974	84	8026	50
	44	11	2421	70	7579	0109	14	9891	2312	84	7688	49
	48	12	2702	70	7298	0052	14	9946	2650	84	7350	48
	52	13	2983	70	7017	995955	14	0.040005	2988	84	7012	47
	56	14	3264	70	6736	9938	14	0062	3326	84	6674	46
37	0	15	3545	70	6455	9882	14	0118	3663	84	6337	45
	4	16	3825	70	6175	9825	14	0175	4000	84	6000	44
	8	17	4105	70	5895	9768	14	0232	4337	84	5663	43
	12	18	4385	70	5615	9711	14	0289	4674	84	5326	42
	16	19	4665	70	5335	9654	14	0346	5011	84	4989	41
	20	20	4944	70	5056	9596	14	0404	5348	84	4652	40
	24	21	5223	70	4777	9539	14	0461	5684	84	4316	39
	28	22	5502	70	4498	9482	14	0518	6020	84	3980	38
	32	23	5781	70	4219	9425	14	0575	6356	84	3644	37
	36	24	6060	69	3940	9368	14	0632	6692	84	3308	36
	40	25	6339	69	3662	9310	14	0690	7029	84	2972	35
	44	26	6616	69	3384	9253	14	0747	7363	84	2637	34
	48	27	6894	69	3106	9195	14	0805	7699	84	2301	33
	52	28	7172	69	2828	9138	14	0862	8034	84	1966	32
	56	29	7450	69	2550	9081	14	0919	8369	84	1631	31
38	0	30	7727	69	2273	9023	14	0977	8704	84	1296	30
	4	31	8004	69	1996	8965	14	1035	9039	84	0961	29
	8	32	8281	69	1719	8908	14	1092	9373	84	0627	28
	12	33	8558	69	1442	8850	14	1150	9708	83	0292	27
	16	34	8834	69	1166	8792	14	1208	6.60642	83	339958	26
	20	35	9110	69	0890	8734	14	1266	0376	83	9624	25
	24	36	9386	69	0614	8677	14	1323	0709	83	9291	24
	28	37	9662	69	0338	8619	14	1381	1043	83	8957	23
	32	38	9938	69	0062	8561	14	1439	1377	83	8623	22
	36	39	6.90213	69	3.79787	8503	14	1497	1710	83	8290	21
	40	40	0488	69	9512	8445	14	1555	2043	83	7957	20
	44	41	0763	69	9237	8388	14	1614	2377	83	7623	19
	48	42	1038	69	8962	8330	14	1671	2703	83	7291	18
	52	43	1313	68	8687	8271	14	1729	3049	83	6959	17
	56	44	1587	68	8413	8212	15	1788	3375	83	6625	16
39	0	45	1861	68	8139	8154	14	1846	3707	83	6293	15
	4	46	2135	68	7865	8096	14	1904	4039	83	5961	14
	8	47	2409	68	7591	8039	14	1962	4371	83	5630	13
	12	48	2682	68	7318	7979	15	2021	4703	83	5297	12
	16	49	2956	68	7044	7921	14	2079	5035	83	4965	11
	20	50	3229	68	6771	7863	15	2137	5368	83	4634	10
	24	51	3502	68	6498	7804	15	2196	5698	83	4302	9
	28	52	3774	68	6226	7745	15	2255	6029	83	3971	8
	32	53	4047	68	5953	7687	15	2313	6360	83	3640	7
	36	54	4319	68	5681	7628	14	2372	6691	83	3309	6
	40	55	4591	68	5409	7570	15	2430	7021	83	2979	5
	44	56	4863	68	5137	7511	15	2489	7352	83	2648	4
	48	57	5134	68	4866	7452	15	2548	7682	83	2318	3
	52	58	5406	68	4594	7393	15	2607	8013	82	1987	2
	56	59	5677	68	4323	7334	15	2666	8343	82	1657	1
39	0	60	6.925948	68	0.374052	9.957276	15	0.042734	9.668672	82	0.331323	0
Hours.	Ueg.	L. Cos.	Diff. for 15'' or 1'	L. Sec.	L. Sin.	Diff. for 15'' or 1'	L. Cossec.	L. Cot.	Diff. for 15'' or 1'	L. Tang.	Ueg.	Hours.

74 1140

1' - 4' 10 = 4"

650 4"

TABLE II.—LOG. SINES, TANG'S, &c.

1° 29'		1° - 15' 1" - 15'						153°		10°		
Hours	Deg.	L. Sin.	Diff. for 15" or 1'	L. Cossec.	L. Cos.	Diff. for 15" or 1'	L. Sec.	L. Tang.	Diff. for 15" or 1'	L. Cot.	Reg.	Hours
44	0	0.011412	65	0.333153	9.933660	15	0.016340	0.081822	80	0.311316	60	15
	4	2101	65	7593	3510	15	6401	8502	80	1478	53	53
	8	2140	65	7040	3537	15	6433	8623	81	1177	58	45
	12	2114	65	7332	3475	15	6525	9143	80	857	57	52
	16	2276	64	7124	3413	15	6587	9463	80	6537	56	44
	20	5	64	6865	3352	15	6648	9783	80	5217	55	40
	24	6	64	6607	32 0	15	6710	69010	80	3087	54	35
	28	7	64	6349	3228	15	6772	0423	80	9577	53	32
	32	8	64	6092	3166	15	6834	0742	80	925	52	28
	36	9	64	5834	3104	15	6896	1062	80	8936	51	24
	40	10	64	5577	3042	15	6958	1381	80	8619	50	20
	44	11	64	5320	2980	15	7020	1709	80	8300	49	16
	48	12	64	5063	2918	15	7082	2011	80	7981	48	12
	52	13	64	4807	2855	15	7145	2312	80	7662	47	8
	56	14	64	4550	2793	15	7207	2657	79	7343	46	4
45	0	15	64	4294	2731	15	7269	2975	79	7025	45	15
	4	15	64	4037	2669	15	7331	3293	79	6707	44	53
	8	17	64	3782	2607	16	7394	3612	79	6386	43	52
	12	18	64	3527	2545	15	7456	3930	79	6066	42	48
	16	18	64	3271	2483	16	7519	4248	79	5752	41	44
	20	20	64	3015	2421	16	7581	4566	79	5434	40	40
	24	21	64	2759	2359	15	7644	4883	79	5117	39	36
	28	22	64	2503	2297	16	7706	5201	79	4771	38	32
	32	23	64	2247	2235	16	7769	5518	79	4422	37	28
	36	24	63	1990	2173	16	7832	5836	79	4104	36	24
	40	25	63	1734	2111	16	7895	6153	79	3787	35	20
	44	26	63	1478	2049	16	7957	6470	79	3500	34	16
	48	27	63	1222	1987	16	8020	6786	79	3213	33	12
	52	28	63	966	1925	16	8083	7101	79	2927	32	8
	56	29	63	710	1863	16	8146	7420	79	2650	31	4
46	0	30	63	454	1801	16	8209	7736	79	2364	30	14
	4	31	63	200	1739	16	8272	8053	79	1947	29	53
	8	32	63	34	1686	16	8335	8369	79	1631	27	52
	12	33	63	9713	1624	16	8398	8685	79	1315	26	48
	16	34	63	9460	1562	16	8461	9001	79	999	25	44
	20	35	63	9207	1500	16	8524	9316	79	6584	25	40
	24	36	63	8954	1438	16	8587	9632	79	3184	24	36
	28	37	63	8701	1376	16	8650	9948	79	0052	23	32
	32	38	63	8448	1314	16	8713	70123	79	39737	22	28
	36	39	63	8195	1252	16	8776	0578	79	9422	21	24
	40	40	63	7942	1190	16	8839	0893	79	9107	20	20
	44	41	63	7689	1128	16	8902	1208	79	8792	19	16
	48	42	63	7436	1066	16	8965	1523	79	8477	18	12
	52	43	63	7183	1004	16	9028	1837	78	8163	17	8
	56	44	63	6930	942	16	9091	2152	78	7848	16	4
47	0	45	62	6677	880	16	9154	2466	78	7534	15	13
	4	46	62	6424	818	16	9217	2780	78	7221	14	53
	8	47	62	6171	756	16	9280	3095	78	6905	13	52
	12	48	62	5918	694	16	9343	3410	78	6591	12	48
	16	49	62	5665	632	16	9406	3724	78	6277	11	44
	20	50	62	5412	570	16	9469	4038	78	5961	10	40
	24	51	62	5159	508	16	9532	4352	78	5645	9	36
	28	52	62	4906	446	16	9595	4666	78	5329	8	32
	32	53	62	4653	384	16	9658	4979	78	5013	7	28
	36	54	62	4400	322	16	9721	5293	78	4710	6	24
	40	55	62	4147	260	16	9784	5607	78	4397	5	20
	44	56	62	3894	198	16	9847	5921	78	4084	4	16
	48	57	62	3641	136	16	9910	6235	78	3772	3	12
	52	58	62	3388	74	16	9973	6549	78	345	2	8
	56	59	62	3135	12	16	10036	6863	78	314	1	4
48	0	60	62	2882	50	16	010000	7176	78	039231	0	12

116°

1° - 4' 10" - 4"

63° 4'

TABLE II.—LOG. SINES, TANG'S, &c.

1° 28°			1° - 15' 1" - 15' 1" - 15°						151° 10°			
Hours.	Deg.	L. Sin.	Diff. for 15' or 1"	L. Cossec.	L. Cos.	Diff. for 15' or 1"	L. Sec.	L. Tang.	Diff. for 15' or 1"	L. Cot.	Deg.	Ho
52	0	9.6710	59	0.328311	9.945935	17	0.054065	9.725674	76	0.274329	60	7
	1	1847	59	8153	5848	17	4132	5979	76	4021	59	
	2	3684	59	7916	5800	17	4300	6294	76	3711	58	
	3	5521	59	7679	5733	17	4207	6589	76	3412	57	
	4	7358	59	7442	5666	17	4334	6892	76	3108	56	
	5	9195	59	7205	5598	17	4402	7197	76	2803	55	
	6	3032	59	6968	5531	17	4469	7501	76	2499	54	
	7	3219	59	6731	5464	17	4536	7805	76	2195	53	
	8	3505	59	6495	5396	17	4604	8109	76	1891	52	
	9	3741	59	6259	5322	17	4671	84.9	76	1588	51	
	10	3977	59	6023	5261	17	4739	8716	76	1284	50	
	11	4213	59	5787	5193	17	4807	9020	76	0980	49	
	12	4449	59	5551	5126	17	4874	9323	76	0677	48	
	13	4684	59	5316	5058	17	4942	9626	76	0374	47	
	14	4919	59	5081	4990	17	5010	9929	76	0071	46	
53	0	5155	59	4845	4922	17	5078	730233	76	269767	45	7
	1	5390	59	4609	4854	17	5146	0536	76	9464	44	
	2	5624	59	4373	4786	17	5214	0838	76	9162	43	
	3	5859	59	4137	4718	17	5282	1141	76	8859	42	
	4	6094	58	3900	4650	17	5350	1444	75	8556	41	
	5	6328	58	3663	4582	17	5418	1746	75	8254	40	
	6	6562	58	3426	4514	17	5486	2048	75	7952	39	
	7	6796	58	3189	4446	17	5554	2350	75	7650	38	
	8	7030	58	2952	4377	17	5623	2653	75	7347	37	
	9	7264	58	2715	4309	17	5691	2955	75	7045	36	
	10	7498	58	2478	4241	17	5759	3257	75	6743	35	
	11	7731	58	2241	4173	17	5827	3559	75	6442	34	
	12	7964	58	2004	4104	17	5896	3860	75	6140	33	
	13	8197	58	1767	4036	17	5964	4161	75	5837	32	
	14	8430	58	1530	3967	17	6033	4463	75	5537	31	
54	0	8663	58	1293	3899	17	6101	4764	75	5236	30	7
	1	8896	58	1056	3830	17	6170	5066	75	4934	29	
	2	9129	58	819	3761	17	6239	5367	75	4633	28	
	3	9362	58	582	3692	17	6308	5668	75	4332	27	
	4	9595	58	345	3624	17	6376	5968	75	4032	26	
	5	9828	58	108	3555	17	6445	6269	75	3731	25	
	6	0061	58	1176	3486	17	6514	6570	75	3430	24	
	7	0294	58	879	3417	17	6583	6871	75	3129	23	
	8	0527	58	642	3348	17	6652	7171	75	2827	22	
	9	0760	58	405	3279	17	6721	7471	75	2526	21	
	10	0993	58	168	3210	17	6790	7772	75	2225	20	
	11	1226	58	111	3141	17	6859	8072	75	1924	19	
	12	1459	58	111	3072	17	6928	8371	75	1623	18	
	13	1692	58	111	3003	17	6997	8671	75	1322	17	
	14	1925	57	111	2934	17	7066	8971	75	1021	16	
55	0	2158	57	785	2864	17	7136	9271	75	0720	15	7
	1	2391	57	548	2795	17	7205	9570	75	0419	14	
	2	2624	57	311	2725	17	7275	9870	75	0118	13	
	3	2857	57	74	2656	17	7344	740169	75	28931	12	
	4	3090	57	6945	2587	17	7413	0468	75	9532	11	
	5	3323	57	6718	2517	17	7483	0767	75	9233	10	
	6	3556	57	6491	2448	17	7552	1066	75	8934	9	
	7	3789	57	6264	2378	17	7622	1365	75	8635	8	
	8	4022	57	6037	2308	17	7692	1664	75	8336	7	
	9	4255	57	5810	2239	17	7761	1962	75	8037	6	
	10	4488	57	5583	2169	17	7831	2261	75	7738	5	
	11	4721	57	5356	2099	17	7901	2559	75	7439	4	
	12	4954	57	5129	2029	17	7971	2858	74	7142	3	
	13	5187	57	4902	1959	17	8041	3156	74	6844	2	
	14	5420	57	4675	1889	17	8111	3454	74	6547	1	
55	15	9.68571	57	0.314429	9.941819	17	0.058181	9.743752	74	0.259246	0	4

118°

1° - 4' 10" - 4°

61° 4'

TABLE II.—LOG. SINES, TANG'S, &c.

29°		1° - 15' 1" - 15' 1" - 15°				150° - 10°					
PR.	L. Sin.	Diff. for 15" or 1'	L. Cossec.	L. Cos.	Diff. for 15" or 1'	L. Sec.	L. Tang.	Diff. for 15" or 1'	L. Cot.	Deg.	Hours.
0	9.685571		0.31442	9.941819		0.058481	9.743755		0.250248	10	3 00
1	57:9	57	4201	1749	17	8251	4056	74	5650	50	58
2	027	57	3973	1679	17	8221	4344	74	5452	59	58
3	0254	57	3746	1610	17	8201	4645	74	5255	57	47
4	64e2	57	3518	1539	17	8181	4945	74	5057	56	44
5	6709	57	3291	1469	18	8151	5240	74	470	55	40
6	6936	57	3064	1398	18	8122	5535	74	4412	54	36
7	7163	57	2837	1328	18	8092	5830	74	4125	53	32
8	730	57	2610	1258	18	8072	6125	74	3836	52	28
9	760	57	2382	1187	18	8043	6420	74	3571	51	24
10	7843	56	2157	1117	18	8023	6720	74	3274	50	20
11	8089	56	1931	1046	18	8004	7020	74	2977	49	16
12	825	56	1705	0:78	18	7984	7320	74	2680	48	12
13	8521	56	1479	0:05	18	7965	7620	74	2382	47	8
14	8747	56	1253	0:34	18	7946	7920	74	2085	46	4
15	8972	56	1028	0:63	17	7927	8220	74	1791	45	0
16	9118	56	0:02	0:63	17	7907	8520	74	1495	44	30
17	0423	56	0:57	0:22	18	7878	8820	74	1198	43	52
18	9147	56	0:52	0:51	18	7849	9120	74	0:0	42	47
19	9373	56	0:127	04:0	18	7820	9420	74	0:0	41	44
20	6000F	56	30902	0409	18	7791	9720	74	0:11	40	40
21	0322	56	9677	0338	18	7772	10020	74	0:0	39	36
22	0547	56	9452	0267	18	7753	7502-1	74	34371	38	32
23	0772	56	9227	0196	18	7734	0578	74	9484	37	28
24	057	56	9003	0125	18	7715	0872	74	9126	36	24
25	1221	56	8777	0054	18	7696	1172	74	8733	35	20
26	1444	56	8556	93922	18	7677	1472	74	8333	34	16
27	1668	56	8332	8911	18	7658	1772	74	7943	33	12
28	182	56	8108	8640	18	7639	2072	74	7544	32	8
29	2115	56	7885	7768	18	7620	2372	74	7153	31	4
30	233	56	7661	6897	18	7601	2672	74	6758	30	0
31	252	56	7438	6025	18	7582	2972	74	6359	29	56
32	275	56	7215	5154	18	7563	3272	74	5959	28	52
33	3009	56	6992	4282	18	7544	3572	74	5559	27	47
34	3231	56	6769	3411	18	7525	3872	74	5159	26	44
35	3454	55	6546	2539	18	7506	4172	73	4758	25	40
36	3676	55	6324	1667	18	7487	4472	73	4357	24	36
37	3898	55	6102	915	18	7468	4772	73	3956	23	32
38	4120	55	5880	9123	18	7449	5072	73	3555	22	28
39	4342	55	5658	9051	18	7430	5372	73	3154	21	24
40	4564	55	5436	8980	18	7411	5672	73	2753	20	20
41	4786	55	5214	8908	18	7392	5972	73	2352	19	16
42	5008	55	4992	8836	18	7373	6272	73	1951	18	12
43	5230	55	4771	8764	18	7354	6572	73	1550	17	8
44	5450	55	4550	8691	18	7335	6872	73	1149	16	4
45	5671	55	4328	8619	18	7316	7172	73	748	15	0
46	5892	55	4106	8547	18	7297	7472	73	348	14	54
47	6113	55	3884	8475	18	7278	7772	73	28	13	52
48	6334	55	3662	8403	18	7259	8072	73	208	12	48
49	6554	55	3440	8330	18	7240	8372	73	127	11	44
50	6775	55	3218	8258	18	7221	8672	73	48	10	40
51	6995	55	3005	8185	18	7202	8972	73	119	9	36
52	7215	55	2793	8113	18	7183	9272	73	089	8	32
53	7435	55	2580	8040	18	7164	9572	73	0:05	7	28
54	7654	55	2368	7967	18	7145	9872	73	0:13	6	24
55	7874	55	2156	7895	18	7126	10172	73	0:21	5	20
56	8094	55	1944	7822	18	7107	76272	73	3312	4	17
57	8313	55	1732	7749	18	7088	0572	73	8426	3	12
58	8532	55	1520	7676	18	7069	0872	73	0:44	2	8
59	8751	55	1308	7603	18	7050	1172	73	0:52	1	4
60	8970	55	1096	7530	18	7031	1472	73	0:0	0	0
		Diff. for 15" or 1'			Diff. for 15" or 1'			Diff. for 15" or 1'		Deg.	Hours
	L. Cos.		L. Sec.	L. Sin.		L. Cossec.	L. Cot.		L. Tang.		

TABLE II.—LOG. SINES, TANG'S, &c.

2° — 30°

1° — 15' 1" — 15' 1" — 15°

149° 2'

Hours.	Deg.	L. Sin.	Diff. for 15" or 1'	L. Cosec.	L. Cos.	Diff. for 15" or 1'	L. Sec.	L. Tang.	Diff. for 15" or 1'	L. Cot.	Deg.	Hours.
0	0	0.69370	55	0.301030	0.937531	18	0.062469	0.761438	71	0.238561	60	0
	1	918	55	0811	7458	18	2549	1731	71	2819	59	56
	2	9407	55	0503	7385	18	2615	2029	71	7178	58	52
	3	962	54	0374	7312	18	2688	2314	71	7186	57	48
	4	9844	54	0156	7238	18	2762	2606	71	7194	56	44
	5	703069	54	99938	7165	18	2835	2897	71	7103	55	40
	6	028	54	9730	7092	18	2908	3188	71	6812	54	36
	7	048	54	9502	7019	18	2981	3479	71	6521	53	32
	8	0711	54	9284	6946	18	3054	3770	71	6230	52	28
	9	0335	54	9067	6872	18	3128	4061	71	5939	51	24
	10	1151	54	8849	6799	18	3201	4352	71	5748	50	20
	11	138	54	8632	6725	18	3275	4643	71	5557	49	16
	12	1545	54	8415	6652	18	3348	4934	71	5367	48	12
	13	1802	54	8198	6578	18	3422	5224	71	4776	47	8
	14	2011	54	7981	6505	18	3495	5514	71	4486	46	4
1	0	2226	54	7764	6431	18	3568	5805	71	4195	45	0
	1	2452	54	7548	6357	18	3643	605	71	3905	44	56
	2	2689	54	7331	6284	18	3716	6385	71	3615	43	52
	3	2925	54	7115	6210	18	3790	6675	71	3325	42	48
	4	3101	54	6899	6136	18	3864	6965	71	3035	41	44
	5	3317	54	6683	6062	18	3938	7255	71	2745	40	40
	6	3533	54	6467	5988	18	4012	7545	71	2455	39	36
	7	3748	54	6251	5914	18	4086	7834	71	2164	38	32
	8	3964	54	6036	5840	18	4160	8124	71	1873	37	28
	9	4180	54	5820	5766	18	4234	8414	71	1581	36	24
	10	4395	54	5605	5692	18	4308	8703	71	1290	35	20
	11	4610	54	5390	5618	18	4382	8992	71	1000	34	16
	12	4825	54	5175	5543	18	4457	9282	71	0710	33	12
	13	5040	54	4960	5469	18	4531	9571	71	0420	32	8
	14	5254	54	4746	5395	19	4605	9859	71	0131	31	4
2	0	5469	53	4531	5320	18	4680	770149	72	299851	30	56 0
	1	5683	53	4317	5246	18	4754	0437	72	9563	29	52
	2	5898	53	4102	5172	18	4828	0726	72	9274	28	50
	3	6112	53	3888	5097	19	4903	1015	72	8985	27	48
	4	6327	53	3674	5023	19	4977	1303	72	8697	26	44
	5	6540	53	3460	4948	19	5052	1592	72	8408	25	40
	6	6753	53	3247	4873	19	5127	1880	72	8120	24	36
	7	6967	53	3033	4798	19	5202	2169	72	7831	23	32
	8	7180	53	2820	4723	19	5277	2457	72	7541	22	28
	9	7393	53	2607	4648	19	5352	2745	72	7255	21	24
	10	7606	53	2394	4573	19	5427	3033	72	6967	20	20
	11	7819	53	2181	4498	19	5502	3321	72	6677	19	16
	12	8032	53	1968	4424	19	5576	3608	72	6389	18	12
	13	8245	53	1755	4349	19	5651	3896	72	6104	17	8
	14	8458	53	1542	4274	19	5726	4184	72	5811	16	4
3	0	8670	53	1330	4199	19	5801	4471	72	5520	15	57 0
	1	8882	53	1118	4123	19	5877	4759	72	5231	14	56
	2	9074	53	0906	4048	19	5952	5046	72	4943	13	52
	3	9301	53	0694	3973	19	6027	5333	72	4657	12	48
	4	9518	53	0482	3898	19	6102	5620	72	4369	11	44
	5	9730	53	0270	3822	19	6178	5908	72	4082	10	40
	6	9942	53	0058	3747	19	6253	6195	72	3795	9	36
	7	710153	53	28847	3671	19	6328	6482	72	3518	8	32
	8	074	53	9636	3596	19	6404	6768	72	3231	7	28
	9	0575	53	9425	3520	19	6480	7055	72	2945	6	24
	10	0788	53	9214	3444	19	6556	7342	71	2658	5	20
	11	0997	53	9003	3368	19	6631	7628	71	2372	4	16
	12	1209	53	8792	3293	19	6707	7915	71	2085	3	12
	13	1419	53	8582	3217	19	6783	8201	71	1799	2	8
	14	1631	53	8371	3141	19	6859	8488	71	1512	1	4
3	0	971839	52	0288161	0.933065		0.066935	0.778774		0.221237	0	56 0

2° — 120°

1° — 4' 10" — 4'

56° 3'

TABLE II.—LOG. SINES, TANG'S, &c.

34°		1° - 15' 1" - 15' 1" - 15°						145°					
Hours.	Deg.	L. Sin.	Diff. for 15'' or 1'	L. Cossec.	L. Cos.	Diff. for 15'' or 1'	L. Sec.	L. Tang.	Diff. for 15'' or 1'	L. Cot.	Deg.	Hours.	
16	0	9.747539	47	0.332458	9.918574	21	0.061426	9.822988	68	0.171012	60	42	00
	1	7749	47	2251	8489	21	1511	9260	68	0740	59		56
	2	7936	47	2034	8404	21	1536	9532	68	0408	58		52
	3	8123	47	1877	8318	21	1622	9805	68	0195	57		48
	4	8310	47	1690	8233	21	1767	930077	68	169223	56		44
	5	8497	46	1503	8148	21	1852	0349	68	9651	55		40
	6	8683	46	1317	8063	21	1938	0621	68	9379	54		36
	7	8870	46	1130	7977	21	2023	0643	68	9107	53		32
	8	9056	46	944	7891	21	2109	1165	68	8835	52		28
	9	9243	46	758	7805	21	2195	1437	68	8563	51		24
	10	9429	46	571	7720	21	2280	1709	68	8291	50		20
	11	9615	46	385	7634	21	2366	1981	68	8019	49		16
	12	9801	46	0199	7548	21	2452	2253	68	7747	48		12
	13	9987	46	0013	7462	21	2538	2525	68	7475	47		8
	14	750172	46	249222	7376	21	2624	2796	68	7204	46		4
17	0	0358	46	9649	7290	21	2710	3068	68	6932	45	43	0
	1	0543	46	9457	7204	21	2796	3339	68	6661	44		56
	2	0729	46	9271	7118	21	2882	3611	68	6389	43		52
	3	0914	46	9086	7032	21	2968	3882	68	6118	42		48
	4	1099	46	8901	6945	21	3055	4154	68	5846	41		44
	5	1284	46	8716	6859	21	3141	4425	68	5575	40		40
	6	1469	46	8531	6773	21	3227	4696	68	5304	39		36
	7	1654	46	8346	6687	21	3313	4967	68	5033	38		32
	8	1838	46	8161	6600	21	3400	5238	68	4762	37		28
	9	2023	46	7977	6514	21	3486	5509	68	4491	36		24
	10	2209	46	7793	6427	21	3573	5780	68	4220	35		20
	11	2396	46	7608	6341	21	3659	6051	68	3949	34		16
	12	2576	46	7424	6254	21	3746	6322	68	3678	33		12
	13	2760	46	7240	6167	21	3833	6593	68	3407	32		8
	14	2944	46	7056	6080	21	3920	6864	68	3136	31		4
18	0	3128	46	6872	5994	21	4006	7134	68	2866	30	43	0
	1	3313	46	6688	5907	21	4093	7405	68	2595	29		56
	2	3495	46	6505	5820	21	4180	7675	68	2325	28		52
	3	3679	46	6321	5733	21	4267	7946	68	2054	27		48
	4	3862	46	6138	5646	21	4354	8216	68	1784	26		44
	5	4046	46	5954	5559	21	4441	8487	68	1513	25		40
	6	4229	46	5771	5472	21	4528	8757	68	1243	24		36
	7	4412	46	5588	5385	21	4615	9027	68	0973	23		32
	8	4595	46	5405	5297	21	4703	9298	68	0702	22		28
	9	4778	46	5222	5210	21	4790	9568	68	0432	21		24
	10	4961	46	5039	5123	21	4877	9838	68	0162	20		20
	11	5143	46	4857	5035	21	4965	94010	68	159822	19		16
	12	5326	46	4674	4948	21	5052	0378	68	9622	18		12
	13	5509	46	4492	4860	21	5140	0148	68	9352	17		8
	14	5690	46	4310	4773	21	5227	0917	68	9083	16		4
19	0	5872	45	4128	4685	21	5315	1187	67	8813	15	41	0
	1	6054	45	3946	4597	21	5403	1457	67	8543	14		56
	2	6237	45	3764	4510	21	5490	1728	67	8274	13		52
	3	6418	45	3582	4422	21	5578	1999	67	8004	12		48
	4	6600	45	3400	4334	21	5666	2269	67	7734	11		44
	5	6781	45	3219	4246	21	5754	2535	67	7465	10		40
	6	6963	45	3037	4159	21	5843	2805	67	7195	9		36
	7	7144	45	2856	4070	21	5930	3074	67	6924	8		32
	8	7325	45	2675	3982	21	6018	3343	67	6657	7		28
	9	7507	45	2493	3894	21	6106	3613	67	6387	6		24
	10	7688	45	2312	3805	21	6194	3882	67	6118	5		20
	11	7869	45	2131	3717	21	6282	4151	67	5848	4		16
	12	8050	45	1950	3630	21	6370	4420	67	5579	3		12
	13	8230	45	1770	3541	21	6458	4689	67	5311	2		8
	14	8411	45	1589	3453	21	6547	4958	67	5042	1		4
19	0	9.753511	45	0.24140	9.913314	21	0.086636	9.845227	67	0.154773	0	40	0

350		1° - 15' 1" - 15' 1" - 15'										350	
Hours.	Deg.	L. Sin.	Diff. for 15" or 1'	L. Cossec.	L. Cos.	Diff. for 15" or 1'	L. Sec.	L. Tang.	Diff. for 15" or 1'	L. Cot.	Deg.	Hours.	
20	0	0.75851		0.24140	9.913304	22	0.080636	0.845227		0.154773	10	30	
	4	8772	45	1.28	3270	22	6724	5490	67	450	59	26	
	8	8452	45	1048	3148	22	6812	5704	67	4730	58	22	
	12	8132	45	6862	3018	22	6900	6021	67	387	57	48	
	16	9312	45	6888	3010	22	6990	6302	67	338	56	54	
	20	9492	45	659	2921	22	7079	6571	67	3429	55	40	
	24	9672	45	0428	2833	22	7167	6839	67	3470	54	36	
	28	9852	45	0148	2744	22	7255	7108	67	292	53	32	
	32	760011	45	33069	2655	22	7343	7376	67	2624	52	28	
	36	9241	45	9789	2566	22	7431	7645	67	2335	51	24	
	40	0390	45	9610	2477	22	7523	7913	67	2077	50	20	
	44	0549	45	9431	2388	22	7612	8181	67	1819	49	16	
	48	0748	45	9252	2299	22	7701	8449	67	1551	48	12	
	52	0927	45	9073	2210	22	7790	8717	67	1283	47	8	
	56	1106	45	8894	2121	22	7879	8985	67	1015	46	4	
31	0	1285	45	8715	2031	22	7969	9254	67	0747	45	30	
	4	1464	45	8536	1942	22	8058	9522	67	0479	44	26	
	8	1643	44	8357	1853	22	8147	9790	67	0210	43	22	
	12	1821	44	8179	1763	22	8237	856058	67	147942	42	18	
	16	1999	44	8001	1674	22	8326	0325	67	9675	41	14	
	20	2177	44	7823	1584	21	8415	0593	67	9407	40	10	
	24	2356	44	7644	1495	22	8505	0861	67	9139	39	6	
	28	2534	44	7466	1405	22	8595	1129	67	8871	38	2	
	32	2712	44	7288	1316	22	8684	1396	67	8604	37	22	
	36	2890	44	7111	1225	22	8774	1664	67	8336	36	18	
	40	3067	44	6933	1136	22	8864	1931	67	8068	35	14	
	44	3245	44	6755	1046	22	8954	2199	67	7801	34	10	
	48	3422	44	6578	0956	22	9044	2466	67	7534	33	6	
	52	3600	44	6400	0866	22	9134	2734	67	7266	32	2	
	56	3777	44	6223	0776	22	9224	3001	67	6999	31	4	
32	0	3954	44	6046	0686	22	9314	3268	67	6731	30	30	
	4	4131	44	5869	0596	22	9404	3535	67	6463	29	26	
	8	4309	44	5692	0506	22	9494	3802	67	6196	28	22	
	12	4485	44	5515	0416	22	9584	4069	67	5929	27	18	
	16	4662	44	5338	0325	22	9675	4337	67	5663	26	14	
	20	4839	44	5161	0235	22	9765	4603	67	5397	25	10	
	24	5015	44	4985	0145	22	9855	4870	67	5130	24	6	
	28	5191	44	4809	0054	21	9946	5137	67	4863	23	2	
	32	5367	44	4633	906931	22	00367	5404	67	4596	22	22	
	36	5544	44	4458	9873	23	0127	5671	67	4329	21	18	
	40	5720	44	4280	8779	22	0218	5938	67	4062	20	14	
	44	5896	44	4104	8682	22	0309	6204	67	3794	19	10	
	48	6072	44	3928	8585	22	0400	6471	67	3527	18	6	
	52	6247	44	3751	8488	21	0490	6737	67	3260	17	2	
	56	6423	44	3575	8391	21	0581	7004	67	2993	16	4	
33	0	6598	44	3402	8294	21	0672	7270	60	2725	15	30	
	4	6774	44	3226	8207	21	0763	7537	60	2458	14	26	
	8	6949	44	3051	8116	21	0854	7804	60	2191	13	22	
	12	7124	44	2874	8025	21	0945	8071	60	1924	12	18	
	16	7300	44	2700	8934	21	1036	8338	60	1657	11	14	
	20	7475	44	2523	8843	21	1127	8602	60	1390	10	10	
	24	7651	44	2347	8751	21	1218	8867	60	1122	9	6	
	28	7826	44	2171	8660	21	1310	9131	60	855	8	2	
	32	7999	44	2000	8569	21	1401	9395	60	600	7	22	
	36	8173	44	1827	8478	21	1493	9660	60	344	6	18	
	40	8348	44	1651	8387	21	1584	9924	60	60	5	14	
	44	8522	43	1477	8295	21	1675	860322	60	13097	4	10	
	48	8697	43	1300	8203	21	1767	8484	60	953	3	6	
	52	8871	43	1124	8111	21	1859	0730	60	9270	2	2	
	56	9045	43	0951	8019	21	1951	0130	60	9004	1	4	
34	0	9219	43	0777	7927	21	003912	085123	60	013873	0	30	

TABLE II.—LOG. SINES, TANG'S, &c.

24 36°

P = 15' P = 15' P = 15°

143° 24

H. hrs.	Deg.	L. Sin.	Diff. for 15' or 1"	L. Cos.	L. Cos.	Diff. for 15' or 1"	L. Sec.	L. Tang.	Diff. for 15' or 1"	L. Cot.	Deg.	Hours
24	0	9 70 219	43	0.230781	0.907138	23	0.004942	0.861261	66	0.137339	60	30
	4	930.3	43	0.607	786.6	23	2.1.4	1527	66	8473	59	5
	8	956.6	43	0.434	777.4	23	2.1.26	1792	66	8302	58	52
	12	9740	43	0.25.0	768.2	23	2.1.8	2058	66	8131	57	46
	16	9913	43	0.067	759.0	23	2410	2323	66	7967	56	40
	20	.770087	43	.322913	7498	23	2502	2589	66	7811	55	40
	24	6	43	9740	7406	23	2564	2654	66	7644	54	36
	28	7	43	9567	7314	23	2626	2749	66	7481	53	32
	32	8	43	9394	7221	23	2688	2879	66	7322	52	28
	36	9	43	9221	7129	23	2751	3010	66	7169	51	24
	40	70	43	9048	7037	23	2813	3140	66	7015	50	20
	44	11	43	8875	6945	23	2875	3270	66	6860	49	16
	48	12	43	8703	6852	23	2938	3400	66	6705	48	12
	52	13	43	8530	6760	23	3000	3530	66	6550	47	8
	56	14	43	8357	6667	23	3063	3660	66	6395	46	4
25	0	15	43	8185	6575	23	3125	3790	66	6240	45	35
	4	16	43	8013	6482	23	3188	3920	66	6085	44	30
	8	17	43	7841	6389	23	3251	4050	66	5930	43	26
	12	18	43	7669	6296	23	3314	4180	66	5775	42	22
	16	19	43	7497	6203	23	3377	4310	66	5620	41	18
	20	20	43	7325	6111	23	3440	4440	66	5465	40	14
	24	21	43	7153	6018	23	3503	4570	66	5310	39	10
	28	22	43	6981	5925	23	3566	4700	66	5155	38	6
	32	23	43	6809	5832	23	3629	4830	66	5000	37	2
	36	24	43	6637	5739	23	3692	4960	66	4845	36	0
	40	25	43	6465	5646	23	3755	5090	66	4690	35	0
	44	26	43	6293	5552	23	3818	5220	66	4535	34	0
	48	27	43	6121	5459	23	3881	5350	66	4380	33	0
	52	28	43	5949	5366	23	3944	5480	66	4225	32	0
	56	29	43	5777	5272	23	4007	5610	66	4070	31	0
26	0	30	43	5605	5179	23	4070	5740	66	3915	30	34
	4	31	43	5433	5085	23	4133	5870	66	3760	29	30
	8	32	43	5261	4992	23	4196	6000	66	3605	28	26
	12	33	43	5089	4898	23	4259	6130	66	3450	27	22
	16	34	43	4917	4805	23	4322	6260	66	3295	26	18
	20	35	43	4745	4711	23	4385	6390	66	3140	25	14
	24	36	43	4573	4617	23	4448	6520	66	2985	24	10
	28	37	43	4401	4523	23	4511	6650	66	2830	23	6
	32	38	43	4229	4429	23	4574	6780	66	2675	22	2
	36	39	43	4057	4335	23	4637	6910	66	2520	21	0
	40	40	43	3885	4241	23	4700	7040	66	2365	20	0
	44	41	43	3713	4147	23	4763	7170	66	2210	19	0
	48	42	43	3541	4053	23	4826	7300	66	2055	18	0
	52	43	43	3369	3959	23	4889	7430	66	1900	17	0
	56	44	43	3197	3865	23	4952	7560	66	1745	16	0
27	0	45	43	3025	3771	23	5015	7690	66	1590	15	33
	4	46	43	2853	3677	23	5078	7820	66	1435	14	29
	8	47	43	2681	3583	23	5141	7950	66	1280	13	25
	12	48	43	2509	3489	23	5204	8080	66	1125	12	21
	16	49	43	2337	3395	23	5267	8210	66	970	11	17
	20	50	43	2165	3301	24	5330	8340	66	815	10	13
	24	51	43	1993	3207	24	5393	8470	66	660	9	9
	28	52	43	1821	3113	24	5456	8600	66	505	8	5
	32	53	43	1649	3019	24	5519	8730	66	350	7	1
	36	54	43	1477	2925	24	5582	8860	66	195	6	0
	40	55	43	1305	2831	24	5645	8990	66	40	5	0
	44	56	43	1133	2737	24	5708	9120	66	0	4	0
	48	57	43	961	2643	24	5771	9250	66	0	3	0
	52	58	43	789	2549	24	5834	9380	66	0	2	0
	56	59	43	617	2455	24	5897	9510	66	0	1	0
27	0	0	43	0.220537	0.902349	24	0.097651	0.877114	66	0.122960	0	32

28 138°

P = 4' P = 4'

53° 34

28°		37°		1° - 15' 1" - 15' 1" - 15°		142°		9°					
Hours.	Deg.	L. Sin.	Diff. for 15' or 1"	L. Cossec.	L. Cos.	Diff. for 15' or 1"	L. Sec.	L. Tang.	Diff. for 15' or 1"	L. Cot.	Deg.	Hours.	
28	0	0	9.77463	42	0.92037	9.90234	94	0.00975	9.87714	66	0.12284	31	0
	4	1	9630	42	0370	2.53	94	7747	7377	66	32	50	
	8	2	9708	42	0202	2158	94	7842	7646	66	23.0	58	
	12	3	9660	42	0034	2063	94	7937	7503	66	20.7	57	
	16	4	780133	42	91867	1967	94	8033	8164	66	1834	56	
	20	5	0300	42	9700	1879	94	8128	8428	66	1572	55	
	24	6	0467	42	9533	1776	94	8224	8691	66	130	54	
	28	7	0634	42	9366	1681	94	8319	8953	66	1047	53	
	32	8	0801	42	9199	1585	94	8415	9216	66	8764	52	
	36	9	0368	42	9032	1490	94	8510	9478	66	8522	51	
	40	10	1135	42	8865	1394	94	8606	9741	65	8250	50	
	44	11	1301	42	8699	1298	94	8702	980003	65	11997	49	
	48	12	1467	42	8533	1.02	94	8798	02.5	65	9735	48	
	52	13	1634	42	8366	1106	94	8894	0522	65	9472	47	
	56	14	1800	41	8200	1010	94	8990	0760	65	9210	46	
29	0	15	1966	41	8034	0914	94	9086	1052	65	8945	45	
	4	16	2132	41	7868	0818	94	9182	1314	65	8684	44	
	8	17	2298	41	7702	0722	94	9278	1576	65	8424	43	
	12	14	2464	41	7536	0626	94	9374	1838	65	8162	42	
	16	19	2630	41	7370	05.2	94	9471	2101	65	7900	41	
	20	20	2796	41	7204	0433	94	9567	2363	65	7637	40	
	24	21	2962	41	7038	0337	94	9663	2625	65	7375	39	
	28	22	3127	41	6873	0240	94	9759	2887	65	7113	38	
	32	23	3292	41	6708	0144	94	9856	3148	65	6852	37	
	36	24	3457	41	6543	0047	94	9953	3410	65	65.0	36	
	40	25	3623	41	6377	899851	94	1.00049	3672	65	6328	35	
	44	26	3788	41	6212	8954	94	0146	3934	65	6064	34	
	48	27	3953	41	6047	8917	94	0243	4196	65	5804	33	
	52	28	4118	41	5882	8881	94	0339	4457	65	5543	32	
	56	29	4283	41	5717	8846	94	0436	4719	65	5281	31	
30	0	30	4447	41	5553	8810	94	0533	4980	65	5020	30	
	4	31	4612	41	5388	8774	94	0630	5242	65	4758	29	
	8	32	4776	41	5224	8737	94	0727	5503	65	4497	28	
	12	33	4941	41	5059	8701	94	0824	5765	65	4235	27	
	16	34	5105	41	4895	8665	94	0921	6026	65	3974	26	
	20	35	5269	41	4731	8629	94	1019	6288	65	3712	25	
	24	36	5433	41	4567	8594	94	1116	6549	65	3451	24	
	28	37	5597	41	4403	8559	94	1213	6810	65	3190	23	
	32	38	5761	41	4239	8524	94	1311	7072	65	2928	22	
	36	39	5925	41	4075	8489	94	1408	7333	65	2667	21	
	40	40	6089	41	3911	8454	94	1505	7594	65	2406	20	
	44	41	6253	41	3748	8419	94	1603	7855	65	2145	19	
	48	42	6418	41	3584	8384	94	1701	8117	65	1883	18	
	52	43	6579	41	3421	8349	94	1799	8378	65	1622	17	
	56	44	6743	41	3257	8314	94	1896	8639	65	1361	16	
31	0	45	6906	41	3094	8279	94	1994	8900	65	1100	15	
	4	46	7069	41	2931	8244	94	2092	9161	65	829	14	
	8	47	7232	41	2768	8209	94	2190	9422	65	5578	13	
	12	48	7394	41	2606	8174	95	2288	9682	65	2858	12	
	16	49	7557	41	2443	8139	94	2386	9943	65	0057	11	
	20	50	7720	41	2280	8104	94	2484	990304	65	109796	10	
	24	51	7883	41	2117	8069	95	2582	0465	65	9535	9	
	28	52	8045	41	1955	8034	95	2680	0725	65	9975	8	
	32	53	8208	40	1792	7999	94	2778	0986	65	9914	7	
	36	54	8370	40	1630	7964	95	2877	1247	65	8753	6	
	40	55	8532	40	1468	7929	94	2975	1507	65	8493	5	
	44	56	8694	40	1306	8896	95	3074	1768	65	8232	4	
	48	57	8857	40	1143	8861	95	3172	2029	65	7971	3	
	52	58	9018	40	9992	8826	95	3271	2290	65	7711	2	
	56	59	9180	40	8620	8791	94	3369	2551	65	7451	1	
31	60	60	9342	40	7248	8756	25	0.103468	9.892810	65	0.107190	0	28

TABLE II.—LOG. SINES, TANG'S, &c.

30		30°		1' — 15' 1" — 15'		15°		141°		90			
Hours.	Deg.	L. Sin.	Diff. for 15" or 1'	L. Cosec.	L. Cos.	Diff. for 15" or 1'	L. Sec.	L. Tang.	Diff. for 15" or 1'	L. Cot.	Deg.	Hours.	
34	0	0.789312	40	0.210658	0.995332	25	0.103466	0.892810	65	0.107190	60	27	60
	4	9533	40	04:7	6431	25	3597	3070	65	6930	59		56
	8	9665	40	03:5	6334	25	3366	3331	65	6669	58		52
	12	9827	40	01:7	6236	25	3764	3531	65	6409	57		48
	16	9968	40	00:12	6137	25	3803	3851	65	6149	56		44
	20	79014'	40	903851	6038	25	3962	4111	65	5889	55		40
	24	6310	40	9890	5939	25	4061	4371	65	5629	54		36
	28	0472	40	9528	5840	25	4160	4639	65	5368	53		32
	32	0633	40	9367	5741	25	4259	4892	65	5108	52		28
	36	0793	40	9207	5641	25	4359	5152	65	4848	51		24
	40	0954	40	9046	5542	25	4458	5412	65	4588	50		20
	44	1115	40	8885	5443	25	4557	5672	65	4328	49		16
	48	1275	40	8725	5343	25	4657	5932	65	4068	48		12
	52	1436	40	8564	5244	25	4756	6192	65	3808	47		8
	56	1596	40	8404	5144	25	4856	6452	65	3548	46		4
33	0	1757	40	8243	5045	25	4955	6712	65	3288	45	27	0
	4	1917	40	8083	4945	25	5055	6972	65	3028	44		56
	8	2077	40	7923	4846	25	5154	7231	65	2769	43		52
	12	2237	40	7763	4746	25	5254	7491	65	2509	42		48
	16	2397	40	7603	4646	25	5354	7751	65	2249	41		44
	20	2557	40	7443	4546	25	5454	8011	65	1989	40		40
	24	2716	40	7284	4446	25	5554	8270	65	1730	39		36
	28	2876	40	7124	4346	25	5654	8530	65	1470	38		32
	32	3035	40	6965	4246	25	5754	8789	65	1211	37		28
	36	3195	40	6805	4146	25	5854	9049	65	0951	36		24
	40	3354	40	6646	4046	25	5954	9308	65	0692	35		20
	44	3514	40	6486	3946	25	6054	9568	65	0432	34		16
	48	3673	40	6327	3846	25	6154	9827	65	0173	33		12
	52	3832	40	6168	3745	25	6255	90087	65	0913	32		8
	56	3991	40	6009	3645	25	6355	0346	65	9654	31		4
34	0	4150	40	5850	3544	25	6456	0605	65	9395	30	26	0
	4	4308	40	5692	3444	25	6556	0864	65	9136	29		56
	8	4467	40	5533	3343	25	6657	1124	65	8877	28		52
	12	4626	40	5374	3243	25	6757	1383	65	8618	27		48
	16	4784	40	5216	3142	25	6858	1642	65	8358	26		44
	20	4942	39	5058	3041	25	6959	1901	65	8099	25		40
	24	5101	39	4899	2940	25	7060	2161	65	7839	24		36
	28	5259	39	4741	2839	25	7161	2420	65	7580	23		32
	32	5417	39	4583	2738	25	7262	2679	65	7321	22		28
	36	5575	39	4425	2637	25	7363	2938	65	7062	21		24
	40	5733	39	4267	2536	25	7464	3197	65	6803	20		20
	44	5891	39	4109	2435	25	7565	3456	65	6544	19		16
	48	6049	39	3951	2334	25	7666	3715	65	6285	18		12
	52	6206	39	3794	2233	25	7767	3973	65	6027	17		8
	56	6364	39	3636	2132	25	7868	4232	65	5768	16		4
35	0	6521	39	3479	2030	25	7970	4491	65	5509	15	25	0
	4	6679	39	3321	1929	25	8071	4750	65	5250	14		56
	8	6836	39	3164	1827	25	8173	5009	65	4991	13		52
	12	6993	39	3007	1726	25	8274	5267	65	4733	12		48
	16	7150	39	2850	1624	25	8376	5526	65	4474	11		44
	20	7307	39	2693	1523	25	8477	5784	64	4216	10		40
	24	7464	39	2536	1421	25	8579	6043	65	3957	9		36
	28	7621	39	2379	1319	25	8681	6302	65	3698	8		32
	32	7777	39	2222	1217	25	8783	6560	65	3440	7		28
	36	7934	39	2066	1115	25	8885	6819	65	3181	6		24
	40	8090	39	1910	1013	25	8987	7077	65	2923	5		20
	44	8247	39	1753	911	25	9089	7336	65	2664	4		16
	48	8404	39	1597	809	25	9191	7594	65	2407	3		12
	52	8560	39	1440	707	25	9293	7853	65	2147	2		8
	56	8716	39	1284	605	25	9395	8111	64	1889	1		4
	60	0.793672	39	0.201129	0.995053	25	0.103477	0.995367	65	0.091631	0	24	0

TABLE II.—LOG. SINES, TANG'S, &c.

39°		1° = 15' 1" = 15' 1" = 15°										140°	
Hours.	D: g.	L. Sin.	Diff. for 15' or 1"	L. Cossec.	L. Cos.	Diff. for 15' or 1"	L. Sec.	L. Tang.	Diff. for 15' or 1"	L. Cot.	Deg.	Hours.	
36	0	0	9.79872	3.	0.20112	0.890503	26	0.109497	0.903369	65	0.091031	60	23
	4	1	9029	39	0.772	0.400	25	9600	8628	64	1772	59	5
	8	2	9184	39	0816	02.28	26	9702	8686	64	1114	58	5
	12	3	9339	39	0861	01.95	25	9805	9144	64	0257	57	4
	16	4	9495	39	0905	00.93	26	9907	9402	64	0390	56	4
	20	5	9651	39	0349	089991	25	110009	9660	64	0340	55	4
	24	6	9806	39	0194	9888	26	0112	9918	64	0082	54	3
	28	7	9962	39	0039	9785	26	0215	010177	64	08922	53	2
	32	8	000117	39	199683	9682	26	0318	0435	64	9565	52	2
	36	9	0272	39	9728	9579	26	0421	0893	64	9307	51	2
	40	10	0427	39	9573	9476	26	0524	0951	64	9049	50	2
	44	11	0582	39	9418	9373	26	0627	1309	64	8791	49	1
	48	12	0737	39	9263	9270	26	0730	1467	64	8533	48	1
	52	13	0892	39	9108	9167	26	0833	1725	64	8272	47	1
	56	14	1047	39	8953	9064	26	0936	1983	64	8017	46	1
37	0	15	1201	39	8799	8961	26	1030	2240	64	7760	45	23
	4	16	1356	39	8644	8858	26	1142	2498	64	7502	44	5
	8	17	1511	39	8489	8755	26	1245	2756	64	7244	43	5
	12	18	1665	39	8335	8651	26	1349	3014	64	6986	42	4
	16	19	1819	39	8181	8548	26	1452	3271	64	6729	41	4
	20	20	1973	39	8027	8444	26	1556	3529	64	6471	40	4
	24	21	2128	39	7872	8341	26	1659	3787	64	6213	39	3
	28	22	2282	39	7718	8237	26	1763	4045	64	5955	38	2
	32	23	2436	39	7564	8134	26	1866	4302	64	5697	37	2
	36	24	2590	39	7410	8030	26	1970	4560	64	5440	36	2
	40	25	2743	39	7257	7926	26	2074	4817	64	5183	35	2
	44	26	2897	39	7103	7822	26	2178	5075	64	4925	34	1
	48	27	3050	39	6950	7718	26	2282	5332	64	4667	33	1
	52	28	3204	39	6796	7614	26	2386	5590	64	4410	32	1
	56	29	3357	39	6643	7510	26	2490	5847	64	4153	31	1
38	0	30	3511	39	6489	7406	26	2594	6105	64	3895	30	23
	4	31	3664	39	6336	7302	26	2698	6362	64	3637	29	5
	8	32	3817	39	6183	7198	26	2802	6619	64	3381	28	5
	12	33	3970	39	6030	7093	26	2907	6877	64	3123	27	4
	16	34	4123	39	5877	6989	26	3011	7134	64	2866	26	4
	20	35	4276	39	5724	6885	26	3115	7391	64	2607	25	4
	24	36	4428	39	5572	6780	26	3220	7648	64	2349	24	3
	28	37	4581	39	5419	6676	26	3324	7905	64	2092	23	3
	32	38	4734	39	5266	6571	26	3429	8163	64	1837	22	2
	36	39	4886	39	5114	6466	26	3534	8420	64	1580	21	2
	40	40	5039	39	4961	6362	26	3638	8677	64	1323	20	2
	44	41	5191	39	4809	6257	26	3743	8934	64	1066	19	1
	48	42	5343	39	4657	6152	26	3848	9191	64	809	18	1
	52	43	5495	39	4505	6047	26	3953	9448	64	552	17	1
	56	44	5647	39	4353	5942	26	4058	9705	64	295	16	1
39	0	45	5799	39	4201	5837	26	4163	9962	64	0039	15	21
	4	46	5951	39	4049	5732	26	4268	02019	64	072781	14	5
	8	47	6103	39	3897	5627	26	4373	0476	64	9324	13	5
	12	48	6254	39	3746	5521	26	4479	0733	64	9867	12	4
	16	49	6406	39	3594	5416	26	4584	0990	64	9010	11	4
	20	50	6558	39	3442	5311	26	4689	1247	64	8753	10	4
	24	51	6709	39	3291	5206	26	4794	1503	64	8497	9	3
	28	52	6860	39	3140	5100	26	4900	1760	64	8240	8	2
	32	53	7011	39	2988	4994	26	5006	2017	64	7983	7	2
	36	54	7163	39	2837	4889	26	5111	2274	64	7726	6	2
	40	55	7314	39	2686	4783	26	5217	2531	64	7469	5	2
	44	56	7465	39	2535	4678	26	5322	2787	64	7213	4	1
	48	57	7616	39	2384	4572	26	5428	3044	64	6957	3	1
	52	58	7766	39	2234	4466	26	5534	3300	64	6700	2	1
	56	59	7917	39	2083	4360	26	5640	3557	64	6443	1	1
39	60	60	8068	39	0.191932	0.894254	26	0.115746	0.923814	64	0.076186	0	20

130°

1° = 4' 10" = 4"

50°

TABLE II.—LOG. SINES, TANG'S, &c.

20 40°

P = 15' 1" = 15' 1" = 150

130° 0'

Hours.	Deg.	L. Sin.	Diff. for 15" or 1'	L. Cosec.	L. Cos.	Diff. for 15" or 1'	L. Sec.	L. Tang.	Diff. for 15" or 1'	L. Cot.	Deg.	Hours.
40	0	0.9030.8	37	0.191332	0.684954	96	0.115748	9.923814	64	0.076186	60	19
	4	.8818	37	.1768	.4148	96	.5552	.4070	64	.5930	5	0
	8	.8369	37	.1631	.4042	93	.5154	.4327	64	.5673	5	3
	12	.8519	37	.1481	.3936	93	.6064	.4583	64	.5417	5	5
	16	.8669	37	.1331	.3829	98	.6171	.4840	64	.4160	5	11
	20	.8819	37	.1181	.3723	96	.6277	.5096	64	.5904	5	16
	24	.8969	37	.1031	.3617	97	.6383	.5353	64	.4648	5	20
	28	.9119	37	.0881	.3510	97	.6490	.5609	64	.4391	5	23
	32	.9269	37	.0731	.3404	97	.6596	.5865	64	.4135	5	25
	36	.9419	37	.0581	.3297	97	.6703	.6122	64	.3878	5	28
	40	.9569	37	.0431	.3191	97	.6809	.6378	64	.3622	5	30
	44	.9718	37	.0282	.3085	97	.6916	.6634	64	.3366	4	16
	48	.9868	37	.0132	.2978	97	.7022	.6890	64	.3110	4	18
	52	.10017	37	.18943	.2871	97	.7128	.7146	64	.2854	4	21
	56	.0167	37	.9333	.2764	97	.7236	.7403	64	.2597	4	24
41	0	.0316	37	.9684	.2657	97	.7343	.765	64	.2341	4	19
	4	.0463	37	.9533	.2550	97	.7450	.7915	64	.2085	4	20
	8	.0614	37	.9386	.2443	97	.7557	.8171	64	.1829	4	23
	12	.0763	37	.9238	.2336	97	.7664	.8427	64	.1573	4	25
	16	.0912	37	.9084	.2229	97	.7771	.8683	64	.1317	4	28
	20	.1061	37	.8939	.2121	97	.7879	.8940	64	.1060	4	30
	24	.1210	37	.8790	.2014	97	.7986	.9196	64	.0804	3	16
	28	.1354	37	.8643	.1907	97	.8093	.9451	64	.0548	3	19
	32	.1507	37	.8493	.1799	97	.8201	.9708	64	.0292	3	22
	36	.1656	37	.8344	.1692	97	.8308	.9964	64	.0036	3	24
	40	.1804	37	.8196	.1584	97	.8416	.030220	64	.069780	3	20
	44	.1952	37	.8048	.1477	97	.8523	.0475	64	.8525	3	16
	48	.2100	37	.7900	.1370	97	.8631	.0731	64	.9289	3	18
	52	.2248	37	.7752	.1263	97	.8739	.0987	64	.9013	2	8
	56	.2396	37	.7604	.1155	97	.8847	.1243	64	.8737	2	4
42	0	.2544	37	.7456	.1048	97	.8955	.1499	64	.8501	2	18
	4	.2692	37	.7308	.0940	97	.9063	.1755	64	.8245	2	20
	8	.2840	37	.7160	.0833	97	.9170	.2010	64	.7990	2	23
	12	.2988	37	.7011	.0725	97	.9278	.2266	64	.7734	2	25
	16	.3135	37	.6863	.0618	97	.9387	.2522	64	.7478	2	28
	20	.3283	37	.6717	.0510	97	.9495	.2778	64	.7222	2	30
	24	.3430	37	.6570	.0403	97	.9603	.3033	64	.6967	2	16
	28	.3578	37	.6423	.0295	97	.9711	.3289	64	.6711	2	19
	32	.3725	37	.6275	.0188	97	.9820	.3545	64	.6455	2	22
	36	.3873	37	.6128	.0079	97	.9928	.3800	64	.6200	2	24
	40	.4019	37	.5981	.0073969	97	.100037	.4056	64	.5944	2	30
	44	.4166	37	.5834	.9655	97	.0145	.4311	64	.5689	1	16
	48	.4313	37	.5687	.9746	97	.0254	.4567	64	.5433	1	18
	52	.4460	37	.5540	.9837	97	.0363	.4823	64	.5177	1	21
	56	.4607	37	.5393	.9928	97	.0471	.5078	64	.4922	1	24
43	0	.4753	37	.5247	.9490	97	.0580	.5333	64	.4667	1	17
	4	.4900	37	.5100	.9311	97	.0689	.5599	64	.4411	1	18
	8	.5048	37	.4954	.9202	97	.0798	.5844	64	.4156	1	20
	12	.5195	37	.4807	.9033	97	.0907	.6100	64	.3900	1	23
	16	.5343	37	.4661	.8984	97	.1016	.6355	64	.3645	1	24
	20	.5495	37	.4515	.8875	97	.1125	.6610	64	.3390	1	40
	24	.5646	36	.4368	.8766	97	.1234	.6866	64	.3134	9	36
	28	.5798	36	.4222	.8657	97	.1343	.7121	64	.2879	8	32
	32	.5953	36	.4076	.8547	97	.1453	.7377	64	.2623	7	28
	36	.6070	36	.3930	.8438	97	.1562	.7632	64	.2368	6	24
	40	.6215	36	.3785	.8328	97	.1672	.7887	64	.2113	5	20
	44	.6361	36	.3639	.8219	97	.1781	.8142	64	.1858	4	16
	48	.6507	36	.3493	.8110	97	.1891	.8398	64	.1603	3	12
	52	.6653	36	.3348	.7999	97	.2001	.8653	64	.1347	2	8
	56	.6798	36	.3202	.7890	97	.2110	.8908	64	.1092	1	4
43	40	.6816942	36	.0.183057	.9.877780	97	.0.123220	.9.929163	64	.0.080837	0	16

Hours. Deg. L. Cos. Diff. for 15" or 1' L. Sec. L. Sin. Diff. for 15" or 1' L. Cosec. L. Cot. Diff. for 15" or 1' L. Tang. Deg. Hours.

72		41°		1° - 15' 1" - 15' 1" - 15°						138°		9	
Hours.	Deg.	L. Sin.	Diff. for 15' or 1"	L. Cosec.	L. Cos.	Diff. for 15' or 1"	L. Sec.	L. Tang.	Diff. for 15' or 1"	L. Cot.	Deg.	Sec.	
14	0	0	9.818943	0.183057	9.877780	97	0.129230	9.939163	64	0.066047	69	15	
	4	1	7038	2912	7670	97	2330	9418	64	0523	30		
	8	2	7233	2767	7560	97	2440	9673	64	0327	36		
	12	3	7378	2632	7450	97	2550	9928	64	0172	37		
	16	4	7524	2476	7340	97	2660	940184	64	059016	38		
	20	5	7669	2331	7230	97	2770	0439	64	8361	35		
	24	6	7813	2187	7120	97	2880	0693	64	8207	34		
	28	7	7958	2042	7010	97	2990	0048	64	9052	33		
	32	8	8103	1897	6899	98	3101	1904	64	8796	32		
	36	9	8247	1753	6789	98	3211	1458	64	8542	31		
	40	10	8392	1608	6678	98	3322	1714	64	8294	30		
	44	11	8536	1464	6568	98	3432	1968	64	8052	29		
	48	12	8681	1319	6458	98	3542	2223	64	7777	28		
	52	13	8825	1175	6347	98	3653	2478	64	7522	27		
	56	14	8969	1031	6236	98	3764	2733	64	7267	26		
	15	0	15	9113	0887	6125	98	3875	2988	64	7012	45	15
4		16	9257	0743	6014	98	3986	3243	64	6757	44		
8		17	9401	0599	5904	98	4096	3497	64	6502	43		
12		18	9545	0455	5793	98	4207	3752	64	6248	42		
16		19	9689	0311	5682	98	4318	4007	64	5993	41		
20		20	9833	0167	5571	98	4429	4262	64	5738	40		
24		21	9977	0024	5459	98	4541	4517	64	5483	39		
28		22	820120	179880	5348	98	4652	4772	64	5228	38		
32		23	0263	9737	5237	98	4763	5027	64	4974	37		
36		24	0406	9594	5125	98	4875	5281	64	4719	36		
40		25	0550	9450	5014	98	4986	5536	64	4464	35		
44		26	0693	9307	4903	98	5097	5790	64	4210	34		
48		27	0836	9164	4791	98	5209	6045	64	3956	33		
52		28	0979	9021	4680	98	5320	6299	64	3701	32		
56		29	1122	8878	4568	98	5432	6554	64	3446	31		
16		0	30	1265	8735	4456	98	5544	6809	64	3191	30	16
	4	31	1407	8593	4344	98	5656	7063	64	2937	29		
	8	32	1550	8450	4232	98	5768	7317	64	2682	28		
	12	33	1693	8307	4121	98	5879	7572	64	2427	27		
	16	34	1835	8165	4009	98	5991	7826	64	2172	26		
	20	35	1978	8022	3897	98	6103	8081	64	1917	25		
	24	36	2120	7880	3784	98	6216	8336	64	1662	24		
	28	37	2262	7738	3672	98	6328	8590	64	1408	23		
	32	38	2404	7596	3560	98	6440	8844	64	1154	22		
	36	39	2546	7454	3447	98	6553	9099	64	9001	21		
	40	40	2688	7312	3335	98	6665	9353	64	6647	20		
	44	41	2830	7170	3223	98	6777	9607	63	6393	19		
	48	42	2972	7028	3110	98	6890	9862	63	6138	18		
	52	43	3114	6886	2998	98	7002	950116	63	042024	17		
	56	44	3256	6744	2885	98	7115	0371	63	9621	16		
	17	0	45	3397	6603	2772	98	7228	0625	63	9375	15	17
4		46	3538	6462	2659	98	7341	0879	63	9121	14		
8		47	3680	6320	2547	98	7453	1133	63	8867	13		
12		48	3821	6179	2434	98	7566	1387	63	8613	12		
16		49	3963	6037	2321	98	7679	1642	63	8358	11		
20		50	4104	5896	2208	98	7792	1896	63	8104	10		
24		51	4245	5755	2095	98	7905	2150	63	7850	9		
28		52	4386	5614	1981	98	8019	2405	63	7595	8		
32		53	4527	5473	1868	98	8132	2659	63	7341	7		
36		54	4668	5332	1755	98	8245	2913	63	7087	6		
40		55	4809	5192	1641	98	8359	3167	63	6833	5		
44		56	4949	5051	1528	98	8472	3421	63	6579	4		
48		57	5090	4910	1415	98	8585	3675	63	6325	3		
52		58	5230	4770	1301	98	8699	3929	63	6071	2		
56		59	5370	4630	1187	98	8813	4183	63	5817	1		
17		60	0	9.825511	0.174489	9.871074	98	0.128926	9.954437	63	0.065513	0	17

43°		1° - 15' 1° - 15' 1° - 15°						137°		9°			
ours.	Deg.	L. Sin.	Diff. for 15' or 1'	L. Cos.ec.	L. Cos.	Diff. for 15' or 1'	L. Sec.	L. Tang.	Diff. for 15' or 1'	L. Cot.	Deg.	Hours.	
8	0	9.825511	35	0.174489	9.871074	28	0.128926	9.954437	63	0.045503	69	11	50
	1	3651	35	4349	0890	28	3040	4091	63	5309	59		56
	2	3791	35	4269	0846	28	9154	4045	63	5055	58		52
	3	3931	35	4069	0792	28	9268	5199	63	4801	57		48
	4	4072	35	3925	0618	28	9392	5454	63	4546	56		44
	5	4211	35	3739	0504	28	9496	5707	63	4293	55		40
	6	4351	35	3649	0390	28	9610	5961	63	4039	54		36
	7	4491	35	3509	0276	28	9724	6215	63	3785	53		32
	8	4630	35	3370	0161	28	9839	6469	63	3531	52		28
	9	4770	35	3230	0047	28	9953	6723	63	3277	51		24
	10	4910	35	3090	869933	29	130067	6977	63	3023	50		20
	11	5049	35	2951	9818	29	0182	7231	63	2769	49		16
	12	5189	35	2811	9704	29	0296	7485	63	2515	48		12
	13	5328	35	2672	9589	29	0411	7739	63	2261	47		8
	14	5467	35	2533	9474	29	0526	7993	63	2007	46		4
10	0	5606	35	2394	9360	29	0640	8246	63	1754	45	11	0
	1	5745	35	2255	9245	29	0755	8500	63	1500	44		56
	2	5884	35	2116	9130	29	0870	8754	63	1246	43		52
	3	6023	35	1977	9015	29	0985	9008	63	0992	42		48
	4	6162	35	1838	8900	29	1100	9262	63	0738	41		44
	5	6301	35	1699	8785	29	1215	9516	63	0484	40		40
	6	6440	35	1561	8670	29	1330	9769	63	0231	39		36
	7	6579	35	1422	8555	29	1445	060023	63	030977	38		32
	8	6718	35	1284	8439	29	1561	0277	63	0723	37		28
	9	6857	34	1145	8324	29	1676	0531	63	0469	36		24
	10	6996	34	1007	8209	29	1791	0784	63	0216	35		20
	11	7135	34	869	8093	29	1907	1038	63	0962	34		16
	12	7274	34	731	7978	29	2022	1291	63	0709	33		12
	13	7413	34	593	7862	29	2138	1545	63	0455	32		8
	14	7552	34	454	7747	29	2253	1799	63	0201	31		4
50	0	7691	34	317	7631	29	2369	2054	63	0747	30	10	0
	1	7830	34	179	7515	29	2485	2308	63	0494	29		56
	2	7969	34	41	7399	29	2601	2562	63	0240	28		52
	3	8108	34	109964	7283	29	2717	2816	63	07187	27		48
	4	8247	34	9766	7167	29	2833	3067	63	0463	26		44
	5	8386	34	8628	7051	29	2949	3321	63	0679	25		40
	6	8525	34	7491	6935	29	3065	3574	63	0426	24		36
	7	8664	34	6354	6819	29	3181	3827	63	0173	23		32
	8	8803	34	5216	6703	29	3297	4081	63	0919	22		28
	9	8942	34	4079	6588	29	3414	4335	63	0665	21		24
	10	9081	34	2942	6472	29	3530	4588	63	0412	20		20
	11	9220	34	1805	6357	29	3647	4842	63	0158	19		16
	12	9359	34	668	6242	29	3763	5095	63	0905	18		12
	13	9498	34	8531	6127	29	3880	5349	63	0651	17		8
	14	9637	34	7394	6012	29	3996	5602	63	0398	16		4
51	0	9776	34	6257	5897	29	4113	5855	63	0145	15	9	0
	1	9915	34	5121	5782	29	4230	6109	63	0891	14		56
	2	10054	34	3985	5667	29	4347	6362	63	0638	13		52
	3	10193	34	2848	5552	29	4464	6616	63	0384	12		48
	4	10332	34	1712	5437	29	4581	6869	63	0131	11		44
	5	10471	34	575	5322	29	4698	7123	63	0877	10		40
	6	10610	34	7439	5207	29	4815	7376	63	0624	9		36
	7	10749	34	6303	5092	29	4932	7629	63	0371	8		32
	8	10888	34	5167	4977	29	5049	7883	63	0117	7		28
	9	11027	34	4031	4862	29	5166	8136	63	0864	6		24
	10	11166	34	2895	4747	29	5284	8389	63	0611	5		20
	11	11305	34	1759	4632	29	5402	8643	63	0357	4		16
	12	11444	34	623	4517	29	5520	8896	63	0104	3		12
	13	11583	34	6488	4402	29	5637	9149	63	0851	2		8
	14	11722	34	5352	4287	29	5755	9403	63	0597	1		4
	15	11861	34	4216	4172	29	5873	9656	63	0344	0	8	0
Hours.	Deg.	L. Cos.	Diff. for 15' or 1'	L. Sec.	L. Sin.	Diff. for 15' or 1'	L. Cos.ec.	L. Cot.	Diff. for 15' or 1'	L. Tang.	Deg.	Hours.	

2° 43°

1' - 15' 1" - 15' 1" - 15°

136°

Hours.	Deg.	L. Sin.	Diff. for 15" or 1'	L. Cossec.	L. Cos.	Diff. for 15" or 1'	L. Sec.	L. Tang.	Diff. for 15" or 1'	L. Cot.	Deg.	Hours.
52	0 0	9.833783		0.160217	9.864127		0.135873	9.906656		0.030344	60	7
	4 1	3019	34	6081	4010	29	5990	9609	63	0091	59	
	8 2	4054	34	5946	3892	29	6108	970162	63	028328	58	
	12 3	4190	34	5810	3774	29	6220	0419	63	9584	57	
	16 4	4325	34	5675	3650	29	6344	0069	63	9331	56	
	20 5	4460	34	5540	3538	30	6462	0022	63	9078	55	
	24 6	4595	34	5405	3419	30	6581	1177	63	8824	54	
	28 7	4730	34	5270	3301	29	6699	1429	63	8571	53	
	32 8	4865	34	5135	3183	29	6817	1682	63	8318	52	
	36 9	4999	34	5001	3064	29	6936	1935	63	8065	51	
	40 10	5134	34	4866	2946	30	7054	2188	63	7812	50	
	44 11	5269	34	4731	2828	30	7172	2441	63	7559	49	
	48 12	5403	34	4597	2709	29	7291	2694	63	7306	48	
	52 13	5538	34	4462	2590	30	7410	2948	63	7052	47	
	56 14	5672	34	4328	2471	30	7529	3201	63	6799	46	
53	0 15	5807	33	4193	2353	30	7647	3454	63	6546	45	7
	4 16	5941	33	4059	2234	30	7766	3707	63	6293	44	
	8 17	6075	33	3925	2115	30	7885	3960	63	6040	43	
	12 18	6209	33	3791	1996	30	8004	4213	63	5787	42	
	16 19	6343	33	3657	1877	30	8123	4466	63	5534	41	
	20 20	6477	33	3523	1758	30	8242	4719	63	5281	40	
	24 21	6611	33	3389	1639	30	8362	4972	63	5027	39	
	28 22	6745	33	3255	1519	30	8481	5226	63	4774	38	
	32 23	6879	33	3121	1400	30	8600	5479	63	4521	37	
	36 24	7012	33	2988	1280	30	8720	5732	63	4268	36	
	40 25	7146	33	2854	1161	30	8839	5985	63	4015	35	
	44 26	7279	33	2721	1041	30	8959	6238	63	3762	34	
	48 27	7413	33	2587	922	30	9078	6491	63	3509	33	
	52 28	7546	33	2454	802	30	9198	6744	63	3256	32	
	56 29	7679	33	2321	683	30	9318	6997	63	3003	31	
54	0 30	7812	33	2188	562	30	9438	7250	63	2750	30	8
	4 31	7945	33	2055	442	30	9558	7503	63	2497	29	
	8 32	8078	33	1922	322	30	9678	7756	63	2244	28	
	12 33	8211	33	1789	202	30	9798	8009	63	1991	27	
	16 34	8344	33	1656	82	30	9918	8262	63	1738	26	
	20 35	8477	33	1523	59902	30	10038	8515	63	1485	25	
	24 36	8610	33	1390	9842	30	10158	8768	63	1232	24	
	28 37	8742	33	1258	9721	30	10279	9021	63	0979	23	
	32 38	8875	33	1125	9601	30	10399	9274	63	0726	22	
	36 39	9007	33	993	9480	30	10520	9527	63	0473	21	
	40 40	9140	33	860	9360	30	10640	9780	63	0220	20	
	44 41	9272	33	728	9239	30	10761	10033	63	01967	19	
	48 42	9404	33	596	9118	30	10882	10286	63	9714	18	
	52 43	9536	33	464	8998	30	11002	10538	63	9402	17	
	56 44	9668	33	332	8877	30	11123	10791	63	9090	16	
55	0 45	9800	33	200	8756	30	11244	11044	63	8956	15	5
	4 46	9932	33	669	8635	30	11365	11297	63	8703	14	
	8 47	840044	31	159936	8514	30	11486	11550	63	8450	13	
	12 48	0196	33	984	8393	30	11607	11803	63	8197	12	
	16 49	0328	33	9672	8272	30	11728	12056	63	7944	11	
	20 50	0459	33	9541	8150	30	11850	12309	63	7691	10	
	24 51	0591	33	9409	8029	30	11971	12562	63	7438	9	
	28 52	0722	33	9278	7908	30	12092	12815	63	7185	8	
	32 53	0853	33	9147	7786	30	12214	13068	63	6932	7	
	36 54	0985	33	9015	7665	30	12335	13321	63	6680	6	
	40 55	1116	33	8884	7543	30	12457	13574	63	6427	5	
	44 56	1247	33	8753	7421	30	12579	13827	63	6174	4	
	48 57	1379	33	8621	7300	30	12700	14080	63	5921	3	
	52 58	1510	33	8490	7178	30	12822	14333	63	5668	2	
	56 59	1640	33	8360	7056	30	12944	14586	63	5415	1	
55 00	60	9.841771		0.158229	9.856934		0.143066	9.984627		0.015162	0	4

88 133°

1' - 1' 15" - 1'

140° 3'

TABLE II.—LOG. SINES, TANG'S, &c.

29 44°			1' — 15'			1° — 15'			15° — 9°			
Hours.	Deg.	L. Sin.	Diff. for 15" or 1'	L. Cos.	L. Cos.	Diff. for 15" or 1'	L. Sec.	L. Tang.	Diff. for 15" or 1'	L. Cot.	Deg.	Hours.
56	0	0.841771	33	0.158229	0.856934	30	0.143066	0.984837	63	0.015163	60	3
	1	1902	33	8098	6812	30	3188	5000	63	4910	59	
	2	2033	33	7987	6690	30	3310	5343	63	4657	58	
	3	2164	33	7836	6568	30	3439	5596	63	4404	57	
	4	2294	33	7708	6446	30	3554	5848	63	4159	56	
	5	2424	33	7576	6323	31	3677	6101	63	3909	55	
	6	2555	33	7445	6201	31	3799	6354	63	3646	54	
	7	2685	33	7315	6078	31	3923	6607	63	3393	53	
	8	2816	33	7184	5956	31	4044	6860	63	3140	52	
	9	2946	33	7054	5834	31	4166	7112	63	2888	51	
	10	3076	33	6924	5711	31	4289	7365	63	2635	50	
	11	3206	33	6794	5588	31	4412	7618	63	2382	49	
	12	3336	33	6664	5465	31	4535	7871	63	2129	48	
	13	3465	33	6535	5342	31	4658	8123	63	1877	47	
	14	3595	33	6405	5219	31	4781	8376	63	1624	46	
57	0	3725	33	6275	5096	31	4904	8629	63	1371	45	3
	1	3855	33	6145	4973	31	5027	8882	63	1118	44	
	2	3984	33	6016	4850	31	5150	9134	63	866	43	
	3	4114	33	5886	4727	31	5273	9387	63	613	42	
	4	4244	33	5757	4603	31	5397	9640	63	360	41	
	5	4373	33	5627	4480	31	5520	9892	63	107	40	
	6	4502	33	5498	4357	31	5643	10145	63	154	39	
	7	4631	33	5369	4233	31	5767	10398	63	100	38	
	8	4760	33	5240	4109	31	5891	10651	63	55	37	
	9	4889	33	5111	3986	31	6014	10903	63	10	36	
	10	5018	33	4982	3862	31	6138	11156	63	34	35	
	11	5147	33	4853	3738	31	6262	11409	63	1	34	
	12	5276	33	4724	3614	31	6386	11662	63	533	33	
	13	5404	33	4596	3490	31	6510	11914	63	1066	32	
	14	5533	33	4467	3366	31	6634	12167	63	1600	31	
58	0	5662	33	4338	3242	31	6758	12420	63	2134	30	3
	1	5790	33	4210	3118	31	6882	12672	63	1668	29	
	2	5919	33	4081	2994	31	7006	12925	63	1202	28	
	3	6047	33	3953	2869	31	7131	13178	63	736	27	
	4	6175	33	3825	2745	31	7255	13430	63	270	26	
	5	6303	33	3697	2620	31	7380	13683	63	104	25	
	6	6432	33	3568	2496	31	7504	13936	63	154	24	
	7	6560	33	3440	2371	31	7629	14189	63	103	23	
	8	6688	33	3312	2247	31	7753	14441	63	55	22	
	9	6816	33	3184	2123	31	7878	14694	63	5	21	
	10	6944	33	3056	1998	31	8003	14947	63	249	20	
	11	7071	33	2929	1873	31	8128	15199	63	300	19	
	12	7198	33	2801	1747	31	8253	15452	63	351	18	
	13	7327	33	2673	1622	31	8378	15705	63	402	17	
	14	7454	33	2546	1497	31	8503	15957	63	454	16	
59	0	7582	33	2418	1372	31	8628	16210	63	505	15	1
	1	7709	33	2291	1246	31	8754	16462	63	557	14	
	2	7836	33	2164	1121	31	8879	16715	63	608	13	
	3	7964	33	2036	996	31	9004	16968	63	659	12	
	4	8091	33	1909	870	31	9130	17221	63	711	11	
	5	8218	33	1782	745	31	9255	17473	63	762	10	
	6	8345	33	1655	619	31	9381	17726	63	813	9	
	7	8472	33	1528	493	31	9507	17979	63	864	8	
	8	8599	33	1401	368	31	9632	18232	63	915	7	
	9	8726	33	1274	242	31	9758	18484	63	966	6	
	10	8853	33	1147	116	31	9884	18737	63	1017	5	
	11	8979	33	1021	90	31	10010	18990	63	1068	4	
	12	9106	33	894	64	31	10136	19243	63	1119	3	
	13	9232	33	768	38	31	10262	19496	63	1170	2	
	14	9359	33	642	12	31	10389	19749	63	1221	1	
59 50	0	9485	33	0.150515	0.849485	31	0.150515	0.000000	63	0.000000	0	0
Hours.	Deg.	L. Cos.	Diff. for 15" or 1'	L. Sec.	L. Sin.	Diff. for 15" or 1'	L. Cos.	L. Cot.	Diff. for 15" or 1'	L. Tang.	Deg.	Hours.

TABLE III.

PROPORTIONAL LOGARITHMS.

TABLE III.—PROPORTIONAL LOGARITHMS.

"	0° 0'	0° 1'	0° 2'	0° 3'	0° 4'	0° 5'	0° 6'	0° 7'	0° 8'	"
0		2.2553	1.9542	1.7782	1.6332	1.5583	1.4771	1.4102	1.3522	0
1	4.0334	.9481	.9506	.7757	.6514	.5549	.4759	.4091	.3513	1
2	3.7394	.9410	.9471	.7734	.6496	.5534	.4747	.4081	.3504	2
3	.5663	.9341	.9435	.7710	.6478	.5520	.4735	.4071	.3493	3
4	.4214	.9272	.9400	.7686	.6460	.5506	.4723	.4061	.3486	4
5	.3245	.9205	.9365	.7663	.6443	.5491	.4711	.4050	.3477	5
6	.2553	.9139	.9331	.7639	.6425	.5477	.4699	.4040	.3468	6
7	.1883	.9073	.9296	.7616	.6407	.5463	.4688	.4030	.3459	7
8	.1303	.9009	.9262	.7593	.6390	.5449	.4676	.4020	.3450	8
9	.0793	.8946	.9228	.7570	.6372	.5435	.4664	.4010	.3441	9
10	.0334	.8883	.9195	.7547	.6355	.5421	.4652	.4000	.3432	10
11	2.9920	.8822	.9162	.7524	.6338	.5407	.4640	.3989	.3423	11
12	.8542	.8761	.9128	.7501	.6320	.5393	.4628	.3979	.3415	12
13	.8195	.8701	.9096	.7479	.6303	.5379	.4617	.3969	.3406	13
14	.8673	.8642	.9063	.7456	.6286	.5365	.4606	.3959	.3397	14
15	.8573	.8584	.9031	.7434	.6269	.5351	.4594	.3949	.3388	15
16	.8293	.8526	.8999	.7412	.6252	.5337	.4582	.3939	.3379	16
17	.8030	.8469	.8967	.7390	.6235	.5324	.4571	.3929	.3371	17
18	.7782	.8413	.8935	.7368	.6218	.5310	.4559	.3919	.3362	18
19	.7547	.8358	.8904	.7346	.6201	.5296	.4548	.3910	.3353	19
20	.7394	.8303	.8873	.7324	.6185	.5283	.4536	.3900	.3345	20
21	.7112	.8249	.8842	.7302	.6168	.5269	.4525	.3890	.3336	21
22	.6910	.8196	.8811	.7281	.6151	.5256	.4514	.3880	.3327	22
23	.6717	.8143	.8781	.7259	.6135	.5242	.4502	.3870	.3319	23
24	.6532	.8091	.8751	.7238	.6118	.5228	.4491	.3860	.3310	24
25	.6355	.8040	.8721	.7217	.6102	.5215	.4480	.3851	.3302	25
26	.6185	.8000	.8691	.7196	.6085	.5202	.4468	.3841	.3293	26
27	.6021	.7960	.8661	.7175	.6069	.5189	.4457	.3831	.3284	27
28	.5863	.7920	.8632	.7154	.6053	.5174	.4446	.3821	.3276	28
29	.5710	.7880	.8602	.7133	.6037	.5162	.4435	.3812	.3267	29
30	.5563	.7840	.8573	.7112	.6021	.5149	.4424	.3802	.3259	30
31	.5421	.7800	.8544	.7091	.6005	.5136	.4412	.3792	.3250	31
32	.5283	.7760	.8516	.7071	.5989	.5123	.4401	.3782	.3242	32
33	.5149	.7720	.8487	.7050	.5973	.5110	.4390	.3772	.3233	33
34	.5019	.7680	.8459	.7030	.5957	.5097	.4379	.3762	.3225	34
35	.4894	.7640	.8431	.7010	.5941	.5084	.4368	.3754	.3216	35
36	.4771	.7600	.8403	.6990	.5925	.5071	.4357	.3745	.3208	36
37	.4652	.7560	.8375	.6970	.5909	.5058	.4346	.3735	.3199	37
38	.4536	.7520	.8348	.6950	.5894	.5045	.4335	.3726	.3191	38
39	.4424	.7480	.8320	.6930	.5878	.5032	.4325	.3716	.3183	39
40	.4314	.7440	.8293	.6910	.5863	.5019	.4314	.3707	.3174	40
41	.4206	.7400	.8266	.6890	.5847	.5006	.4303	.3697	.3166	41
42	.4102	.7360	.8239	.6871	.5832	.4994	.4292	.3688	.3158	42
43	.4000	.7320	.8212	.6851	.5816	.4981	.4281	.3678	.3149	43
44	.3900	.7280	.8186	.6832	.5801	.4968	.4270	.3669	.3141	44
45	.3802	.7240	.8159	.6812	.5786	.4956	.4260	.3660	.3133	45
46	.3707	.7200	.8133	.6793	.5771	.4943	.4249	.3650	.3124	46
47	.3613	.7160	.8107	.6774	.5755	.4931	.4238	.3641	.3116	47
48	.3522	.7120	.8081	.6755	.5740	.4918	.4228	.3632	.3108	48
49	.3432	.7080	.8055	.6736	.5725	.4906	.4217	.3623	.3100	49
50	.3345	.7040	.8030	.6717	.5710	.4894	.4206	.3613	.3091	50
51	.3259	.7000	.8004	.6698	.5695	.4881	.4196	.3604	.3083	51
52	.3174	.6960	.7979	.6679	.5680	.4869	.4185	.3595	.3075	52
53	.3091	.6920	.7954	.6661	.5666	.4856	.4175	.3586	.3067	53
54	.3010	.6880	.7929	.6642	.5651	.4844	.4164	.3576	.3059	54
55	.2931	.6840	.7904	.6624	.5636	.4832	.4154	.3567	.3051	55
56	.2853	.6800	.7879	.6605	.5621	.4820	.4143	.3558	.3043	56
57	.2775	.6760	.7855	.6587	.5607	.4808	.4133	.3549	.3034	57
58	.2700	.6720	.7830	.6568	.5592	.4795	.4122	.3540	.3026	58
59	1.2626	.9615	.7806	.6550	.5578	.4783	.4112	.3531	.3018	59
"	0° 0'	0° 1'	0° 2'	0° 3'	0° 4'	0° 5'	0° 6'	0° 7'	0° 8'	"

TABLE III.—PROPORTIONAL LOGARITHMS.

"	h =	h =	h =	h =	h =	h =	h =	h =	h =	h =	h =	"
"	00 9'	00 10'	00 11'	00 12'	00 13'	00 14'	00 15'	00 16'	00 17'	00 18'	00 19'	"
0	1.3010	1.3353	1.3139	1.1761	1.1413	1.1091	1.0792	1.0512	1.0249			0
1	.02	.45	.32	.55	.08	.86	.87	.07	.44			1
2	.9934	.38	.96	.49	.02	.81	.82	.02	.40			2
3	.86	.31	.19	.43	.1397	.76	.77	.0498	.35			3
4	.78	.24	.13	.37	.91	.71	.73	.93	.31			4
5	.70	.17	.06	.31	.86	.66	.68	.89	.27			5
6	.62	.10	.9090	.25	.80	.61	.63	.84	.23			6
7	.54	.02	.93	.19	.74	.55	.58	.80	.19			7
8	.46	.9495	.86	.13	.69	.50	.53	.75	.14			8
9	.39	.88	.80	.07	.63	.45	.49	.71	.10			9
10	.31	.81	.73	.01	.58	.40	.44	.67	.08			10
11	.23	.74	.67	.1695	.52	.35	.39	.62	.02			11
12	.15	.67	.61	.89	.47	.30	.34	.58	.0197			12
13	.07	.60	.54	.83	.42	.25	.30	.53	.93			13
14	.9899	.53	.48	.77	.36	.20	.25	.49	.86			14
15	.91	.45	.41	.71	.31	.15	.20	.44	.85			15
16	.83	.38	.35	.65	.25	.09	.15	.40	.81			16
17	.76	.31	.28	.60	.20	.04	.11	.35	.76			17
18	.68	.24	.22	.54	.14	.0999	.06	.31	.72			18
19	.60	.17	.16	.48	.09	.94	.01	.26	.68			19
20	.52	.10	.09	.42	.03	.89	.0696	.22	.64			20
21	.45	.03	.03	.36	.1398	.84	.92	.18	.60			21
22	.37	.9396	.1996	.30	.92	.79	.87	.13	.56			22
23	.29	.89	.90	.24	.87	.74	.89	.09	.51			23
24	.21	.82	.84	.19	.82	.68	.78	.04	.47			24
25	.14	.75	.77	.13	.76	.64	.73	.00	.43			25
26	.06	.68	.71	.07	.71	.59	.68	.0398	.39			26
27	.9798	.62	.65	.01	.66	.54	.63	.91	.35			27
28	.91	.55	.58	.1595	.60	.49	.59	.87	.31			28
29	.83	.48	.52	.89	.55	.44	.54	.82	.28			29
30	.75	.41	.46	.84	.49	.39	.49	.78	.23			30
31	.68	.34	.39	.78	.44	.34	.45	.74	.18			31
32	.60	.27	.33	.72	.39	.29	.40	.69	.14			32
33	.53	.20	.27	.66	.33	.24	.35	.65	.10			33
34	.45	.13	.21	.61	.28	.19	.31	.60	.06			34
35	.38	.07	.14	.55	.23	.14	.26	.56	.03			35
36	.30	.00	.08	.49	.17	.09	.21	.52	.0036			36
37	.22	.9893	.02	.43	.12	.04	.17	.47	.93			37
38	.15	.86	.1696	.38	.07	.0899	.12	.43	.89			38
39	.07	.79	.80	.32	.01	.94	.06	.39	.85			39
40	.00	.72	.83	.26	.1196	.89	.03	.34	.81			40
41	.9892	.66	.77	.20	.91	.84	.9398	.30	.77			41
42	.85	.59	.71	.15	.86	.80	.94	.26	.73			42
43	.78	.52	.65	.09	.80	.75	.89	.21	.69			43
44	.70	.45	.59	.03	.75	.70	.85	.17	.65			44
45	.63	.39	.52	.1498	.70	.65	.80	.13	.61			45
46	.55	.32	.46	.92	.64	.60	.75	.08	.57			46
47	.48	.25	.40	.86	.59	.55	.71	.04	.53			47
48	.40	.18	.34	.81	.54	.50	.66	.00	.49			48
49	.33	.12	.28	.75	.49	.45	.62	.0395	.44			49
50	.26	.05	.22	.69	.43	.40	.57	.91	.40			50
51	.18	.9198	.16	.64	.38	.35	.52	.87	.36			51
52	.11	.99	.09	.58	.33	.31	.46	.82	.32			52
53	.04	.85	.03	.52	.28	.26	.43	.78	.29			53
54	.9596	.78	.1797	.47	.23	.21	.39	.74	.24			54
55	.89	.72	.91	.41	.17	.16	.34	.70	.20			55
56	.82	.65	.85	.36	.12	.11	.30	.65	.16			56
57	.74	.52	.79	.30	.07	.06	.25	.61	.12			57
58	.67	.42	.73	.24	.02	.01	.21	.57	.08			58
59	.60	.35	.67	.19	.1097	.0797	.16	.52	.04			59
"	00 9'	00 10'	00 11'	00 12'	00 13'	00 14'	00 15'	00 16'	00 17'	"		"
"	h =	h =	h =	h =	h =	h =	h =	h =	h =	"		"

"	00 18'	00 19'	00 20'	00 21'	00 22'	00 23'	00 24'	00 25'	00 26'	00 27'	00 28'	00 29'	"
0	10000	9765	9542	9331	9128	8935	8751	8573	8408	8250	8091	7939	0
1	9906	61	39	27	25	32	48	70	90	36	79	96	1
2	92	58	35	24	22	29	45	68	83	97	34	76	2
3	88	54	32	20	19	26	42	65	81	95	31	73	3
4	84	50	28	17	15	23	39	62	78	92	28	71	4
5	80	46	24	13	12	20	36	59	75	89	26	68	5
6	76	42	21	10	09	17	33	56	72	86	23	66	6
7	72	39	17	06	06	13	30	53	69	84	20	63	7
8	68	35	14	03	02	10	27	50	67	81	18	61	8
9	64	31	10	00	00	07	24	47	64	78	15	58	9
10	60	27	06	9706	96	04	21	44	75	12	55	54	10
11	56	23	03	93	92	01	18	42	72	10	53	51	11
12	52	20	00	89	89	00	15	39	70	07	50	7899	12
13	48	16	96	86	86	95	12	36	67	04	48	96	13
14	44	12	92	83	83	92	09	33	64	02	45	94	14
15	40	08	88	79	79	88	06	30	61	5199	43	91	15
16	36	05	85	75	75	85	03	27	59	96	40	89	16
17	32	01	81	72	72	82	00	24	56	94	37	87	17
18	28	98	78	69	70	79	98	22	53	91	35	84	18
19	24	93	74	66	66	76	94	19	50	88	32	82	19
20	20	90	71	63	63	73	91	16	48	86	30	79	20
21	16	86	67	59	60	70	88	13	45	83	27	77	21
22	12	82	64	55	57	67	85	10	42	81	25	74	22
23	08	78	60	52	53	64	82	07	39	78	22	72	23
24	05	75	56	49	50	61	79	04	37	75	20	69	24
25	01	71	53	45	47	57	76	02	34	73	17	67	25
26	98	67	49	42	44	54	73	98	31	70	14	64	26
27	93	64	46	39	41	51	70	96	28	67	12	62	27
28	89	60	43	35	37	48	67	93	26	65	09	59	28
29	85	56	39	32	34	45	64	90	23	62	07	57	29
30	81	52	35	28	31	42	61	87	20	59	04	55	30
31	77	49	32	25	28	39	58	84	18	57	02	52	31
32	73	45	28	22	24	36	55	82	15	54	7919	50	32
33	69	41	25	18	21	33	52	79	12	52	97	47	33
34	65	38	21	15	18	30	49	76	09	49	94	45	34
35	61	34	18	12	15	27	46	73	07	46	92	42	35
36	58	30	14	09	12	24	43	70	04	44	89	40	36
37	54	26	11	05	08	21	40	67	01	41	87	37	37
38	50	23	07	01	05	17	37	65	98	38	84	35	38
39	46	19	04	98	02	14	35	62	96	36	81	32	39
40	42	15	00	95	99	11	32	59	93	33	79	30	40
41	38	12	93	91	96	08	29	56	90	31	76	28	41
42	34	08	93	88	92	05	26	53	88	28	74	25	42
43	30	04	90	85	89	02	23	51	85	25	71	23	43
44	27	01	86	81	86	99	20	48	82	23	69	20	44
45	23	98	83	78	23	96	17	45	79	20	66	18	45
46	19	93	79	75	80	93	14	42	77	17	64	15	46
47	15	90	76	71	77	90	11	39	74	15	61	13	47
48	11	86	72	68	73	87	08	37	71	13	59	10	48
49	07	82	68	65	70	84	05	34	69	10	56	08	49
50	03	79	65	62	67	81	02	31	66	07	54	06	50
51	00	75	62	58	64	78	98	28	63	04	51	03	51
52	9706	71	59	55	61	75	97	25	61	02	49	01	52
53	92	68	55	52	58	72	94	23	59	99	46	7708	53
54	88	64	51	48	54	69	91	20	55	97	44	96	54
55	84	61	48	45	51	66	88	17	53	94	41	94	55
56	80	57	44	42	48	63	85	14	50	91	39	91	56
57	77	53	41	38	45	60	82	11	47	89	36	89	57
58	73	50	37	35	42	57	79	09	44	86	34	86	58
59	69	46	34	32	39	54	76	06	42	84	31	84	59
"	00 18'	00 19'	00 20'	00 21'	00 22'	00 23'	00 24'	00 25'	00 26'	00 27'	00 28'	00 29'	"
"	h =	h =	h =	h =	h =	h =	h =	h =	h =	h =	h =	h =	"

TABLE III.—PROPORTIONAL LOGARITHMS.

"	00 30'	00 31'	00 32'	00 33'	00 34'	00 35'	00 36'	00 37'	00 38'	00 39'	00 40'	00 41'	"
0	7793	7639	7501	7368	7238	7112	6990	6871	6755	6642	6532	6425	0
1	77	70	37	7499	65	36	10	96	53	40	30	23	1
2	77	74	34	97	63	34	08	86	67	51	38	29	2
3	74	32	94	61	32	06	84	65	49	37	27	20	3
4	73	30	92	59	29	04	82	63	47	35	25	18	4
5	69	27	90	57	27	02	80	61	45	33	23	16	5
6	67	25	88	54	25	00	78	59	43	31	21	14	6
7	65	23	85	52	23	7098	76	57	42	29	19	13	7
8	63	21	83	50	21	96	74	55	40	27	18	11	8
9	60	18	81	48	19	93	72	53	38	25	16	09	9
10	57	16	79	46	17	91	70	51	36	24	14	07	10
11	55	13	76	44	15	89	68	49	34	22	13	06	11
12	53	11	74	41	13	87	66	47	32	20	10	04	12
13	50	09	72	39	10	85	64	45	30	18	09	02	13
14	48	07	70	37	08	83	62	43	28	16	07	00	14
15	45	04	67	35	06	81	60	42	26	14	05	6398	15
16	43	02	65	33	04	79	58	40	25	12	03	97	16
17	41	00	63	30	02	77	56	38	23	11	01	95	17
18	38	7597	61	28	00	75	54	36	21	09	00	93	18
19	36	95	58	26	7196	73	52	34	19	07	6496	91	19
20	34	93	56	24	66	71	50	32	17	05	64	90	20
21	31	90	54	22	63	69	48	30	15	03	64	88	21
22	29	88	52	20	61	67	46	28	13	01	62	86	22
23	26	86	50	17	58	65	44	26	11	00	61	84	23
24	24	83	47	15	57	63	42	24	09	6598	59	83	24
25	22	81	45	13	55	61	40	22	08	66	57	81	25
26	19	79	43	11	53	59	38	20	06	64	55	79	26
27	17	77	41	09	51	57	36	18	04	62	54	77	27
28	14	74	38	07	49	55	34	16	02	60	52	75	28
29	12	72	36	04	47	53	32	14	00	58	50	74	29
30	10	70	34	02	45	50	30	12	6696	57	48	72	30
31	07	67	32	00	43	48	28	11	66	55	46	71	31
32	05	65	29	7398	40	46	26	09	64	53	44	69	32
33	03	63	27	96	38	44	24	07	62	51	42	67	33
34	00	60	25	94	36	42	22	05	61	49	41	65	34
35	7698	58	23	91	34	40	20	03	59	47	39	64	35
36	96	56	21	89	32	38	18	01	57	45	37	62	36
37	93	54	18	87	30	36	16	6799	55	44	36	60	37
38	91	51	16	85	28	34	14	97	53	42	34	58	38
39	88	49	14	83	26	32	12	95	51	40	32	57	39
40	86	47	12	81	24	30	10	93	49	38	30	55	40
41	84	44	09	79	22	28	08	91	47	36	28	53	41
42	81	42	07	76	20	26	06	89	45	34	26	51	42
43	79	40	05	74	17	24	04	87	43	32	24	49	43
44	77	38	03	72	15	22	02	85	41	30	22	47	44
45	74	35	01	70	13	20	00	84	40	28	20	45	45
46	72	33	7398	68	11	18	6696	82	38	26	18	44	46
47	70	31	96	66	09	16	96	80	36	24	16	42	47
48	67	28	94	64	07	14	94	78	34	22	14	41	48
49	65	26	92	61	05	12	92	76	32	20	12	39	49
50	63	24	90	59	03	10	90	74	30	18	10	38	50
51	60	22	87	57	01	08	88	72	28	16	08	36	51
52	58	20	85	55	99	06	86	70	26	14	06	34	52
53	55	17	83	53	97	04	84	68	24	12	04	32	53
54	53	15	81	51	94	02	82	66	22	10	02	31	54
55	51	13	79	49	92	00	81	64	20	08	00	29	55
56	48	10	76	46	90	6998	79	62	18	06	38	27	56
57	46	08	74	44	88	18	77	61	16	04	36	25	57
58	44	06	72	42	86	16	75	59	14	02	34	23	58
59	41	03	70	40	84	14	73	57	12	00	32	21	59
"	00 30'	00 31'	00 32'	00 33'	00 34'	00 35'	00 36'	00 37'	00 38'	00 39'	00 40'	00 41'	"

TABLE III.—PROPORTIONAL LOGARITHMS.

"	00 42'	00 43'	00 44'	00 45'	00 46'	00 47'	00 48'	00 49'	00 50'	00 51'	00 52'	00 53'	"
a	a	a	a	a	a	a	a	a	a	a	a	a	a
0	6320	6318	6118	6021	5925	5832	5740	5651	5563	5477	5393	5310	0
1	19	17	17	19	24	30	39	49	62	76	91	09	1
2	17	15	15	17	22	29	37	48	60	74	90	07	2
3	15	13	13	16	20	27	36	46	59	73	89	06	3
4	13	11	11	14	19	26	34	45	57	71	87	05	4
5	12	10	10	13	17	24	33	43	56	70	86	03	5
6	10	08	08	11	16	23	31	41	54	69	84	02	6
7	08	06	07	09	14	21	30	40	53	67	83	00	7
8	06	05	05	08	13	19	28	39	51	66	82	58	8
9	05	03	03	06	11	18	27	37	50	64	80	56	9
10	03	01	02	05	09	16	25	36	49	63	79	96	10
11	01	00	00	03	08	15	24	35	47	61	77	95	11
12	00	00	00	01	06	13	22	33	46	60	76	94	12
13	6398	6196	6099	5998	5898	5798	5698	5598	5498	5398	5298	5198	13
14	96	95	95	96	03	10	19	30	43	57	73	91	14
15	94	93	94	97	02	09	18	29	41	56	72	90	15
16	93	91	92	95	00	07	16	27	40	54	70	88	16
17	91	90	90	93	5998	06	15	26	38	53	69	87	17
18	89	88	89	92	97	04	13	24	37	52	68	85	18
19	88	86	87	90	95	03	12	23	36	50	66	84	19
20	86	85	85	89	94	01	10	21	34	49	65	83	20
21	84	83	84	87	92	00	09	20	33	47	64	81	21
22	82	81	82	85	91	5798	07	18	31	46	62	80	22
23	81	79	81	84	89	96	06	17	30	45	61	79	23
24	79	78	79	82	88	95	04	15	28	43	59	77	24
25	78	76	77	81	86	93	03	14	27	42	58	76	25
26	76	74	76	79	84	92	01	13	26	40	57	75	26
27	74	73	74	77	83	90	00	11	24	39	55	73	27
28	72	71	72	76	81	89	5698	10	23	37	54	72	28
29	71	69	71	74	80	87	97	08	21	36	53	71	29
30	69	68	69	73	78	86	95	07	20	35	51	69	30
31	67	66	67	71	77	84	94	05	18	33	50	68	31
32	65	65	66	69	75	83	92	04	17	32	48	66	32
33	64	63	64	68	74	81	91	03	16	30	47	65	33
34	63	61	63	66	72	80	89	01	14	29	46	64	34
35	60	60	61	65	70	78	88	5598	13	28	44	62	35
36	59	58	59	63	69	77	86	98	11	26	43	61	36
37	57	56	58	61	67	75	85	96	10	25	41	60	37
38	55	55	56	60	66	74	83	95	08	23	40	58	38
39	54	53	55	58	64	72	82	94	07	22	39	57	39
40	52	51	53	57	63	71	80	92	06	21	37	56	40
41	50	50	51	55	61	69	79	91	04	19	36	54	41
42	48	48	50	54	60	68	77	89	03	18	35	53	42
43	47	46	48	52	58	66	76	88	01	16	33	52	43
44	45	45	46	50	56	65	74	86	00	15	32	50	44
45	43	43	45	49	55	63	73	85	5498	14	31	49	45
46	42	41	43	47	53	61	71	83	97	12	29	48	46
47	40	40	42	46	52	60	70	82	96	11	28	46	47
48	38	38	40	44	50	58	68	80	94	09	26	45	48
49	37	36	38	42	49	57	67	79	93	08	25	44	49
50	35	35	37	41	47	55	66	78	91	07	24	42	50
51	33	33	35	39	46	54	64	76	90	05	22	41	51
52	32	31	33	38	44	52	63	75	88	04	21	40	52
53	30	30	32	36	43	51	61	73	87	02	20	38	53
54	28	28	30	35	41	49	60	72	86	01	18	37	54
55	26	26	29	33	39	48	58	70	84	00	17	35	55
56	25	25	27	31	38	46	57	69	83	5398	15	34	56
57	23	23	25	30	36	45	55	67	81	97	14	33	57
58	21	21	24	28	35	43	54	66	80	95	13	31	58
59	20	20	22	27	33	42	53	64	78	94	11	30	59
"	00 42'	00 43'	00 44'	00 45'	00 46'	00 47'	00 48'	00 49'	00 50'	00 51'	00 52'	00 53'	"
a	a	a	a	a	a	a	a	a	a	a	a	a	a

"	00 30'	00 31'	00 32'	00 33'	00 34'	00 35'	00 36'	00 37'	00 38'	00 39'	00 40'	00 41'	"
0	7728	7639	7501	7368	7238	7112	6990	6871	6755	6642	6532	6425	0
1	79	37	7439	65	36	10	86	69	53	40	30	23	1
2	77	34	97	63	34	08	86	67	51	38	29	21	2
3	74	32	94	61	32	06	84	65	49	37	27	20	3
4	72	30	92	59	29	04	82	63	47	35	25	18	4
5	69	27	90	57	27	02	80	61	45	33	23	16	5
6	67	25	88	54	25	00	78	59	43	31	21	14	6
7	65	23	85	52	23	7098	76	57	42	29	19	13	7
8	62	20	83	50	21	96	74	55	40	27	18	11	8
9	60	18	81	48	19	93	72	53	38	25	16	09	9
10	57	16	79	46	17	91	70	51	36	24	14	07	10
11	55	13	76	44	15	89	68	49	34	22	12	06	11
12	53	11	74	41	12	87	66	47	32	20	10	04	12
13	50	09	72	39	10	85	64	45	30	18	09	02	13
14	48	07	70	37	08	83	62	43	28	16	07	00	14
15	45	04	67	35	06	81	60	42	26	14	05	6308	15
16	43	02	65	33	04	79	58	40	25	12	03	97	16
17	41	00	63	30	02	77	56	38	23	11	01	95	17
18	38	7597	61	28	00	75	54	36	21	09	00	93	18
19	36	95	58	26	7198	73	52	34	19	07	6498	91	19
20	34	93	56	24	96	71	50	32	17	05	96	96	20
21	31	90	54	22	93	69	48	30	15	03	94	88	21
22	29	88	52	20	91	67	46	28	13	01	92	86	22
23	26	86	50	17	89	65	44	26	11	00	91	84	23
24	24	83	47	15	87	63	43	24	09	6598	89	83	24
25	22	81	45	13	85	61	40	22	08	96	87	81	25
26	19	79	43	11	83	59	38	20	06	94	85	79	26
27	17	77	41	09	81	57	36	18	04	92	84	77	27
28	14	74	38	07	79	55	34	16	02	90	82	76	28
29	12	72	36	04	77	53	32	14	00	89	80	74	29
30	10	70	34	02	75	50	30	12	6698	87	78	72	30
31	07	67	32	00	72	48	28	11	96	85	76	71	31
32	05	65	29	7298	70	46	26	09	94	83	75	69	32
33	03	63	27	96	68	44	24	07	92	81	73	67	33
34	00	60	25	94	66	42	22	05	91	79	71	65	34
35	7698	58	23	91	64	40	20	03	89	78	69	64	35
36	96	56	21	89	62	38	18	01	87	76	67	62	36
37	93	54	18	87	60	36	16	6798	85	74	66	60	37
38	91	51	16	85	58	34	14	97	83	72	64	58	38
39	88	49	14	83	56	32	12	95	81	70	62	57	39
40	86	47	12	81	54	30	10	93	79	68	60	55	40
41	84	44	00	79	52	28	08	91	77	67	58	53	41
42	81	42	07	76	49	26	06	89	76	65	57	51	42
43	79	40	05	74	47	24	04	87	74	63	55	49	43
44	77	38	03	72	45	22	02	85	72	61	53	46	44
45	74	35	01	70	43	20	00	84	70	59	51	44	45
46	72	33	7398	68	41	18	6898	82	68	58	50	44	46
47	70	31	96	66	39	16	96	80	66	56	48	43	47
48	67	28	94	64	37	14	94	78	64	54	46	41	48
49	65	26	92	61	35	12	92	76	63	52	44	39	49
50	63	24	90	59	33	10	90	74	61	50	43	38	50
51	60	22	87	57	31	08	88	72	59	48	41	36	51
52	58	19	85	55	29	06	86	70	57	47	39	34	52
53	55	17	83	53	27	04	84	68	55	45	37	32	53
54	53	15	81	51	24	02	82	66	53	43	35	31	54
55	51	13	79	49	22	00	81	64	51	41	34	29	55
56	48	10	76	46	20	6998	79	63	50	39	32	27	56
57	46	08	74	44	18	96	77	61	48	38	30	25	57
58	44	06	72	42	16	94	75	59	46	36	28	24	58
59	41	03	70	40	14	92	73	57	44	34	27	22	59
"	00 30'	00 31'	00 32'	00 33'	00 34'	00 35'	00 36'	00 37'	00 38'	00 39'	00 40'	00 41'	"

"	00 42'	00 43'	00 44'	00 45'	00 46'	00 47'	00 48'	00 49'	00 50'	00 51'	00 52'	00 53'	"	
0	6330	6318	6118	6021	5925	5832	5740	5651	5563	5477	5393	5310	0	
1	17	16	17	19	24	30	39	49	62	76	91	09	1	
2	19	15	15	17	23	29	37	48	60	74	90	07	2	
3	15	13	13	16	20	27	36	46	59	73	89	06	3	
4	13	11	12	14	19	26	34	45	57	71	87	05	4	
5	19	10	10	13	17	24	33	43	56	70	86	03	5	
6	10	08	08	11	16	23	31	42	54	69	84	02	6	
7	08	06	07	09	14	21	30	40	53	67	83	00	7	
8	06	05	05	08	13	19	28	39	51	66	82	5899	8	
9	05	03	03	06	11	18	27	37	50	64	80	96	9	
10	03	01	02	05	09	16	25	36	49	63	79	96	10	
11	01	00	00	03	08	15	24	35	47	61	77	95	11	
12	00	00	00	01	06	13	22	33	46	60	76	94	12	
13	6338	6198	6099	5998	5898	5798	5698	5598	5498	5398	5298	5198	13	
14	96	95	95	97	00	05	12	21	32	44	59	75	92	14
15	94	93	94	97	02	09	18	29	41	56	72	90	15	
16	93	91	92	95	00	07	16	27	40	54	70	88	16	
17	91	90	90	93	5898	06	15	26	38	53	69	87	17	
18	89	88	89	92	97	04	13	24	37	52	68	85	18	
19	88	86	87	90	95	03	12	23	36	50	66	84	19	
20	86	85	85	89	94	01	10	21	34	49	65	83	20	
21	84	83	84	87	92	00	09	20	33	47	64	81	21	
22	82	81	82	85	91	5798	07	18	31	46	62	80	22	
23	81	79	81	84	89	96	06	17	30	45	61	79	23	
24	79	78	79	82	88	95	04	15	28	43	59	77	24	
25	78	76	77	81	86	93	03	14	27	42	58	76	25	
26	76	74	76	79	84	92	01	13	26	40	57	75	26	
27	74	73	74	77	83	90	00	11	24	39	55	73	27	
28	72	71	72	76	81	89	5698	10	23	37	54	72	28	
29	71	69	71	74	80	87	97	08	21	36	53	71	29	
30	69	68	69	73	78	86	95	07	20	35	51	69	30	
31	67	66	67	71	77	84	94	05	18	33	50	68	31	
32	65	65	66	69	75	83	92	04	17	32	48	66	32	
33	64	63	64	68	74	81	91	03	16	30	47	65	33	
34	62	61	63	66	73	80	89	01	14	29	46	64	34	
35	60	60	61	65	70	78	86	5598	13	28	44	62	35	
36	59	58	59	63	69	77	86	11	26	43	61	61	36	
37	57	56	58	61	67	75	85	96	10	25	41	60	37	
38	55	55	56	60	66	74	83	95	08	23	40	58	38	
39	54	53	55	58	64	72	81	94	07	22	39	57	39	
40	52	51	53	57	63	71	80	92	06	21	37	56	40	
41	50	50	51	55	61	69	79	91	04	19	36	54	41	
42	48	48	49	54	60	68	77	89	03	18	35	53	42	
43	47	46	48	52	58	66	76	88	01	16	33	52	43	
44	45	45	46	50	56	65	74	86	00	15	32	50	44	
45	43	43	45	49	55	63	73	85	5498	14	31	49	45	
46	42	41	43	47	53	61	71	83	97	13	29	48	46	
47	40	40	42	46	52	60	70	82	96	11	28	46	47	
48	38	38	40	44	50	58	68	80	94	09	26	45	48	
49	37	36	38	42	49	57	67	79	93	08	25	44	49	
50	35	35	37	41	47	55	66	78	91	07	24	42	50	
51	33	33	35	39	46	54	64	76	90	05	23	41	51	
52	32	31	33	38	44	52	63	75	88	04	21	40	52	
53	30	30	32	36	43	51	61	73	87	03	20	38	53	
54	28	28	30	35	41	49	60	72	86	01	18	37	54	
55	26	26	29	33	39	48	58	70	84	00	17	35	55	
56	25	25	27	31	38	46	57	69	83	5398	15	34	56	
57	23	23	25	30	36	45	55	67	81	97	14	33	57	
58	21	21	24	28	35	43	54	66	80	95	13	31	58	
59	20	20	22	27	33	42	53	64	78	94	11	30	59	

TABLE III.—PROPORTIONAL LOGARITHMS.

"	00 54'	00 55'	00 56'	00 57'	00 58'	00 59'	10 0'	10 1'	10 2'	10 3'	10 4'	10 5'	"
0	5239	5149	5071	4924	4818	4844	4771	4689	4628	4559	4491	4424	0
1	27	48	70	93	17	43	70	98	27	58	90	22	1
2	26	46	68	91	16	42	69	67	26	57	89	21	2
3	25	45	67	90	15	41	68	96	25	56	88	20	3
4	23	44	66	89	13	39	66	95	24	55	86	19	4
5	22	43	64	88	12	38	65	93	23	54	85	18	5
6	21	41	63	86	11	37	64	92	22	53	84	17	6
7	19	40	62	85	10	36	63	91	21	51	83	16	7
8	18	39	61	84	08	34	62	90	19	50	82	15	8
9	17	37	59	83	07	33	60	89	18	49	81	14	9
10	15	36	58	81	06	32	59	88	17	48	80	13	10
11	14	35	57	80	05	31	58	86	16	47	79	11	11
12	13	33	55	79	03	30	57	85	15	46	77	10	12
13	11	32	54	77	02	28	56	84	14	44	76	09	13
14	10	31	53	76	01	27	54	83	13	43	75	08	14
15	00	29	51	75	00	26	53	82	11	42	74	07	15
16	07	28	50	74	4929	25	52	80	10	41	73	06	16
17	06	27	49	73	97	23	51	79	09	40	72	05	17
18	05	25	48	71	96	22	50	78	08	39	71	04	18
19	03	24	46	70	95	21	48	77	07	38	69	03	19
20	02	23	45	69	94	20	47	76	06	36	68	01	20
21	01	22	44	67	92	19	46	75	04	35	67	00	21
22	5199	20	43	66	91	17	45	73	03	34	66	4399	22
23	98	19	41	65	90	16	44	72	02	33	65	96	23
24	97	18	40	64	89	15	43	71	01	32	64	97	24
25	96	16	39	62	87	14	41	70	00	31	63	96	25
26	94	15	37	61	86	12	40	69	4599	30	62	95	26
27	93	14	36	60	85	11	39	68	97	28	60	94	27
28	91	12	35	59	84	10	38	66	96	27	59	93	28
29	90	11	34	57	82	09	36	65	95	26	58	91	29
30	89	10	32	56	81	07	35	64	94	25	57	90	30
31	87	08	31	55	80	06	34	63	93	24	56	89	31
32	86	07	30	54	79	05	33	62	92	23	55	88	32
33	85	06	29	52	77	04	32	60	90	22	54	87	33
34	83	05	27	51	76	03	30	59	89	20	53	86	34
35	82	03	26	50	75	01	29	58	88	19	52	85	35
36	81	02	25	49	74	00	28	57	87	18	50	84	36
37	79	01	24	47	73	4799	27	56	86	17	49	83	37
38	78	5099	22	46	71	98	26	55	85	16	48	81	38
39	77	98	21	45	70	97	24	53	84	15	47	80	39
40	75	97	19	43	69	95	23	52	83	14	46	79	40
41	74	95	18	42	68	94	22	51	81	12	45	78	41
42	73	94	17	41	66	93	21	50	80	11	44	77	42
43	72	93	16	40	65	92	20	49	79	10	43	76	43
44	70	92	14	38	64	91	18	48	78	09	41	75	44
45	69	90	13	37	63	89	17	46	77	08	40	74	45
46	68	89	12	36	61	88	16	45	75	07	39	73	46
47	66	88	11	35	60	87	15	44	74	06	38	72	47
48	65	86	09	33	59	86	14	43	73	05	37	70	48
49	64	85	08	32	58	85	12	42	72	03	36	69	49
50	62	84	07	31	56	83	11	40	71	02	35	68	50
51	61	82	05	30	55	82	10	39	70	01	34	67	51
52	60	81	04	29	54	81	09	38	69	00	33	66	52
53	58	80	03	27	53	80	08	37	67	4599	31	65	53
54	57	79	02	26	52	78	06	36	66	98	30	64	54
55	56	77	00	25	50	77	05	35	65	97	29	63	55
56	54	76	4999	23	49	76	04	33	64	95	28	62	56
57	53	75	98	22	48	75	03	32	63	94	27	61	57
58	52	73	97	21	47	74	02	31	62	93	26	59	58
59	50	72	95	20	45	73	01	30	60	92	25	58	59
"	00 54'	00 55'	00 56'	00 57'	00 58'	00 59'	10 0'	10 1'	10 2'	10 3'	10 4'	10 5'	"

"	10 6'	10 7'	10 8'	10 9'	10 10'	10 11'	10 12'	10 13'	10 14'	10 15'	10 16'	10 17'	"
0	4357	4302	4238	4164	4102	4040	3979	3919	3860	3802	3745	3688	0
1	56	91	37	63	01	39	78	18	59	01	44	87	1
2	55	90	36	62	00	38	77	18	58	00	43	86	2
3	54	89	34	61	4009	37	76	17	57	3799	42	85	3
4	53	88	33	60	98	36	75	16	56	98	41	84	4
5	52	86	29	59	97	35	74	15	55	97	40	83	5
6	51	85	21	58	96	34	73	14	55	96	39	82	6
7	50	84	20	57	95	33	72	13	54	95	38	81	7
8	49	83	19	56	93	32	71	12	53	94	37	80	8
9	47	82	18	55	92	31	70	11	52	93	36	79	9
10	46	81	17	54	91	30	69	10	51	92	35	78	10
11	45	80	16	53	90	29	68	09	50	91	34	77	11
12	44	79	15	52	89	28	67	08	49	90	33	77	12
13	43	78	14	51	88	27	66	07	48	89	32	76	13
14	42	77	13	50	87	26	65	06	47	88	31	75	14
15	41	76	12	48	86	25	64	05	46	86	30	74	15
16	40	75	11	47	85	24	63	04	45	87	29	73	16
17	39	74	10	46	84	23	62	03	44	86	28	72	17
18	38	73	09	45	83	22	61	02	43	85	27	71	18
19	36	71	07	44	82	21	60	01	42	84	27	70	19
20	35	70	06	43	81	20	59	00	41	83	26	69	20
21	34	69	05	42	80	19	58	3899	40	82	25	68	21
22	33	68	04	41	79	18	57	98	39	81	24	67	22
23	32	67	03	40	78	17	56	97	38	80	23	66	23
24	31	66	02	39	77	16	55	96	37	79	22	65	24
25	30	65	01	38	76	15	54	95	36	78	21	64	25
26	29	64	00	37	75	14	53	94	35	77	20	63	26
27	28	63	4199	36	74	13	52	93	34	76	19	62	27
28	27	62	98	35	73	12	51	92	33	75	18	61	28
29	26	61	97	34	72	11	50	91	32	74	17	61	29
30	25	60	96	33	71	10	49	90	31	73	16	60	30
31	23	59	95	32	70	09	48	89	30	72	15	59	31
32	22	58	94	31	69	08	47	88	29	71	14	58	32
33	21	57	93	30	68	07	46	87	28	70	13	57	33
34	20	56	92	29	67	06	45	86	27	69	12	56	34
35	19	54	91	28	66	05	44	85	26	68	11	55	35
36	18	53	89	27	65	04	43	84	25	68	10	54	36
37	17	52	88	26	64	03	42	83	24	67	09	53	37
38	16	51	87	25	63	02	41	82	23	66	08	52	38
39	15	50	86	23	62	01	40	81	22	65	07	51	39
40	14	49	85	22	61	00	39	80	21	64	07	50	40
41	13	48	84	21	60	3999	38	79	21	63	06	49	41
42	11	47	83	20	59	98	37	78	20	62	05	49	42
43	10	46	82	19	58	97	36	77	19	61	04	48	43
44	09	45	81	18	57	96	35	76	18	60	03	47	44
45	08	44	80	17	55	94	34	75	17	59	02	46	45
46	07	43	79	16	54	93	33	74	16	58	01	45	46
47	06	41	78	15	53	92	32	73	15	57	00	44	47
48	05	40	77	14	52	91	31	72	14	56	3899	43	48
49	04	39	76	13	51	90	30	71	13	55	98	42	49
50	03	38	75	12	50	89	29	70	12	54	97	41	50
51	02	37	74	11	49	88	28	69	11	53	96	40	51
52	01	36	73	10	48	87	27	68	10	52	95	39	52
53	00	35	72	09	47	86	26	67	09	51	94	38	53
54	4399	34	71	08	46	85	25	66	08	50	93	37	54
55	98	32	69	07	45	84	24	65	07	49	92	36	55
56	96	30	68	06	44	83	23	64	06	48	91	35	56
57	95	31	67	05	43	82	22	63	05	47	91	35	57
58	94	30	66	04	42	81	21	62	04	46	90	34	58
59	93	29	65	03	41	80	20	61	03	46	89	33	59
"	10 6'	10 7'	10 8'	10 9'	10 10'	10 11'	10 12'	10 13'	10 14'	10 15'	10 16'	10 17'	"

"	10 18'	10 19'	10 20'	10 21'	10 22'	10 23'	10 24'	10 25'	10 26'	10 27'	10 28'	10 29'	"
0	3632	3378	3523	3468	3415	3362	3310	3259	3208	3158	3108	3059	0
1	31	76	21	67	14	61	09	58	07	57	07	58	1
2	30	75	20	66	13	60	08	57	06	56	06	57	2
3	29	74	19	65	12	59	07	56	05	55	05	56	3
4	28	73	18	64	11	58	06	55	04	54	05	56	4
5	27	72	17	63	10	58	06	54	04	53	04	55	5
6	26	71	16	63	09	57	05	53	03	53	03	54	6
7	25	70	15	62	08	56	04	53	02	52	02	53	7
8	24	69	15	61	08	55	03	52	01	51	01	52	8
9	23	68	14	60	07	54	02	51	00	50	01	52	9
10	23	67	13	59	06	53	01	50	3199	49	00	51	10
11	22	66	12	58	05	52	00	49	98	48	3090	50	11
12	21	65	11	57	04	51	00	48	98	48	98	49	12
13	20	65	10	56	03	51	3299	47	97	47	97	48	13
14	19	64	09	55	02	50	98	47	96	46	96	47	14
15	18	63	08	54	01	49	97	46	95	45	95	47	15
16	17	62	07	54	00	48	96	45	94	44	95	46	16
17	16	61	06	53	00	47	95	44	93	43	94	45	17
18	15	60	06	52	3399	46	94	43	93	43	93	44	18
19	14	59	05	51	98	45	94	42	92	42	92	43	19
20	13	58	04	50	97	45	93	42	91	41	91	43	20
21	12	57	03	49	96	44	92	41	90	40	91	42	21
22	11	56	02	48	95	43	91	40	89	39	90	41	22
23	10	55	01	47	94	42	90	39	88	38	89	40	23
24	10	55	00	46	93	41	89	38	88	38	88	39	24
25	09	54	3499	46	93	40	88	37	87	37	87	38	25
26	08	53	98	45	92	39	88	36	86	36	87	38	26
27	07	52	97	44	91	38	87	35	85	35	86	37	27
28	06	51	97	43	90	38	86	35	84	34	85	36	28
29	05	50	96	42	89	37	85	34	83	33	84	35	29
30	04	49	95	41	88	36	84	33	83	33	83	34	30
31	03	48	94	40	87	35	83	32	82	32	82	34	31
32	02	47	93	39	86	34	82	31	81	31	82	33	32
33	01	46	92	38	85	33	82	31	80	30	81	32	33
34	00	45	91	38	85	32	81	30	79	29	80	31	34
35	3599	45	90	37	84	32	80	29	78	29	79	30	35
36	98	44	89	36	83	31	79	28	78	28	78	30	36
37	98	43	88	35	82	30	78	27	77	27	78	29	37
38	97	42	88	34	81	29	77	26	76	26	77	28	38
39	96	41	87	33	80	28	76	25	75	25	76	27	39
40	95	40	86	32	79	27	75	25	74	24	75	26	40
41	94	39	85	31	79	26	75	24	73	24	74	26	41
42	93	38	84	31	78	25	74	23	73	23	73	25	42
43	92	37	83	30	77	25	73	22	72	22	73	24	43
44	91	36	82	29	76	24	73	21	71	21	72	23	44
45	90	35	81	28	75	23	71	20	70	20	71	22	45
46	89	35	80	27	74	22	70	20	69	19	70	22	46
47	88	34	80	26	73	21	70	19	68	19	69	21	47
48	87	33	79	25	72	20	69	18	68	18	69	20	48
49	87	32	78	24	72	19	68	17	67	17	68	19	49
50	86	31	77	23	71	19	67	16	66	16	67	18	50
51	85	30	76	22	70	18	66	15	65	15	66	18	51
52	84	29	75	22	69	17	65	14	64	14	65	17	52
53	83	28	74	21	68	16	65	14	63	14	65	16	53
54	82	27	73	20	67	15	64	13	63	13	64	15	54
55	81	26	72	19	66	14	63	12	62	12	63	14	55
56	80	25	71	18	65	13	62	11	61	11	62	14	56
57	79	25	71	17	65	13	61	10	60	10	61	13	57
58	78	24	70	16	64	12	60	09	59	10	60	12	58
59	77	23	69	15	63	11	59	08	58	09	60	11	59
"	10 18'	10 19'	10 20'	10 21'	10 22'	10 23'	10 24'	10 25'	10 26'	10 27'	10 28'	10 29'	"

TABLE III.—PROPORTIONAL LOGARITHMS.

"	a m	a m	a m	a m	a m	a m	a m	a m	a m	a m	a m	a m	a m	"
10 30'	10 31'	10 32'	10 33'	10 34'	10 35'	10 36'	10 37'	10 38'	10 39'	10 40'	10 41'	10 40'	10 41'	"
0	3010	2968	2915	2868	2821	2775	2730	2685	2640	2596	2553	2510	0	
1	09	08	14	07	21	75	99	84	40	06	52	09	09	
2	03	01	13	06	20	74	99	84	39	05	51	08	02	
3	06	00	12	06	19	73	28	83	38	04	51	07	07	
4	07	59	12	05	18	73	27	82	38	03	50	07	04	
5	06	58	11	04	18	72	26	81	37	03	49	06	05	
6	05	58	10	03	17	71	25	81	36	02	48	05	07	
7	05	57	09	02	16	70	25	80	35	01	48	04	07	
8	04	56	09	02	15	69	24	79	35	01	47	04	04	
9	03	55	08	01	15	69	23	78	34	00	46	03	03	
10	02	54	07	00	14	68	22	78	33	00	45	02	10	
11	01	54	06	59	13	67	22	77	32	00	45	02	09	
12	01	53	05	59	12	66	21	76	32	00	44	01	12	
13	00	52	05	58	11	66	20	75	31	00	43	00	13	
14	2999	51	04	57	11	65	19	75	30	00	43	00	14	
15	96	50	03	56	10	64	19	74	29	00	42	99	15	
16	97	50	02	55	09	63	18	73	29	00	41	98	16	
17	97	49	01	55	08	63	17	72	28	00	40	97	17	
18	96	48	01	54	08	62	16	72	27	00	40	97	18	
19	95	47	00	53	07	61	16	71	26	00	39	96	19	
20	94	46	2899	52	06	60	15	70	26	00	38	95	20	
21	93	45	98	52	05	60	14	69	25	00	38	94	21	
22	93	45	98	51	05	59	13	69	24	00	37	94	22	
23	92	44	97	50	04	58	13	68	24	00	36	93	23	
24	91	43	96	49	03	57	12	67	23	00	35	92	24	
25	90	42	95	48	02	56	11	66	22	00	35	92	25	
26	89	42	94	48	01	56	10	66	21	00	34	91	26	
27	89	41	94	47	01	55	10	65	21	00	33	90	27	
28	88	40	93	46	00	54	09	64	20	00	33	89	28	
29	87	39	92	45	2999	53	08	63	19	00	32	89	29	
30	86	39	91	45	98	53	07	63	18	00	31	88	30	
31	85	38	91	44	98	52	07	62	18	00	30	87	31	
32	85	37	90	43	97	51	06	61	17	00	30	87	32	
33	84	36	89	42	96	50	05	60	16	00	29	86	33	
34	83	35	88	42	95	50	04	60	15	00	28	85	34	
35	82	35	87	41	95	49	04	59	15	00	27	85	35	
36	81	34	87	40	94	48	03	58	14	00	27	84	36	
37	81	33	86	39	93	47	02	57	13	00	26	83	37	
38	80	32	85	38	92	47	01	57	12	00	25	82	38	
39	79	31	84	38	92	46	01	56	12	00	25	82	39	
40	78	31	83	37	91	45	00	55	11	00	24	81	40	
41	77	30	83	36	90	44	2999	55	10	00	23	80	41	
42	77	29	82	35	89	44	98	54	10	00	22	80	42	
43	76	28	81	35	88	43	98	53	09	00	22	79	43	
44	75	27	80	34	88	42	97	52	08	00	21	78	44	
45	74	27	80	33	87	41	96	52	07	00	20	77	45	
46	73	26	79	32	86	41	95	51	07	00	19	77	46	
47	73	25	78	31	85	40	95	50	06	00	19	76	47	
48	72	24	77	31	85	39	94	49	05	00	18	75	48	
49	71	24	76	30	84	38	93	49	04	00	17	75	49	
50	70	23	76	29	83	38	92	48	04	00	16	74	50	
51	69	22	75	28	82	37	92	47	03	00	15	73	51	
52	69	21	74	28	82	36	91	46	02	00	15	72	52	
53	68	20	73	27	81	35	90	46	01	00	15	72	53	
54	67	20	73	26	80	35	89	45	01	00	14	71	54	
55	66	19	72	25	79	34	89	44	00	00	13	70	55	
56	65	18	71	25	79	33	88	43	2999	56	12	70	56	
57	65	17	70	24	78	32	87	43	99	55	11	69	57	
58	64	16	69	23	77	32	87	42	98	54	11	68	58	
59	63	16	69	22	76	31	86	41	97	53	10	67	59	
"	a m	a m	a m	a m	a m	a m	a m	a m	a m	a m	a m	a m	a m	"

"	10 42'	10 43'	10 44'	10 45'	10 46'	10 47'	10 48'	10 49'	10 50'	10 51'	10 52'	10 53'	"
0	24 57	24 54	24 52	24 51	24 50	24 49	24 48	24 47	24 46	24 45	24 44	24 43	0
1	66	64	62	60	58	56	54	52	50	48	46	44	1
2	65	63	61	59	57	55	53	51	49	47	45	43	2
3	65	62	60	58	56	54	52	50	48	46	44	42	3
4	64	62	60	58	56	54	52	50	48	46	44	42	4
5	63	61	59	57	55	53	51	49	47	45	43	41	5
6	62	60	58	56	54	52	50	48	46	44	42	40	6
7	62	59	57	55	53	51	49	47	45	43	41	39	7
8	61	59	57	55	53	51	49	47	45	43	41	39	8
9	60	58	56	54	52	50	48	46	44	42	40	38	9
10	60	57	55	53	51	49	47	45	43	41	39	37	10
11	59	57	55	53	51	49	47	45	43	41	39	37	11
12	58	56	54	52	50	48	46	44	42	40	38	36	12
13	58	56	54	52	50	48	46	44	42	40	38	36	13
14	57	55	53	51	49	47	45	43	41	39	37	35	14
15	56	54	52	50	48	46	44	42	40	38	36	34	15
16	55	53	51	49	47	45	43	41	39	37	35	33	16
17	55	53	51	49	47	45	43	41	39	37	35	33	17
18	54	52	50	48	46	44	42	40	38	36	34	32	18
19	53	51	49	47	45	43	41	39	37	35	33	31	19
20	53	51	49	47	45	43	41	39	37	35	33	31	20
21	52	50	48	46	44	42	40	38	36	34	32	30	21
22	51	49	47	45	43	41	39	37	35	33	31	29	22
23	50	48	46	44	42	40	38	36	34	32	30	28	23
24	50	48	46	44	42	40	38	36	34	32	30	28	24
25	49	47	45	43	41	39	37	35	33	31	29	27	25
26	48	46	44	42	40	38	36	34	32	30	28	26	26
27	48	46	44	42	40	38	36	34	32	30	28	26	27
28	47	45	43	41	39	37	35	33	31	29	27	25	28
29	46	44	42	40	38	36	34	32	30	28	26	24	29
30	45	43	41	39	37	35	33	31	29	27	25	23	30
31	45	43	41	39	37	35	33	31	29	27	25	23	31
32	44	42	40	38	36	34	32	30	28	26	24	22	32
33	43	41	39	37	35	33	31	29	27	25	23	21	33
34	43	41	39	37	35	33	31	29	27	25	23	21	34
35	42	40	38	36	34	32	30	28	26	24	22	20	35
36	41	39	37	35	33	31	29	27	25	23	21	19	36
37	41	39	37	35	33	31	29	27	25	23	21	19	37
38	40	38	36	34	32	30	28	26	24	22	20	18	38
39	39	37	35	33	31	29	27	25	23	21	19	17	39
40	39	37	35	33	31	29	27	25	23	21	19	17	40
41	38	36	34	32	30	28	26	24	22	20	18	16	41
42	37	35	33	31	29	27	25	23	21	19	17	15	42
43	36	34	32	30	28	26	24	22	20	18	16	14	43
44	36	34	32	30	28	26	24	22	20	18	16	14	44
45	35	33	31	29	27	25	23	21	19	17	15	13	45
46	34	32	30	28	26	24	22	20	18	16	14	12	46
47	33	31	29	27	25	23	21	19	17	15	13	11	47
48	33	31	29	27	25	23	21	19	17	15	13	11	48
49	32	30	28	26	24	22	20	18	16	14	12	10	49
50	31	29	27	25	23	21	19	17	15	13	11	9	50
51	31	29	27	25	23	21	19	17	15	13	11	9	51
52	30	28	26	24	22	20	18	16	14	12	10	8	52
53	29	27	25	23	21	19	17	15	13	11	9	7	53
54	29	27	25	23	21	19	17	15	13	11	9	7	54
55	28	26	24	22	20	18	16	14	12	10	8	6	55
56	27	25	23	21	19	17	15	13	11	9	7	5	56
57	26	24	22	20	18	16	14	12	10	8	6	4	57
58	26	24	22	20	18	16	14	12	10	8	6	4	58
59	25	23	21	19	17	15	13	11	9	7	5	3	59

TABLE III.—PROPORTIONAL LOGARITHMS.

"	a = 10 54'	a = 10 55'	a = 10 56'	a = 10 57'	a = 10 58'	a = 10 59'	a = 20 0'	a = 20 1'	a = 20 2'	a = 20 3'	a = 20 4'	"
0	1964	1946	1908	1871	1834	1797	1761	1725	1689	1654	1619	0
1	63	45	08	70	33	97	60	24	89	53	18	1
2	82	44	07	70	33	96	60	24	88	52	17	2
3	82	44	06	69	32	95	59	23	87	52	17	3
4	81	43	06	68	31	95	59	22	87	51	16	4
5	81	43	05	68	31	94	58	22	86	51	16	5
6	80	42	04	67	30	94	57	21	86	50	15	6
7	79	41	04	67	30	93	57	21	85	50	14	7
8	79	41	03	66	29	92	56	20	84	49	14	8
9	78	40	03	65	28	92	55	19	84	48	13	9
10	77	39	02	65	28	91	55	19	83	48	13	10
11	77	39	01	64	27	91	54	18	83	47	12	11
12	76	38	01	63	27	90	54	18	82	47	12	12
13	75	38	00	63	26	89	53	17	81	46	11	13
14	75	37	1099	62	25	89	52	17	81	45	10	14
15	74	36	99	62	25	88	52	16	80	45	10	15
16	74	36	98	61	24	88	51	15	80	44	09	16
17	73	35	98	60	23	87	51	15	79	44	09	17
18	72	34	97	60	23	86	50	14	78	43	08	18
19	72	34	96	59	22	86	49	14	78	43	07	19
20	71	33	96	59	22	85	49	13	77	42	07	20
21	70	33	95	58	21	85	48	12	77	41	06	21
22	70	32	94	57	20	84	48	12	76	41	06	22
23	69	31	94	57	20	83	47	11	76	40	05	23
24	68	31	93	56	19	83	46	11	75	40	05	24
25	68	30	93	55	19	82	46	10	74	39	04	25
26	67	29	92	55	18	81	45	09	74	38	03	26
27	67	29	91	54	17	81	45	09	73	38	03	27
28	66	28	91	54	17	80	44	08	73	37	02	28
29	65	28	90	53	16	80	43	08	72	37	02	29
30	65	27	89	52	16	79	43	07	71	36	01	30
31	64	26	89	52	15	78	42	06	71	35	00	31
32	63	26	88	51	14	78	42	06	70	35	00	32
33	63	25	88	50	14	77	41	05	70	34	1599	33
34	62	24	87	50	13	77	40	05	69	34	99	34
35	62	24	86	49	12	76	40	04	68	33	98	35
36	61	23	86	49	12	75	39	03	68	33	98	36
37	60	23	85	48	11	75	39	03	67	32	97	37
38	60	22	84	47	11	74	38	02	67	31	96	38
39	59	21	84	47	10	74	37	02	66	31	96	39
40	58	21	83	46	09	73	37	01	65	30	95	40
41	58	20	83	46	09	72	36	00	65	30	95	41
42	57	19	82	45	08	72	36	00	64	29	94	42
43	56	19	81	44	08	71	35	1099	64	28	93	43
44	56	18	81	44	07	71	34	99	63	28	93	44
45	55	18	80	43	06	70	34	98	63	27	92	45
46	55	17	80	43	06	69	33	97	62	27	92	46
47	54	16	79	42	05	69	33	97	61	26	91	47
48	53	16	78	41	05	68	32	96	61	26	91	48
49	53	15	78	41	04	68	31	96	60	25	90	49
50	52	14	77	40	03	67	31	95	60	24	89	50
51	51	14	76	39	03	66	30	94	59	24	89	51
52	51	13	76	39	02	66	30	94	58	23	88	52
53	50	13	75	38	02	65	29	93	58	23	88	53
54	50	12	75	38	01	65	28	93	57	22	87	54
55	49	11	74	37	00	64	28	92	57	21	87	55
56	49	11	73	36	00	63	27	92	56	21	86	56
57	48	10	73	36	1709	63	27	91	55	20	85	57
58	47	09	72	35	98	62	26	90	55	20	85	58
59	46	09	71	35	98	62	25	90	54	19	84	59
"	10 54'	10 55'	10 56'	10 57'	10 58'	10 59'	20 0'	20 1'	20 2'	20 3'	20 4'	"
.	a =	a =	a =	a =	a =	a =	a =	a =	a =	a =	a =	.

°	' 30		' 30		' 30		' 30		' 30		' 30		' 30		°
	5'	6'	7'	8'	9'	10'	11'	12'	13'	14'	15'	16'	17'		
0	1564	1549	1515	1481	1447	1413	1380	1347	1314	1282	1249			0	
1	83	48	14	80	46	13	79	46	14	81	49			1	
2	82	48	14	79	46	12	78	46	13	81	48			2	
3	82	47	13	79	45	12	78	45	13	80	48			3	
4	81	47	12	78	45	11	78	45	12	80	47			4	
5	81	46	12	78	44	11	77	44	11	79	47			5	
6	80	46	11	77	43	10	77	44	11	78	46			6	
7	80	45	11	77	43	09	76	43	10	78	46			7	
8	79	44	10	76	42	09	76	43	10	77	45			8	
9	78	44	10	76	42	08	75	42	09	77	45			9	
10	78	43	09	75	41	08	74	42	09	76	44			10	
11	77	43	08	74	41	07	74	41	08	76	43			11	
12	77	42	08	74	40	07	73	41	08	75	43			12	
13	76	42	07	73	40	06	73	40	07	75	42			13	
14	76	41	07	73	39	06	72	39	07	74	42			14	
15	75	40	06	72	38	05	72	39	06	74	41			15	
16	74	40	06	72	38	04	71	38	06	73	41			16	
17	74	39	05	71	37	04	71	38	05	73	40			17	
18	73	39	04	70	37	03	70	37	04	72	40			18	
19	73	38	04	70	36	03	70	37	04	71	39			19	
20	72	38	03	69	36	02	69	36	03	71	39			20	
21	71	37	03	69	35	02	68	35	03	70	38			21	
22	71	36	02	68	35	01	68	35	02	70	38			22	
23	70	36	02	68	34	01	67	34	02	69	37			23	
24	70	35	01	67	33	00	67	34	01	69	37			24	
25	69	35	00	67	33	1399	66	33	01	68	36			25	
26	69	34	00	66	32	99	66	33	00	68	35			26	
27	68	34	1400	65	32	98	65	32	00	67	35			27	
28	67	33	99	65	31	98	65	32	1399	67	34			28	
29	67	32	98	64	31	97	64	31	98	66	34			29	
30	66	32	98	64	30	97	63	31	98	66	33			30	
31	66	31	97	63	29	96	63	30	97	65	33			31	
32	65	31	96	63	29	96	62	29	97	64	32			32	
33	65	30	96	62	28	95	62	29	96	64	32			33	
34	64	30	95	61	28	94	61	28	96	63	31			34	
35	63	29	95	61	27	94	61	28	95	63	31			35	
36	63	28	94	60	27	93	60	27	95	62	30			36	
37	62	28	94	60	26	93	60	27	94	62	30			37	
38	62	27	93	59	26	92	59	26	94	61	29			38	
39	61	27	93	59	25	92	59	26	93	61	29			39	
40	61	26	92	58	24	91	58	25	92	60	28			40	
41	60	26	91	58	24	91	57	25	92	60	27			41	
42	59	25	91	57	23	90	57	24	91	59	27			42	
43	59	24	90	56	23	89	56	23	91	59	26			43	
44	58	24	90	56	22	89	56	23	90	58	26			44	
45	58	23	89	55	22	88	55	22	90	57	25			45	
46	57	23	89	55	21	88	55	22	89	57	25			46	
47	56	22	88	54	21	87	54	21	89	56	24			47	
48	56	22	87	54	20	87	54	21	88	56	24			48	
49	55	21	87	53	19	86	53	20	88	55	23			49	
50	55	20	86	52	19	86	52	20	87	55	23			50	
51	54	20	86	52	18	85	52	19	87	54	22			51	
52	54	19	85	51	18	84	51	18	86	54	22			52	
53	53	19	85	51	17	84	51	18	85	53	21			53	
54	52	18	84	50	17	83	50	17	85	53	21			54	
55	52	18	83	50	16	83	50	17	84	52	20			55	
56	51	17	83	49	16	82	49	16	84	52	19			56	
57	51	16	82	49	15	82	49	16	83	51	19			57	
58	50	16	82	48	14	81	48	15	83	50	18			58	
59	50	15	81	47	14	81	48	15	82	50	18			59	

TABLE III.—PROPORTIONAL LOGARITHMS.

"	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	"
"	30 16'	30 17'	30 18'	30 19'	30 20'	30 21'	30 22'	30 23'	30 24'	30 25'	30 26'	30 27'	30 28'	"
0	12817	1186	1154	1123	109	1061	1030	0999	0968	0939	0901			0
1	17	85	33	32	91	60	29	99	69	39	01			1
2	16	84	53	32	90	60	29	98	68	38	06			2
3	16	84	52	21	90	59	28	98	68	38	01			3
4	15	83	52	20	89	58	28	97	67	37	07			4
5	15	83	51	20	89	58	27	97	67	37	07			5
6	14	82	51	19	88	57	27	96	66	36	06			6
7	14	82	50	19	88	57	26	96	66	36	06			7
8	13	81	50	18	87	56	26	95	65	35	05			8
9	13	81	49	18	87	56	25	95	65	35	05			9
10	12	80	49	17	86	55	25	94	64	34	04			10
11	11	80	48	17	86	55	24	94	64	34	04			11
12	11	79	48	16	85	54	24	93	63	33	03			12
13	10	79	47	16	85	54	23	93	63	33	03			13
14	10	78	47	15	84	53	23	92	62	32	03			14
15	09	78	46	15	84	53	22	92	62	32	02			15
16	09	77	46	14	83	52	22	91	61	31	01			16
17	08	77	45	14	83	52	21	91	61	31	01			17
18	08	76	45	13	82	51	21	90	60	30	00			18
19	07	75	44	13	82	51	20	90	60	30	00			19
20	07	75	43	12	81	50	20	89	59	29	00	00		20
21	06	74	43	12	81	50	19	89	59	29	00			21
22	06	74	42	11	80	49	19	89	58	28	00			22
23	05	73	42	11	80	49	18	88	58	28	00			23
24	05	73	41	10	79	48	18	87	57	27	00			24
25	04	72	41	10	79	48	17	87	57	27	00			25
26	04	72	40	09	78	47	17	86	56	26	00			26
27	03	71	40	09	78	47	16	86	56	26	00			27
28	02	71	39	08	77	46	16	85	55	25	00			28
29	02	70	39	08	76	46	15	85	55	25	00			29
30	01	70	38	07	76	45	15	84	54	24	00			30
31	01	69	38	06	75	45	14	84	54	24	00			31
32	00	69	37	06	75	44	14	83	53	23	00			32
33	00	68	37	05	74	44	13	83	53	23	00			33
34	1199	68	36	05	74	43	13	82	52	22	00			34
35	00	67	36	04	73	43	12	82	52	22	00			35
36	00	67	35	04	73	42	12	81	51	21	00			36
37	00	66	35	03	72	42	11	81	51	21	00			37
38	00	65	34	03	72	41	11	80	50	20	00			38
39	00	65	34	02	71	41	10	80	50	20	00			39
40	00	64	33	02	71	40	09	79	49	19	00			40
41	00	64	32	01	70	40	08	79	49	19	00			41
42	00	63	32	01	70	39	08	78	48	18	00			42
43	00	63	31	00	69	39	08	78	48	18	00			43
44	00	62	31	00	69	38	07	77	47	17	00			44
45	00	62	30	1099	68	37	07	77	47	17	00			45
46	00	61	30	99	68	37	06	76	46	16	00			46
47	00	61	29	98	67	36	06	76	46	16	00			47
48	00	60	29	98	67	36	05	75	45	15	00			48
49	00	60	28	97	66	35	05	75	45	15	00			49
50	00	59	28	97	66	35	04	74	44	14	00			50
51	00	59	27	96	65	34	04	74	44	14	00			51
52	00	58	27	96	65	34	03	73	43	13	00			52
53	00	58	26	95	64	33	03	73	43	13	00			53
54	00	57	26	95	64	33	02	72	42	12	00			54
55	00	57	25	94	63	32	02	72	42	12	00			55
56	00	56	25	94	63	32	01	71	41	11	00			56
57	00	56	24	93	62	31	01	71	41	11	00			57
58	00	55	24	93	62	31	00	70	40	10	00			58
59	00	54	23	92	61	30	00	70	40	10	00			59
"	30 16'	30 17'	30 18'	30 19'	30 20'	30 21'	30 22'	30 23'	30 24'	30 25'	30 26'			"
"	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m			"

TABLE III.—PROPORTIONAL LOGARITHMS.

°	' 27'		' 28'		' 29'		' 30'		' 31'		' 32'		' 33'		' 34'		' 35'		' 36'		' 37'		°
	h m	a m	h m	a m	h m	a m	h m	a m	h m	a m	h m	a m	h m	a m	h m	a m	h m	a m	h m	a m	h m	a m	
0	0820	0850	0821	0792	0763	0734	0706	0678	0649	0621	0594	0	0										
1	79	50	90	91	62	34	05	77	49	21	93	1	1										
2	79	49	90	91	62	33	05	77	48	21	93	2	2										
3	78	49	19	90	62	33	04	76	48	20	92	3	3										
4	78	48	19	90	61	32	04	76	48	20	92	4	4										
5	77	48	18	89	61	32	03	75	47	19	91	5	5										
6	77	47	18	89	60	31	03	75	47	19	91	6	6										
7	76	47	17	88	60	31	03	74	46	18	91	7	7										
8	76	46	17	88	59	30	02	74	46	18	90	8	8										
9	75	46	16	87	59	30	02	73	45	17	90	9	9										
10	75	45	16	87	58	30	01	73	45	17	89	10	10										
11	74	45	16	87	58	29	01	72	44	16	89	11	11										
12	74	44	15	86	57	29	00	72	44	16	88	12	12										
13	73	44	15	86	57	28	00	71	43	15	88	13	13										
14	73	43	14	85	56	28	00	71	43	15	87	14	14										
15	72	43	14	85	56	27	99	70	42	15	87	15	15										
16	72	42	13	84	55	27	98	70	42	14	86	16	16										
17	71	42	13	84	55	26	98	70	41	14	86	17	17										
18	71	41	12	83	54	26	97	69	41	13	85	18	18										
19	70	41	12	83	54	25	97	69	41	13	85	19	19										
20	70	40	11	82	53	25	96	68	40	12	85	20	20										
21	69	40	11	82	53	24	96	68	40	12	84	21	21										
22	69	39	10	81	52	24	95	67	39	11	84	22	22										
23	68	39	10	81	52	23	95	67	39	11	83	23	23										
24	68	38	09	80	51	23	94	66	38	10	83	24	24										
25	67	38	09	80	51	22	94	66	38	10	82	25	25										
26	67	37	08	79	51	22	94	65	37	09	82	26	26										
27	66	37	08	79	50	21	93	65	37	09	81	27	27										
28	66	36	07	78	50	21	93	64	36	09	81	28	28										
29	65	36	07	78	49	21	92	64	36	08	80	29	29										
30	65	35	06	77	49	20	92	63	35	08	80	30	30										
31	64	35	06	77	48	20	91	63	35	07	79	31	31										
32	64	34	05	76	48	19	91	63	34	07	79	32	32										
33	63	34	05	76	47	19	90	62	34	06	79	33	33										
34	63	34	04	75	47	18	90	62	34	06	78	34	34										
35	62	33	04	75	46	18	89	61	33	05	78	35	35										
36	62	33	03	74	46	17	89	61	33	05	77	36	36										
37	61	32	03	74	45	17	88	60	32	04	77	37	37										
38	61	32	02	74	45	16	88	60	32	04	76	38	38										
39	60	31	02	73	44	16	87	59	31	03	76	39	39										
40	60	31	01	73	44	15	87	59	31	03	75	40	40										
41	59	30	01	72	43	15	86	58	30	02	75	41	41										
42	59	30	01	72	43	14	86	58	30	02	74	42	42										
43	58	29	00	71	42	14	86	57	29	02	74	43	43										
44	58	29	00	71	42	13	85	57	29	01	73	44	44										
45	57	28	0799	70	41	13	85	56	28	01	73	45	45										
46	57	28	99	70	41	12	84	56	28	00	73	46	46										
47	56	27	98	69	40	12	84	55	28	00	72	47	47										
48	56	27	98	69	40	11	83	55	27	0599	72	48	48										
49	55	26	97	68	40	11	83	55	27	99	71	49	49										
50	55	26	97	68	39	11	82	54	26	98	71	50	50										
51	55	25	96	67	39	10	82	54	26	98	70	51	51										
52	54	25	96	67	38	10	81	53	25	97	70	52	52										
53	54	24	95	66	38	09	81	53	25	97	69	53	53										
54	53	24	95	66	37	09	80	52	24	96	69	54	54										
55	53	23	94	65	37	08	80	52	24	96	68	55	55										
56	52	23	94	65	36	08	79	51	23	96	68	56	56										
57	52	22	93	64	36	07	79	51	23	95	68	57	57										
58	51	22	93	64	35	07	78	50	22	95	67	58	58										
59	51	21	92	63	35	06	78	50	22	94	67	59	59										

TABLE III.—PROPORTIONAL LOGARITHMS.

"	90 38'	90 39'	90 40'	90 41'	90 42'	90 43'	90 44'	90 45'	90 46'	90 47'	90 48'	"
"	a m	a m	a m	a m	a m	a m	a m	a m	a m	a m	a m	"
0	0566	0530	0512	0494	0458	0431	0404	0378	0352	0326	0300	0
1	66	36	11	84	57	30	04	77	51	25	99	1
2	65	36	11	84	57	30	03	77	51	25	99	2
3	65	37	10	83	56	30	03	77	50	24	98	3
4	64	37	10	83	56	29	03	76	50	24	98	4
5	64	36	09	82	55	29	02	76	49	23	97	5
6	63	36	09	82	55	28	02	75	49	23	97	6
7	63	36	08	81	54	28	01	75	49	23	97	7
8	62	35	08	81	54	27	01	74	48	22	96	8
9	62	35	07	80	54	27	00	74	48	22	96	9
10	62	34	07	80	53	26	00	74	47	21	95	10
11	61	34	07	80	53	26	0399	73	47	21	95	11
12	61	33	06	79	52	26	90	73	46	20	94	12
13	60	33	06	79	52	25	99	72	46	20	94	13
14	60	32	05	78	51	25	98	72	46	19	94	14
15	59	32	05	78	51	24	98	71	45	19	93	15
16	59	31	04	77	50	24	97	71	45	19	93	16
17	58	31	04	77	50	23	97	70	44	18	92	17
18	58	31	03	76	50	23	96	70	44	18	92	18
19	57	30	03	76	49	22	96	70	43	17	91	19
20	57	30	02	75	49	22	95	69	43	17	91	20
21	57	29	02	75	48	22	95	69	42	16	91	21
22	56	29	02	75	48	21	95	68	42	16	90	22
23	56	28	01	74	47	21	94	68	42	16	90	23
24	55	28	01	74	47	20	94	67	41	15	89	24
25	55	27	00	73	46	20	93	67	41	15	89	25
26	54	27	00	73	46	19	93	66	40	14	88	26
27	54	26	0099	72	46	19	92	66	40	14	88	27
28	53	26	99	72	45	18	92	66	39	13	88	28
29	53	26	98	71	45	18	92	65	39	13	87	29
30	52	25	98	71	44	18	91	65	39	13	87	30
31	52	25	98	71	44	17	91	64	38	12	86	31
32	52	24	97	70	43	17	90	64	38	12	86	32
33	51	24	97	70	43	16	90	63	37	11	85	33
34	51	23	96	69	42	16	89	63	37	11	85	34
35	50	23	96	69	42	15	89	63	36	10	85	35
36	50	22	95	68	42	15	88	62	36	10	84	36
37	49	22	95	68	41	14	88	62	36	10	84	37
38	49	21	94	67	41	14	88	61	35	09	83	38
39	48	21	94	67	40	14	87	61	35	09	83	39
40	48	21	93	67	40	13	87	60	34	08	82	40
41	47	20	93	66	39	13	86	60	34	08	82	41
42	47	20	93	66	39	12	86	59	33	07	82	42
43	46	19	92	65	38	12	85	59	33	07	81	43
44	46	19	92	65	38	11	85	59	33	07	81	44
45	46	18	91	64	38	11	84	58	32	06	80	45
46	45	18	91	64	37	10	84	58	32	06	80	46
47	45	17	90	63	37	10	84	57	31	05	79	47
48	44	17	90	63	36	10	83	57	31	05	79	48
49	44	17	89	62	36	09	83	56	30	04	79	49
50	43	16	89	62	35	09	82	56	30	04	78	50
51	43	16	89	62	35	08	82	56	29	04	78	51
52	42	15	88	61	34	08	81	55	29	03	77	52
53	42	15	88	61	34	07	81	55	29	03	77	53
54	41	14	87	60	34	07	81	54	28	02	76	54
55	41	14	87	60	33	06	80	54	28	02	76	55
56	41	13	86	59	33	06	80	53	27	01	76	56
57	40	13	86	59	32	06	79	53	27	01	75	57
58	40	12	85	58	32	05	79	53	26	00	75	58
59	39	12	85	58	31	05	78	52	26	00	74	59
"	90 38'	90 39'	90 40'	90 41'	90 42'	90 43'	90 44'	90 45'	90 46'	90 47'	90 48'	"
"	a m	a m	a m	a m	a m	a m	a m	a m	a m	a m	a m	"

TABLE III.—PROPORTIONAL LOGARITHMS.

"	h = 30 40'	h = 30 50'	h = 30 51'	h = 30 52'	h = 30 53'	h = 30 54'	h = 30 55'	h = 30 56'	h = 30 57'	h = 30 58'	h = 30 59'	"
0	0274	0248	0223	0197	0173	0147	0122	0098	0073	0049	0024	0
1	73	48	23	97	73	47	22	97	73	48	24	1
2	73	47	23	97	71	46	22	97	73	48	23	2
3	73	47	21	96	71	46	21	96	73	47	23	3
4	72	47	21	96	71	46	21	96	71	47	23	4
5	72	46	21	95	70	45	20	96	71	46	22	5
6	71	46	20	95	70	45	20	95	71	46	22	6
7	71	45	20	94	69	44	19	95	70	45	21	7
8	70	45	19	94	69	44	19	94	70	45	21	8
9	70	44	19	94	69	43	19	94	69	45	21	9
10	70	44	19	93	68	43	18	93	69	44	20	10
11	69	44	18	93	68	43	18	93	68	44	20	11
12	69	43	18	92	67	42	17	93	68	44	19	12
13	68	43	17	92	67	42	17	92	68	43	19	13
14	68	42	17	92	66	41	17	92	67	43	19	14
15	67	42	16	91	66	41	16	91	67	42	18	15
16	67	41	16	91	66	41	16	91	66	42	18	16
17	67	41	16	90	65	40	15	91	66	42	17	17
18	66	41	15	90	65	40	15	90	66	41	17	18
19	66	40	15	89	64	39	14	90	65	41	17	19
20	65	40	14	89	64	39	14	89	65	40	16	20
21	65	39	14	89	63	38	14	89	64	40	16	21
22	64	39	13	88	63	38	13	89	64	40	15	22
23	64	38	13	88	63	38	13	88	64	39	15	23
24	64	38	13	87	62	37	12	88	63	39	15	24
25	63	38	12	87	62	37	12	87	63	38	14	25
26	63	37	12	87	61	36	12	87	62	38	14	26
27	62	37	11	86	61	36	11	87	62	38	13	27
28	62	36	11	86	61	36	11	86	62	37	13	28
29	61	36	11	85	60	35	10	86	61	37	12	29
30	61	35	10	85	60	35	10	85	61	36	12	30
31	61	35	10	84	59	34	10	85	60	36	12	31
32	60	35	09	84	59	34	09	84	60	36	11	32
33	60	34	09	84	58	34	09	84	60	35	11	33
34	60	34	08	83	58	33	08	84	59	35	10	34
35	59	33	08	83	58	33	08	83	59	34	10	35
36	59	33	08	82	57	32	07	83	58	34	10	36
37	58	33	07	82	57	32	07	82	58	34	09	37
38	58	32	07	81	56	31	07	82	57	33	09	38
39	57	32	06	81	56	31	06	82	57	33	08	39
40	57	31	06	81	56	31	06	81	57	32	08	40
41	56	31	05	80	55	30	05	81	56	32	08	41
42	56	30	05	80	55	30	05	80	56	31	07	42
43	55	30	05	79	54	29	05	80	55	31	07	43
44	55	30	04	79	54	29	04	80	55	31	06	44
45	55	29	04	79	53	29	04	79	55	30	06	45
46	54	29	03	78	53	28	03	79	54	30	06	46
47	54	28	03	78	53	28	03	78	54	29	05	47
48	53	28	02	77	52	27	03	78	53	29	05	48
49	53	27	02	77	52	27	02	77	53	29	04	49
50	52	27	02	76	51	26	02	77	52	28	04	50
51	52	27	01	76	51	26	01	77	52	28	04	51
52	52	26	01	76	51	26	01	76	52	27	03	52
53	51	26	00	75	50	25	00	76	51	27	03	53
54	51	25	00	75	50	25	00	75	51	27	02	54
55	50	25	00	74	49	24	00	75	50	26	02	55
56	50	24	00	74	49	24	00	75	50	26	02	56
57	50	24	00	74	48	24	00	74	50	25	01	57
58	49	24	00	73	48	23	00	74	49	25	01	58
59	49	23	00	73	48	23	00	73	49	25	00	59

TABLE IV.

**COURSES, DISTANCE, DEPARTURE, AND
DIFFERENCE OF LATITUDE.**

Distance.	C. 1 Pt.		C. 2 Pt.		C. 3 Pt.		C. 4 Pt.		C. 5 Pt.		C. 6 Pt.		C. 7 Pt.		C. 8 Pt.	
	N. by E.	S. by W.	N. by E.	S. by W.	N. by E.	S. by W.	N. by E.	S. by W.	N. by E.	S. by W.	N. by E.	S. by W.	N. by E.	S. by W.	N. by E.	S. by W.
1	1	0	1	.1	1	0.1	1	0	1	0	1	0	1	0	1	0
2	2	0.1	2	0.2	2	0.3	2	0.4	2	0.5	2	0.6	2	0.7	2	0.8
3	3	0.1	3	0.3	3	0.4	3	0.5	3	0.6	3	0.7	3	0.8	3	0.9
4	4	0.2	4	0.4	4	0.6	4	0.8	4	1.0	4	1.2	4	1.4	4	1.6
5	5	0.2	5	0.5	5	0.7	5	1.0	5	1.3	5	1.6	5	1.9	5	2.2
6	6	0.3	6	0.6	6	0.9	6	1.2	6	1.6	6	2.0	6	2.4	6	2.8
7	7	0.3	7	0.7	7	1.0	7	1.4	7	1.8	7	2.2	7	2.6	7	3.0
8	8	0.4	8	0.8	8	1.2	8	1.6	8	2.0	8	2.4	8	2.8	8	3.2
9	9	0.4	9	0.9	9	1.3	9	1.8	9	2.3	9	2.8	9	3.3	9	3.8
10	10	0.5	10	1	10	1.5	10	2	10	2.5	10	3	10	3.5	10	4
11	11	0.5	10.9	1.1	10.9	1.6	10.8	2.1	10.7	2.7	10.5	3.2	10.4	3.7	10.2	4.2
12	12	0.6	11.9	1.2	11.9	1.8	11.8	2.3	11.6	2.9	11.5	3.5	11.3	4	11.1	4.6
13	13	0.6	12.9	1.3	12.9	1.9	12.8	2.5	12.6	3.2	12.4	3.8	12.2	4.4	12	5
14	14	0.7	13.9	1.4	13.8	2.1	13.7	2.8	13.6	3.4	13.4	4.1	13.2	4.7	13	5.4
15	15	0.7	14.9	1.5	14.8	2.2	14.7	2.9	14.6	3.6	14.4	4.4	14.1	5.1	13.9	5.7
16	16	0.8	15.9	1.6	15.8	2.3	15.7	3.1	15.5	3.9	15.3	4.6	15.1	5.4	14.9	6.1
17	17	0.8	16.9	1.7	16.8	2.5	16.7	3.3	16.5	4.1	16.3	4.9	16	5.7	15.8	6.5
18	18	0.9	17.9	1.8	17.8	2.6	17.7	3.5	17.5	4.4	17.3	5.2	17.1	6	16.8	6.9
19	19	0.9	18.9	1.9	18.8	2.8	18.6	3.7	18.4	4.6	18.2	5.5	17.9	6.4	17.6	7.1
20	20	1	19.9	2	19.8	2.9	19.6	3.9	19.4	4.9	19.1	5.8	18.8	6.7	18.5	7.7
21	21	1.1	20.9	2.1	20.8	3.1	20.6	4.1	20.4	5.1	20.1	6.1	19.8	7.1	19.4	8
22	22	1.1	21.9	2.2	21.8	3.3	21.6	4.3	21.3	5.3	21.1	6.4	20.7	7.4	20.3	8.4
23	23	1.1	22.9	2.3	22.8	3.4	22.6	4.5	22.3	5.6	22	6.7	21.7	7.7	21.3	8.9
24	24	1.2	23.9	2.4	23.7	3.5	23.5	4.7	23.3	5.8	23	7	22.6	8.1	22.2	9.2
25	25	1.2	24.9	2.5	24.7	3.7	24.5	4.9	24.3	6.1	23.9	7.3	23.5	8.4	23.1	9.6
26	26	1.3	25.9	2.6	25.7	3.8	25.5	5.1	25.3	6.3	24.9	7.5	24.5	8.6	24	9.9
27	27	1.3	26.9	2.6	26.7	4	26.5	5.3	26.3	6.6	25.8	7.8	25.4	9.1	24.9	10.1
28	28	1.4	27.9	2.7	27.7	4.1	27.5	5.5	27.3	6.8	26.8	8.1	26.4	9.4	25.9	10.7
29	29	1.4	28.9	2.8	28.7	4.3	28.4	5.7	28.1	7	27.8	8.4	27.3	9.6	26.7	11.1
30	30	1.5	29.9	2.9	29.7	4.4	29.4	5.9	29.1	7.3	28.7	8.7	28.2	10.1	27.6	11.5
31	31	1.5	30.8	3	30.7	4.5	30.4	6	30.1	7.5	29.7	9	29.2	10.4	28.6	11.9
32	32	1.6	31.8	3.1	31.7	4.7	31.4	6.2	31.1	7.8	30.6	9.3	30.1	10.8	29	12.2
33	33	1.6	32.8	3.2	32.6	4.8	32.4	6.4	32	8	31.6	9.6	31.1	11.1	30.5	12.6
34	34	1.7	33.8	3.3	33.6	5	33.3	6.6	33	8.3	32.5	9.9	32	11.5	31.4	13
35	35	1.7	34.8	3.4	34.6	5.1	34.3	6.8	34	8.5	33.5	10.2	33	11.8	32.4	13.4
36	36	1.8	35.8	3.5	35.6	5.3	35.3	7	34.9	8.7	34.4	10.5	33.9	12.1	32.3	13.8
37	37	1.8	36.8	3.6	36.6	5.4	36.3	7.2	35.9	9	35.4	10.7	34.8	12.5	32.9	14.2
38	38	1.9	37.8	3.7	37.6	5.6	37.3	7.4	36.9	9.2	36.4	11	35.6	12.9	33.1	14.5
39	39	1.9	38.8	3.8	38.6	5.7	38.3	7.6	37.8	9.5	37.3	11.3	36.7	13.1	33	14.9
40	40	2	39.8	3.9	39.6	5.9	39.2	7.8	38.8	9.7	38.3	11.6	37.7	13.5	37	15.3
41	41	2	40.8	4	40.6	6	40.2	8	39.8	10	39.2	11.9	38.6	13.9	37.9	15.7
42	41.9	2.1	41.8	4.1	41.5	6.2	41.2	8.2	40.7	10.2	40.2	12.3	39.5	14.1	38.9	16.1
43	42.9	2.1	42.8	4.2	42.5	6.3	42.2	8.4	41.7	10.4	41.1	12.5	40.5	14.5	39.7	16.5
44	43.9	2.2	43.8	4.3	43.5	6.5	43.2	8.6	42.7	10.7	42.1	12.8	41.4	14.9	40.7	16.9
45	44.9	2.2	44.8	4.4	44.5	6.6	44.1	8.8	43.7	10.9	43.1	13.1	42.4	15.3	41.6	17.3
46	45.9	2.3	45.8	4.5	45.5	6.7	45.1	9	44.6	11.2	44	13.4	43.3	15.5	42.4	17.6
47	46.9	2.3	46.8	4.6	46.5	6.9	46.1	9.2	45.6	11.4	45	13.6	44.3	15.9	43.5	18
48	47.9	2.4	47.8	4.7	47.5	7	47.1	9.4	46.6	11.7	45.9	13.9	45.2	16.1	44.3	18.4
49	48.9	2.4	48.8	4.8	48.5	7.2	48.1	9.6	47.5	11.9	47	14.2	46.1	16.5	45.3	18.8
50	49.9	2.5	49.8	4.9	49.5	7.3	49	9.8	48.5	12.1	47.8	14.5	47.1	16.8	46.2	19.1
	dep	d. lat.	dep	d. lat.	dep	d. lat.	dep	d. lat.	dep	d. lat.	dep	d. lat.	dep	d. lat.	dep	d. lat.
	W. by N.	W. by S.	E. by N.	E. by S.	W. by N.	W. by S.	E. by N.	E. by S.	W. by N.	W. by S.	E. by N.	E. by S.	W. by N.	W. by S.	E. by N.	E. by S.
	C. 7 1/2 Pts.		C. 7 1/2 Pts.		C. 7 1/2 Pts.		C. 7 Pts.		C. 6 1/2 Pts.		C. 6 1/2 Pts.		C. 6 Pts.		C. 6 Pts.	

Table with 8 main columns for distance intervals (1-2, 2-3, 3-4, 4-5, 5-6, 6-7, 7-8, 8-9 miles) and one final 'Distance' column. Each interval column contains 8 rows of departure/arrival times and directional codes (e.g., N. E. by E., S. E. by E., etc.).

Distance.	C. 1 Pt.		C. 1 Pt.		C. 1 Pt.		C. 1 Pt.		C. 1 Pt.		C. 1 Pt.		C. 2 Pts.	
	N. by E.		S. by W.		N. by E.		S. by W.		N. by E.		S. by W.		N. by E.	
	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.
	N. by E.	S. by W.	N. by E.	S. by W.	N. by E.	S. by W.	N. by E.	S. by W.	N. by E.	S. by W.	N. by E.	S. by W.	N. by E.	S. by W.
51	50.9	2.5	50.8	4.8	50.4	7.5	50.	9.9	49.5	12.4	48.8	14.8	48.	17.2
52	51.0	2.6	51.7	5.1	51.4	7.6	51.	10.1	50.4	12.6	49.8	15.1	49.	17.5
53	51.9	2.6	52.7	5.3	52.4	7.8	52.	10.3	51.4	12.9	50.7	15.4	49.	17.9
54	52.9	2.7	53.7	5.4	53.4	7.9	53.	10.5	52.4	13.1	51.7	15.7	50.6	18.2
55	53.9	2.7	54.7	5.4	54.4	8.1	53.	10.7	53.4	13.4	52.6	16.	51.6	18.5
56	55.0	2.7	55.7	5.5	55.4	8.2	54.	10.9	54.3	13.6	53.6	16.3	52.7	18.9
57	56.0	2.8	56.7	5.5	56.4	8.4	55.	11.1	55.3	13.8	54.6	16.5	53.7	19.2
58	57.0	2.8	57.7	5.6	57.4	8.5	55.	11.3	56.3	14.1	55.5	16.8	54.6	19.5
59	58.0	2.9	58.7	5.6	58.4	8.7	57.	11.5	57.2	14.3	56.5	17.1	55.6	19.8
60	59.0	2.9	59.7	5.7	59.4	8.8	57.	11.7	58.2	14.6	57.4	17.4	56.5	20.1
61	60.0	3.	60.7	5.8	60.3	9.	58.	11.9	59.2	14.8	58.4	17.7	57.4	20.6
62	61.0	3.1	61.7	5.9	61.3	9.1	58.	12.1	60.2	15.1	59.3	18.	58.4	20.9
63	62.0	3.1	62.7	6.	61.8	9.2	59.	12.3	61.1	15.3	60.3	18.3	59.3	21.2
64	63.0	3.1	63.7	6.1	63.3	9.4	59.	12.5	62.1	15.6	61.2	18.6	60.3	21.6
65	64.0	3.2	64.7	6.2	63.8	9.5	59.	12.7	63.1	15.8	62.1	18.9	61.2	21.9
66	65.0	3.2	65.7	6.3	65.3	9.7	64.7	12.9	64.	16.	63.2	19.2	62.1	22.2
67	66.0	3.3	66.7	6.4	66.3	9.8	65.7	13.1	65.	16.3	64.1	19.4	63.1	22.6
68	67.0	3.4	67.7	6.5	67.3	10.	66.7	13.3	66.	16.5	65.1	19.7	64.	22.9
69	68.0	3.4	68.7	6.6	68.3	10.1	67.7	13.5	66.9	16.8	66.	20.	65.	23.2
70	69.0	3.5	69.7	6.7	69.3	10.3	68.7	13.7	67.9	17.	67.	20.3	66.	23.6
71	70.0	3.5	70.7	7.	70.3	10.4	69.6	13.9	68.9	17.3	67.9	20.6	66.8	23.9
72	71.0	3.6	71.7	7.1	71.3	10.6	70.6	14.	69.8	17.5	68.9	20.9	67.8	24.3
73	72.0	3.6	72.6	7.2	72.2	10.7	71.6	14.2	70.8	17.7	69.9	21.2	68.7	24.6
74	73.0	3.7	73.6	7.3	73.3	10.9	72.6	14.4	71.8	18.	70.8	21.5	69.7	24.9
75	74.0	3.7	74.6	7.4	73.9	11.	73.6	14.6	72.8	18.2	71.8	21.8	70.6	25.3
76	75.0	3.8	75.6	7.4	75.2	11.2	74.5	14.8	73.7	18.5	72.7	22.1	71.6	25.6
77	76.0	3.8	76.6	7.5	76.3	11.3	75.5	15.	74.7	18.7	73.7	22.4	72.5	25.9
78	77.0	3.9	77.6	7.6	77.3	11.4	76.5	15.2	75.7	19.	74.6	22.6	73.4	26.3
79	78.0	3.9	78.6	7.7	78.1	11.6	77.5	15.4	76.6	19.2	75.6	22.9	74.4	26.6
80	79.0	4.	79.6	7.8	79.1	11.7	78.5	15.6	77.6	19.4	76.6	23.2	75.3	27.
81	80.0	4.	80.6	7.9	80.1	11.9	79.4	15.8	78.6	19.7	77.5	23.5	76.3	27.3
82	81.0	4.1	81.6	8.	81.1	12.	80.4	16.	79.5	19.9	78.5	23.8	77.2	27.6
83	82.0	4.1	82.6	8.1	82.1	12.2	81.4	16.2	80.5	20.2	79.4	24.1	78.1	28.
84	83.0	4.2	83.6	8.2	83.1	12.3	82.4	16.4	81.5	20.4	80.4	24.4	79.1	28.3
85	84.0	4.3	84.6	8.3	84.1	12.5	83.4	16.6	82.5	20.7	81.3	24.7	80.	28.6
86	85.0	4.3	85.6	8.4	85.1	12.6	84.3	16.8	83.4	20.9	82.3	25.	81.	29.
87	86.0	4.3	86.6	8.5	86.1	12.8	85.3	17.	84.4	21.1	83.3	25.3	82.	29.3
88	87.0	4.4	87.6	8.6	87.	12.9	86.3	17.2	85.4	21.4	84.2	25.5	83.	29.6
89	88.0	4.4	88.6	8.7	88.	13.1	87.3	17.4	86.3	21.6	85.2	25.8	84.	30.
90	89.0	4.5	89.6	8.8	89.	13.2	88.3	17.6	87.3	21.9	86.1	26.	85.	30.3
91	90.0	4.5	90.6	8.9	90.	13.4	89.3	17.8	88.3	22.1	87.1	26.4	86.	30.6
92	91.0	4.6	91.6	9.	91.	13.5	90.2	17.9	89.2	22.4	88.	26.7	87.	31.
93	92.0	4.6	92.6	9.1	92.	13.6	91.2	18.1	90.2	22.6	89.	27.	88.	31.3
94	93.0	4.7	93.6	9.2	93.	13.8	92.2	18.3	91.2	22.8	90.	27.3	89.	31.7
95	94.0	4.7	94.6	9.3	94.	13.9	93.2	18.5	92.2	23.1	90.9	27.6	90.	32.
96	95.0	4.8	95.6	9.5	95.	14.1	94.2	18.7	93.1	23.3	91.9	27.9	90.	32.3
97	96.0	4.8	96.6	9.5	96.	14.2	95.1	18.9	94.1	23.6	92.8	28.2	91.	32.7
98	97.0	4.9	97.6	9.6	97.	14.4	96.1	19.1	95.1	23.8	93.7	28.5	92.	33.
99	98.0	4.9	98.6	9.7	98.	14.5	97.1	19.3	96.	24.1	94.7	28.8	93.	33.3
100	99.0	4.9	99.6	9.8	99.	14.7	98.1	19.5	97.	24.3	95.7	29.	94.	33.7
	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.
	W. 1/4 N.	E. 1/4 N.	W. 1/4 N.	E. 1/4 N.	W. 1/4 N.	E. 1/4 N.	W. by N.	E. by N.	W. by N. 1/2 N.	E. by N. 1/2 N.	W. by N. 1/4 N.	E. by N. 1/4 N.	W. by N. 1/4 N.	E. by N. 1/4 N.
	C. 7 1/2 Pts.	C. 7 1/2 Pts.	C. 7 1/2 Pts.	C. 7 Pts.	C. 6 1/2 Pts.	C. 6 1/2 Pts.	C. 6 1/2 Pts.	C. 6 1/2 Pts.	C. 6 1/2 Pts.	C. 6 1/2 Pts.	C. 6 Pts.	C. 6 Pts.	C. 6 Pts.	C. 6 Pts.

C. 2½ Pts.		C. 2½ Pts.		C. 2½ Pts.		C. 3 Pts.		C. 3½ Pts.		C. 3½ Pts.		C. 4 Pts.		Distance.	
N. N. E. † E.	S. S. E. † E.	S. E. W. † W.	N. N. W. † W.	N. N. E. † E.	S. S. E. † E.	S. E. W. † W.	N. N. W. † W.	N. E. † N.	S. E. † S.	S. W. † S.	N. W. † N.	N. E. † E.	S. E. † S.		S. W. † S.
d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.
46.1	91.8	45.	94.	43.7	95.9	42.4	96.3	41.1	97.4	39.4	98.4	37.8	99.9	36.1	51
47.0	92.7	45.9	94.5	44.7	96.8	43.9	97.2	41.8	98.3	40.3	99.3	38.5	100.3	36.8	52
47.8	93.7	46.7	95.3	45.6	97.7	44.1	98.4	42.6	99.6	41.7	100.6	39.3	101.6	37.5	53
48.6	94.1	47.6	95.5	46.5	98.5	44.9	99.4	43.4	100.9	42.5	101.9	40.0	102.9	38.2	54
49.7	95.5	48.5	96.9	47.4	99.9	45.7	100.6	44.2	102.8	43.3	103.8	40.8	104.8	38.9	55
50.6	96.9	49.4	98.4	48.	101.8	46.6	103.1	45.	105.4	44.1	106.4	41.5	107.6	39.6	56
51.5	98.4	50.3	99.9	48.9	103.8	47.5	104.7	45.8	107.9	44.9	109.9	42.2	110.9	40.3	57
52.4	99.8	51.2	101.3	49.8	105.8	48.4	106.2	46.6	111.4	45.8	112.9	43.	113.9	41.	58
53.3	101.2	52.1	102.8	50.7	107.8	49.3	107.7	47.4	113.9	46.6	114.9	43.7	115.9	41.7	59
54.2	102.7	53.0	104.3	51.6	109.8	50.2	109.2	48.2	116.4	47.4	116.9	44.5	116.9	42.4	60
55.1	104.1	53.9	105.8	52.5	111.8	51.1	110.7	49.	118.9	48.2	117.9	45.2	117.9	43.1	61
56.0	105.6	54.8	107.3	53.4	113.8	52.0	112.2	49.8	121.4	49.0	118.9	45.9	118.9	43.8	62
57.0	107.0	55.7	108.8	54.3	115.8	52.9	113.7	50.6	123.9	49.8	119.9	46.7	119.9	44.5	63
57.9	108.5	56.6	110.3	55.2	117.8	53.8	115.2	51.4	126.4	50.6	120.9	47.4	120.9	45.2	64
58.8	109.9	57.5	111.8	56.1	119.8	54.7	116.7	52.2	128.9	51.4	121.9	48.2	121.9	46.	65
59.7	111.4	58.4	113.3	57.0	121.8	55.6	118.2	53.	131.4	52.2	122.9	48.9	122.9	46.7	66
60.6	112.8	59.3	114.8	57.9	123.8	56.5	119.7	53.8	133.9	53.0	123.9	49.6	123.9	47.4	67
61.5	114.3	60.2	116.3	58.8	125.8	57.4	121.2	54.6	136.4	53.8	124.9	50.4	124.9	48.1	68
62.4	115.7	61.1	117.8	59.7	127.8	58.3	122.7	55.4	138.9	54.6	125.9	51.1	125.9	48.8	69
63.3	117.2	62.0	119.3	60.6	129.8	59.2	124.2	56.2	141.4	55.4	126.9	51.9	126.9	49.5	70
64.2	118.6	62.9	120.8	61.5	131.8	60.1	125.7	57.	143.9	56.2	127.9	52.6	127.9	50.2	71
65.1	120.1	63.8	122.3	62.4	133.8	61.0	127.2	57.8	146.4	57.0	128.9	53.3	128.9	50.9	72
66.0	121.5	64.7	123.8	63.3	135.8	61.9	128.7	58.6	148.9	57.8	129.9	54.1	129.9	51.6	73
66.9	123.0	65.6	125.3	64.2	137.8	62.8	130.2	59.4	151.4	58.6	130.9	54.8	130.9	52.3	74
67.8	124.4	66.5	126.8	65.1	139.8	63.7	131.7	60.2	153.9	59.4	131.9	55.6	131.9	53.	75
68.7	125.9	67.4	128.3	66.0	141.8	64.6	133.2	61.	156.4	60.2	132.9	56.3	132.9	53.7	76
69.6	127.3	68.3	129.8	66.9	143.8	65.5	134.7	61.8	158.9	61.0	133.9	57.1	133.9	54.4	77
70.5	128.8	69.2	131.3	67.8	145.8	66.4	136.2	62.6	161.4	61.8	134.9	57.9	134.9	55.1	78
71.4	130.2	70.1	132.8	68.7	147.8	67.3	137.7	63.4	163.9	62.6	135.9	58.7	135.9	55.8	79
72.3	131.7	71.0	134.3	69.6	149.8	68.2	139.2	64.2	166.4	63.4	136.9	59.5	136.9	56.5	80
73.2	133.1	71.9	135.8	70.5	151.8	69.1	140.7	65.0	168.9	64.2	137.9	60.3	137.9	57.2	81
74.1	134.6	72.8	137.3	71.4	153.8	70.0	142.2	65.8	171.4	65.0	138.9	61.1	138.9	57.9	82
75.0	136.0	73.7	138.8	72.3	155.8	70.9	143.7	66.6	173.9	65.8	139.9	61.9	139.9	58.6	83
75.9	137.5	74.6	140.3	73.2	157.8	71.8	145.2	67.4	176.4	66.6	140.9	62.7	140.9	59.3	84
76.8	138.9	75.5	141.8	74.1	159.8	72.7	146.7	68.2	178.9	67.4	141.9	63.5	141.9	60.0	85
77.7	140.4	76.4	143.3	75.0	161.8	73.6	148.2	69.0	181.4	68.2	142.9	64.3	142.9	60.7	86
78.6	141.8	77.3	144.8	75.9	163.8	74.5	149.7	69.8	183.9	69.0	143.9	65.1	143.9	61.4	87
79.5	143.3	78.2	146.3	76.8	165.8	75.4	151.2	70.6	186.4	69.8	144.9	65.9	144.9	62.1	88
80.4	144.7	79.1	147.8	77.7	167.8	76.3	152.7	71.4	188.9	70.6	145.9	66.7	145.9	62.8	89
81.3	146.2	80.0	149.3	78.6	169.8	77.2	154.2	72.2	191.4	71.4	146.9	67.5	146.9	63.5	90
82.2	147.6	80.9	150.8	79.5	171.8	78.1	155.7	73.0	193.9	72.2	147.9	68.3	147.9	64.2	91
83.1	149.1	81.8	152.3	80.4	173.8	79.0	157.2	73.8	196.4	73.0	148.9	69.1	148.9	64.9	92
84.0	150.5	82.7	153.8	81.3	175.8	79.9	158.7	74.6	198.9	73.8	149.9	69.9	149.9	65.6	93
85.0	151.9	83.6	155.3	82.2	177.8	80.8	160.2	75.4	201.4	74.6	150.9	70.7	150.9	66.3	94
85.9	153.4	84.5	156.8	83.1	179.8	81.7	161.7	76.2	203.9	75.4	151.9	71.5	151.9	67.0	95
86.8	154.8	85.4	158.3	84.0	181.8	82.6	163.2	77.0	206.4	76.2	152.9	72.3	152.9	67.7	96
87.7	156.3	86.3	159.8	84.9	183.8	83.5	164.7	77.8	208.9	77.0	153.9	73.1	153.9	68.4	97
88.6	157.7	87.2	161.3	85.8	185.8	84.4	166.2	78.6	211.4	77.8	154.9	73.9	154.9	69.1	98
89.5	159.2	88.1	162.8	86.7	187.8	85.3	167.7	79.4	213.9	78.6	155.9	74.7	155.9	69.8	99
90.4	160.6	89.0	164.3	87.6	189.8	86.2	169.2	80.2	216.4	79.4	156.9	75.5	156.9	70.5	100

TABLE IV.

C. 1 Pt.		C. 2 Pt.		C. 3 Pt.		C. 4 Pt.		C. 5 Pt.		C. 6 Pt.		C. 7 Pt.		C. 8 Pt.	
S. by E.	S. by W.	N. by E.	N. by W.	S. by E.	S. by W.	N. by E.	N. by W.	S. by E.	S. by W.	N. by E.	N. by W.	S. by E.	S. by W.	N. by E.	N. by W.
d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.
101	101.5	9.9	99.9	14.8	99.1	19.7	98.9	24.5	98.7	29.3	98.5	34.	98.3	38.7	98.1
102	102.5	10.	100.9	15.	100.	19.9	98.9	24.8	97.6	29.6	98.	34.4	97.4	39.	97.2
103	103.5	10.1	101.9	15.1	101.	20.1	98.9	25.	98.6	29.9	97.	34.7	97.3	39.4	97.1
104	104.5	10.2	102.9	15.2	102.	20.2	100.9	25.3	99.5	30.2	97.9	35.	97.1	39.8	96.9
105	105.5	10.3	103.9	15.3	103.	20.3	101.9	25.5	100.5	30.5	98.9	35.4	97.1	40.2	96.9
106	106.5	10.4	104.9	15.4	104.	20.4	102.9	25.8	101.4	30.8	99.8	35.7	97.9	40.6	97.9
107	107.5	10.5	105.9	15.5	105.9	20.5	103.9	26.	102.4	31.1	100.7	36.	98.9	41.	98.9
108	108.5	10.6	106.9	15.6	106.9	20.6	104.9	26.3	103.3	31.4	101.7	36.4	100.7	41.7	99.8
109	109.5	10.7	107.9	15.7	107.9	20.7	105.9	26.5	104.3	31.7	102.6	36.7	100.7	42.1	99.8
110	110.5	10.8	108.9	15.8	107.9	20.8	106.9	26.7	105.3	31.9	103.6	37.1	101.6	42.4	99.8
111	111.5	10.9	109.9	15.9	108.9	20.9	107.7	27.	106.2	32.2	104.5	37.4	102.5	42.7	99.9
112	112.5	11.1	110.9	16.	109.8	21.	108.6	27.2	107.2	32.4	105.4	37.7	103.4	43.	99.9
113	113.5	11.2	111.9	16.1	110.8	21.1	109.6	27.5	108.1	32.7	106.3	38.	104.3	43.3	99.9
114	114.4	11.3	112.8	16.2	111.8	21.2	110.6	27.7	109.1	32.9	107.3	38.4	105.2	43.6	99.9
115	115.4	11.4	113.7	16.3	112.8	21.3	111.6	27.9	110.	33.1	108.3	38.7	106.2	44.	99.9
116	116.4	11.5	114.7	16.4	113.8	21.4	112.5	28.2	111.	33.7	109.2	39.1	107.2	44.4	99.9
117	117.4	11.6	115.7	16.5	114.8	21.5	113.5	28.4	112.	34.	110.2	39.4	108.1	44.7	99.9
118	118.4	11.7	116.7	16.6	115.7	21.6	114.5	28.7	112.9	34.3	111.1	39.8	109.	45.	99.9
119	119.4	11.8	117.7	16.7	116.7	21.7	115.4	28.9	113.9	34.5	112.	40.1	109.9	45.3	99.9
120	119.4	11.8	117.7	16.8	117.7	21.8	116.4	29.2	114.8	34.8	113.	40.4	110.9	45.6	99.9
121	120.4	11.9	118.7	16.9	118.7	21.9	117.4	29.4	115.8	35.1	113.9	40.8	111.8	45.9	99.9
122	121.4	12.	119.7	17.	119.7	22.	118.3	29.6	116.7	35.4	114.9	41.1	112.7	46.2	99.9
123	122.4	12.1	121.7	17.1	120.6	22.1	119.3	29.9	117.7	35.7	115.8	41.4	113.6	46.5	99.9
124	123.4	12.2	122.7	17.2	121.6	22.2	120.3	30.1	118.7	36.	116.8	41.8	114.6	46.8	99.9
125	124.4	12.3	123.6	17.3	122.6	22.3	121.3	30.4	119.6	36.3	117.7	42.1	115.5	47.1	99.9
126	125.4	12.4	124.6	17.4	123.6	22.4	122.3	30.6	120.6	36.6	118.6	42.4	116.4	47.4	99.9
127	126.4	12.5	125.6	17.5	124.6	22.5	123.3	30.9	121.5	36.9	119.6	42.8	117.3	47.7	99.9
128	127.4	12.6	126.6	17.6	125.6	22.6	124.3	31.1	122.5	37.2	120.5	43.1	118.3	48.	99.9
129	128.4	12.7	127.6	17.7	126.6	22.7	125.3	31.3	123.4	37.4	121.5	43.5	119.3	48.3	99.9
130	129.4	12.8	128.6	17.8	127.6	22.8	126.3	31.6	124.4	37.7	122.4	43.8	120.3	48.7	99.9
131	130.4	12.9	129.6	17.9	128.6	22.9	127.3	31.8	125.4	38.	123.3	44.1	121.	49.	99.9
132	131.4	13.	130.6	18.	129.6	23.	128.3	32.	126.3	38.3	124.3	44.5	122.	49.3	99.9
133	132.4	13.1	131.6	18.1	130.6	23.1	129.3	32.3	127.3	38.6	125.3	44.8	123.	49.6	99.9
134	133.4	13.2	132.6	18.2	131.6	23.2	130.3	32.6	128.3	38.9	126.3	45.1	124.	49.9	99.9
135	134.4	13.3	133.6	18.3	132.6	23.3	131.	32.8	129.3	39.2	127.3	45.5	125.4	50.3	99.9
136	135.4	13.4	134.6	18.4	133.6	23.4	132.	33.	130.3	39.5	128.3	45.8	126.4	50.7	99.9
137	136.4	13.5	135.6	18.5	134.6	23.5	133.1	33.3	131.3	39.8	129.3	46.2	127.5	51.1	99.9
138	137.4	13.6	136.6	18.6	135.6	23.6	134.1	33.6	132.3	40.1	130.3	46.6	128.6	51.5	99.9
139	138.4	13.7	137.6	18.7	136.6	23.7	135.1	33.8	133.3	40.4	131.3	47.	129.7	51.9	99.9
140	139.4	13.8	138.6	18.8	137.6	23.8	136.1	34.	134.3	40.7	132.3	47.4	130.8	52.3	99.9
141	140.4	13.9	139.6	18.9	138.6	23.9	137.1	34.3	135.3	41.	133.3	47.8	131.9	52.7	99.9
142	141.4	14.	140.6	19.	139.6	24.	138.1	34.6	136.3	41.3	134.3	48.2	133.	53.1	99.9
143	142.4	14.1	141.6	19.1	140.6	24.1	139.1	34.9	137.3	41.6	135.3	48.6	134.1	53.5	99.9
144	143.4	14.2	142.6	19.2	141.6	24.2	140.1	35.2	138.3	41.9	136.3	49.	135.1	53.9	99.9
145	144.4	14.3	143.6	19.3	142.6	24.3	141.1	35.5	139.3	42.2	137.3	49.4	136.1	54.3	99.9
146	145.4	14.4	144.6	19.4	143.6	24.4	142.1	35.8	140.3	42.5	138.3	49.8	137.1	54.7	99.9
147	146.4	14.5	145.6	19.5	144.6	24.5	143.1	36.1	141.3	42.8	139.3	50.2	138.1	55.1	99.9
148	147.4	14.6	146.6	19.6	145.6	24.6	144.1	36.4	142.3	43.1	140.3	50.6	139.1	55.5	99.9
149	148.4	14.7	147.6	19.7	146.6	24.7	145.1	36.7	143.3	43.4	141.3	51.	140.1	55.9	99.9

Distance.	C. 1 Pt.		C. 1 Pt.		C. 1 Pt.		C. 1 Pt.		C. 1 Pt.		C. 1 Pt.		C. 2 Pts.	
	N. by E.	S. by W.	N. by E.	S. by W.	N. by E.	S. by W.	N. by E.	S. by W.	N. by E.	S. by W.	N. by E.	S. by W.	N. by E.	S. by W.
	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.	d. lat.	dep.
151	150.8	7.4	150.3	14.8	149.4	22.2	148.1	29.5	146.5	36.7	144.5	43.8	142.9	50.9
152	151.8	7.5	151.3	14.9	150.4	22.3	149.1	29.6	147.4	36.8	145.5	44.1	143.9	51.2
153	152.8	7.5	152.3	15.0	151.3	22.4	150.1	29.7	148.4	37.0	146.4	44.4	144.8	51.5
154	153.8	7.6	153.3	15.1	152.3	22.5	151.1	30.0	149.4	37.4	147.4	44.7	145.8	51.8
155	154.8	7.6	154.3	15.2	153.3	22.6	152.1	30.2	150.4	37.7	148.3	45.0	146.7	52.1
156	155.8	7.7	155.3	15.3	154.3	22.7	153.1	30.4	151.3	37.9	149.3	45.3	147.6	52.4
157	156.8	7.7	156.3	15.4	155.3	22.8	154.1	30.6	152.3	38.1	150.2	45.6	148.5	52.7
158	157.8	7.8	157.3	15.5	156.3	22.9	155.1	30.8	153.3	38.4	151.2	45.9	149.4	53.0
159	158.8	7.8	158.3	15.6	157.3	23.0	156.1	31.1	154.3	38.6	152.2	46.2	150.3	53.3
160	159.8	7.9	159.3	15.7	158.3	23.1	157.1	31.2	155.3	38.9	153.1	46.4	151.2	53.6
161	160.8	7.9	160.3	15.8	159.3	23.2	157.9	31.4	156.2	39.1	154.1	46.7	152.1	53.9
162	161.8	7.9	161.3	15.9	160.3	23.3	158.9	31.6	157.1	39.4	155.1	47.0	153.0	54.2
163	162.8	8.0	162.3	16.0	161.3	23.4	159.9	31.8	158.1	39.6	156.1	47.3	154.0	54.5
164	163.8	8.1	163.3	16.1	162.3	23.5	160.8	32.0	159.1	39.9	157.0	47.6	155.0	54.8
165	164.8	8.1	164.3	16.2	163.3	23.6	161.8	32.2	160.1	40.1	157.9	47.9	156.0	55.1
166	165.8	8.1	165.3	16.3	164.3	23.7	162.8	32.4	161.1	40.3	158.9	48.2	157.0	55.4
167	166.8	8.2	166.3	16.4	165.3	23.8	163.8	32.6	162.1	40.6	159.8	48.5	158.0	55.7
168	167.8	8.2	167.3	16.5	166.3	23.9	164.8	32.8	163.1	40.8	160.8	48.8	159.0	56.0
169	168.8	8.3	168.3	16.6	167.3	24.0	165.8	33.0	164.1	41.1	161.7	49.1	160.0	56.3
170	169.8	8.3	169.3	16.7	168.3	24.1	166.7	33.2	165.1	41.3	162.7	49.3	161.0	56.6
171	170.8	8.4	170.3	16.8	169.3	24.2	167.7	33.4	166.1	41.5	163.6	49.6	162.0	56.9
172	171.8	8.4	171.3	16.9	170.3	24.3	168.7	33.6	167.1	41.8	164.6	49.9	163.0	57.2
173	172.8	8.5	172.3	17.0	171.3	24.4	169.7	33.8	168.1	42.0	165.6	50.2	164.0	57.5
174	173.8	8.5	173.3	17.1	172.3	24.5	170.7	33.9	169.1	42.3	166.5	50.5	165.0	57.8
175	174.8	8.6	174.3	17.2	173.3	24.6	171.6	34.1	170.1	42.5	167.5	50.8	166.0	58.1
176	175.8	8.6	175.3	17.3	174.3	24.8	172.6	34.3	171.1	42.8	168.4	51.1	167.0	58.4
177	176.8	8.7	176.3	17.4	175.3	24.9	173.6	34.5	172.1	43.1	169.4	51.4	168.0	58.7
178	177.8	8.7	177.3	17.4	176.3	25.1	174.6	34.7	173.1	43.3	170.3	51.7	169.0	59.0
179	178.8	8.8	178.3	17.5	177.3	25.2	175.6	34.9	174.1	43.5	171.3	52.0	170.0	59.3
180	179.8	8.8	179.3	17.6	178.3	25.4	176.5	35.1	175.1	43.7	172.2	52.3	171.0	59.6
181	180.8	8.9	180.3	17.7	179.3	25.6	177.5	35.3	176.1	44.0	173.2	52.6	172.0	59.9
182	181.8	8.9	181.3	17.8	180.3	25.7	178.5	35.5	177.1	44.2	174.2	52.9	173.0	60.2
183	182.8	9.0	182.3	17.9	181.3	25.9	179.5	35.7	178.1	44.5	175.1	53.2	174.0	60.5
184	183.8	9.0	183.3	18.0	182.3	26.0	180.5	35.9	179.1	44.7	176.1	53.4	175.0	60.8
185	184.8	9.1	184.3	18.1	183.3	26.1	181.4	36.1	180.1	45.0	177.1	53.7	176.0	61.1
186	185.8	9.1	185.3	18.2	184.3	26.3	182.4	36.3	181.1	45.2	178.1	54.0	177.0	61.4
187	186.8	9.2	186.3	18.3	185.3	26.4	183.4	36.5	182.1	45.4	179.1	54.3	178.0	61.7
188	187.8	9.2	187.3	18.4	186.3	26.6	184.4	36.7	183.1	45.6	180.1	54.6	179.0	62.0
189	188.8	9.3	188.3	18.5	187.3	26.7	185.4	36.9	184.1	45.8	181.1	54.9	180.0	62.3
190	189.8	9.3	189.3	18.6	188.3	26.9	186.3	37.1	185.1	46.0	182.1	55.2	181.0	62.6
191	190.8	9.4	190.3	18.7	189.3	27.0	187.3	37.3	186.1	46.2	183.1	55.4	182.0	62.9
192	191.8	9.4	191.3	18.8	190.3	27.2	188.3	37.5	187.1	46.4	184.1	55.7	183.0	63.2
193	192.8	9.5	192.3	18.9	191.3	27.3	189.3	37.7	188.1	46.6	185.1	56.0	184.0	63.5
194	193.8	9.5	193.3	19.0	192.3	27.5	190.3	37.9	189.1	46.8	186.1	56.3	185.0	63.8
195	194.8	9.6	194.3	19.1	193.3	27.6	191.3	38.1	190.1	47.0	187.1	56.6	186.0	64.1
196	195.8	9.6	195.3	19.2	194.3	27.8	192.3	38.3	191.1	47.2	188.1	56.9	187.0	64.4
197	196.8	9.7	196.3	19.3	195.3	27.9	193.3	38.4	192.1	47.4	189.1	57.2	188.0	64.7
198	197.8	9.7	197.3	19.4	196.3	28.1	194.3	38.6	193.1	47.6	190.1	57.5	189.0	65.0
199	198.8	9.8	198.3	19.5	197.3	28.2	195.3	38.8	194.1	47.8	191.1	57.8	190.0	65.3
200	199.8	9.8	199.3	19.6	198.3	28.3	196.3	39.0	195.1	48.0	192.1	58.1	191.0	65.6

TABLE V.

**DISTANCE, DIFFERENCE OF LATITUDE,
AND DEPARTURE.**

TABLE V.

Course 10°.

Distance, Dif. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	1.	0.	61	61.	1.1	121	121.	2.1	181	181.	3.2	241	241.	4.2
2	2.	0.	62	62.	1.1	2	122.	2.1	2	182.	3.3	3	242.	4.3
3	3.	0.1	63	63.	1.1	3	123.	2.1	3	183.	3.3	3	243.	4.3
4	4.	0.1	64	64.	1.1	4	124.	2.2	4	184.	3.4	4	244.	4.3
5	5.	0.1	65	65.	1.1	5	125.	2.2	5	185.	3.4	5	245.	4.3
6	6.	0.1	66	66.	1.2	6	126.	2.2	6	186.	3.4	6	246.	4.3
7	7.	0.1	67	67.	1.2	7	127.	2.2	7	187.	3.3	7	247.	4.3
8	8.	0.1	68	68.	1.2	8	128.	2.2	8	188.	3.3	8	248.	4.3
9	9.	0.2	69	69.	1.2	9	129.	2.3	9	189.	3.3	9	249.	4.3
10	10.	0.2	70	70.	1.2	130	130.	2.3	190	190.	3.3	250	250.	4.4
11	11.	0.2	71	71.	1.2	1	131.	2.3	1	191.	3.3	1	251.	4.4
12	12.	0.2	72	72.	1.2	2	132.	2.3	2	192.	3.4	2	252.	4.4
13	13.	0.2	73	73.	1.3	3	133.	2.3	3	193.	3.4	3	253.	4.4
14	14.	0.2	74	74.	1.3	4	134.	2.4	4	194.	3.4	4	254.	4.4
15	15.	0.3	75	75.	1.3	5	135.	2.4	5	195.	3.4	5	255.	4.5
16	16.	0.3	76	76.	1.3	6	136.	2.4	6	196.	3.4	6	256.	4.5
17	17.	0.3	77	77.	1.3	7	137.	2.4	7	197.	3.4	7	257.	4.5
18	18.	0.3	78	78.	1.4	8	138.	2.4	8	198.	3.5	8	258.	4.5
19	19.	0.3	79	79.	1.4	9	139.	2.4	9	199.	3.5	9	259.	4.5
20	20.	0.3	80	80.	1.4	140	140.	2.4	200	200.	3.5	260	260.	4.5
21	21.	0.4	81	81.	1.4	1	141.	2.5	1	201.	3.5	1	261.	4.6
22	22.	0.4	82	82.	1.4	2	142.	2.5	2	202.	3.5	2	262.	4.6
23	23.	0.4	83	83.	1.4	3	143.	2.5	3	203.	3.5	3	263.	4.6
24	24.	0.4	84	84.	1.5	4	144.	2.5	4	204.	3.6	4	264.	4.6
25	25.	0.4	85	85.	1.5	5	145.	2.5	5	205.	3.6	5	265.	4.6
26	26.	0.5	86	86.	1.5	6	146.	2.5	6	206.	3.6	6	266.	4.6
27	27.	0.5	87	87.	1.5	7	147.	2.6	7	207.	3.6	7	267.	4.7
28	28.	0.5	88	88.	1.5	8	148.	2.6	8	208.	3.6	8	268.	4.7
29	29.	0.5	89	89.	1.6	9	149.	2.6	9	209.	3.6	9	269.	4.7
30	30.	0.5	90	90.	1.6	150	150.	2.6	210	210.	3.7	270	270.	4.7
31	31.	0.5	91	91.	1.6	1	151.	2.6	1	211.	3.7	1	271.	4.7
32	32.	0.6	92	92.	1.6	2	152.	2.7	2	212.	3.7	2	272.	4.7
33	33.	0.6	93	93.	1.6	3	153.	2.7	3	213.	3.7	3	273.	4.8
34	34.	0.6	94	94.	1.6	4	154.	2.7	4	214.	3.7	4	274.	4.8
35	35.	0.6	95	95.	1.7	5	155.	2.7	5	215.	3.8	5	275.	4.8
36	36.	0.6	96	96.	1.7	6	156.	2.7	6	216.	3.8	6	276.	4.8
37	37.	0.6	97	97.	1.7	7	157.	2.7	7	217.	3.8	7	277.	4.8
38	38.	0.7	98	98.	1.7	8	158.	2.8	8	218.	3.8	8	278.	4.9
39	39.	0.7	99	99.	1.7	9	159.	2.8	9	219.	3.8	9	279.	4.9
40	40.	0.7	100	100.	1.7	160	160.	2.8	220	220.	3.8	280	280.	4.9
41	41.	0.7	1	101.	1.8	1	161.	2.8	1	221.	3.9	1	281.	4.9
42	42.	0.7	2	102.	1.8	2	162.	2.8	2	222.	3.9	2	282.	4.9
43	43.	0.8	3	103.	1.8	3	163.	2.8	3	223.	3.9	3	283.	4.9
44	44.	0.8	4	104.	1.8	4	164.	2.9	4	224.	3.9	4	284.	5.
45	45.	0.8	5	105.	1.8	5	165.	2.9	5	225.	3.9	5	285.	5.
46	46.	0.8	6	106.	1.8	6	166.	2.9	6	226.	3.9	6	286.	5.
47	47.	0.8	7	107.	1.9	7	167.	2.9	7	227.	4.	7	287.	5.
48	48.	0.8	8	108.	1.9	8	168.	2.9	8	228.	4.	8	288.	5.
49	49.	0.9	9	109.	1.9	9	169.	2.9	9	229.	4.	9	289.	5.
50	50.	0.9	110	110.	1.9	170	170.	3.	230	230.	4.	290	290.	5.1
51	51.	0.9	1	111.	1.9	1	171.	3.	1	231.	4.	1	291.	5.1
52	52.	0.9	2	112.	2.	2	172.	3.	2	232.	4.	2	292.	5.1
53	53.	0.9	3	113.	2.	3	173.	3.	3	233.	4.1	3	293.	5.1
54	54.	0.9	4	114.	2.	4	174.	3.	4	234.	4.1	4	294.	5.1
55	55.	1.	5	115.	2.	5	175.	3.1	5	235.	4.1	5	295.	5.1
56	56.	1.	6	116.	2.	6	176.	3.1	6	236.	4.1	6	296.	5.2
57	57.	1.	7	117.	2.	7	177.	3.1	7	237.	4.1	7	297.	5.2
58	58.	1.	8	118.	2.1	8	178.	3.1	8	238.	4.2	8	298.	5.2
59	59.	1.	9	119.	2.1	9	179.	3.1	9	239.	4.2	9	299.	5.2
60	60.	1.	120	120.	2.1	180	180.	3.1	240	240.	4.3	300	300.	5.3
dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.

Distance, Departure and Diff. Latitude.

Course 80°.

Course 90°.

Distance, Diff Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	1.	0.	61	61.	2.1	121	120.9	4.2	181	180.9	6.3	241	240.9	8.4
2	2.	0.1	62	62.	2.2	3	121.0	4.3	2	181.0	6.4	3	241.0	8.4
3	3.	0.1	63	63.	2.3	4	122.0	4.3	3	182.0	6.4	4	242.0	8.5
4	4.	0.1	64	64.	2.4	5	123.0	4.4	4	183.0	6.4	5	243.0	8.5
5	5.	0.2	65	65.	2.5	6	124.0	4.4	5	184.0	6.5	6	244.0	8.6
6	6.	0.2	66	66.	2.6	7	125.0	4.4	6	185.0	6.5	7	245.0	8.6
7	7.	0.3	67	67.	2.7	8	126.0	4.4	7	186.0	6.5	8	246.0	8.6
8	8.	0.3	68	68.	2.8	9	127.0	4.5	8	187.0	6.6	9	247.0	8.7
9	9.	0.3	69	69.	2.8		128.0	4.5	9	188.0	6.6		248.0	8.7
10	10.	0.3	70	70.	2.9	130	129.9	4.5	190	189.9	6.6	250	249.9	8.7
11	11.	0.4	71	71.	2.5	1	130.0	4.6	1	190.0	6.7	1	250.0	8.8
12	12.	0.4	72	72.	2.5	2	131.0	4.6	2	191.0	6.7	2	251.0	8.8
13	13.	0.5	73	73.	2.5	3	132.0	4.6	3	192.0	6.7	3	252.0	8.8
14	14.	0.5	74	74.	2.6	4	133.0	4.7	4	193.0	6.8	4	253.0	8.9
15	15.	0.5	75	75.	2.6	5	134.0	4.7	5	194.0	6.8	5	254.0	8.9
16	16.	0.6	76	76.	2.7	6	135.0	4.7	6	195.0	6.8	6	255.0	8.9
17	17.	0.6	77	77.	2.7	7	136.0	4.8	7	196.0	6.9	7	256.0	9.
18	18.	0.6	78	78.	2.7	8	137.0	4.8	8	197.0	6.9	8	257.0	9.
19	19.	0.7	79	79.	2.8	9	138.0	4.9	9	198.0	6.9	9	258.0	9.1
20	20.	0.7	80	80.	2.8	140	139.9	4.9	200	199.9	7.	260	259.9	9.1
21	21.	0.7	81	81.	2.8	1	140.0	4.9	1	200.0	7.	1	260.0	9.1
22	22.	0.8	82	82.	2.9	2	141.0	5.	2	201.0	7.1	2	261.0	9.1
23	23.	0.8	83	83.	2.9	3	142.0	5.	3	202.0	7.1	3	262.0	9.2
24	24.	0.8	84	84.	2.9	4	143.0	5.	4	203.0	7.1	4	263.0	9.2
25	25.	0.9	85	85.	3.	5	144.0	5.1	5	204.0	7.2	5	264.0	9.2
26	26.	0.9	86	86.	3.	6	145.0	5.1	6	205.0	7.2	6	265.0	9.3
27	27.	0.9	87	87.	3.	7	146.0	5.1	7	206.0	7.2	7	266.0	9.3
28	28.	1.	88	88.	3.1	8	147.0	5.2	8	207.0	7.3	8	267.0	9.4
29	29.	1.	89	89.	3.1	9	148.0	5.2	9	208.0	7.3	9	268.0	9.4
30	30.	1.	90	89.9	3.1	150	149.9	5.2	210	209.9	7.3	270	269.9	9.4
31	31.	1.1	91	90.9	3.2	1	150.0	5.3	1	210.0	7.4	1	270.0	9.5
32	32.	1.1	92	91.9	3.2	2	151.0	5.3	2	211.0	7.4	2	271.0	9.5
33	33.	1.2	93	92.9	3.2	3	152.0	5.3	3	212.0	7.4	3	272.0	9.5
34	34.	1.3	94	93.9	3.3	4	153.0	5.4	4	213.0	7.5	4	273.0	9.6
35	35.	1.3	95	94.9	3.3	5	154.0	5.4	5	214.0	7.5	5	274.0	9.6
36	36.	1.3	96	95.9	3.4	6	155.0	5.4	6	215.0	7.5	6	275.0	9.6
37	37.	1.3	97	96.9	3.4	7	156.0	5.5	7	216.0	7.6	7	276.0	9.7
38	38.	1.3	98	97.9	3.4	8	157.0	5.5	8	217.0	7.6	8	277.0	9.7
39	39.	1.4	99	98.9	3.5	9	158.0	5.5	9	218.0	7.6	9	278.0	9.7
40	40.	1.4	100	99.9	3.5	160	159.9	5.6	220	219.9	7.7	280	279.9	9.8
41	41.	1.4	1	100.0	3.5	1	160.0	5.6	1	220.0	7.7	1	280.0	9.8
42	42.	1.5	2	101.0	3.6	2	161.0	5.7	2	221.0	7.7	2	281.0	9.8
43	43.	1.5	3	102.0	3.6	3	162.0	5.7	3	222.0	7.8	3	282.0	9.8
44	44.	1.5	4	103.0	3.6	4	163.0	5.7	4	223.0	7.8	4	283.0	9.9
45	45.	1.6	5	104.0	3.7	5	164.0	5.8	5	224.0	7.9	5	284.0	9.9
46	46.	1.6	6	105.0	3.7	6	165.0	5.8	6	225.0	7.9	6	285.0	10.
47	47.	1.6	7	106.0	3.7	7	166.0	5.8	7	226.0	7.9	7	286.0	10.
48	48.	1.7	8	107.0	3.8	8	167.0	5.9	8	227.0	8.	8	287.0	10.1
49	49.	1.7	9	108.0	3.8	9	168.0	5.9	9	228.0	8.	9	288.0	10.1
50	50.	1.7	110	109.9	3.8	170	169.9	5.9	230	229.9	8.	290	289.9	10.1
51	51.	1.8	1	110.0	3.9	1	170.0	6.	1	230.0	8.1	1	290.0	10.2
52	52.	1.8	2	111.0	3.9	2	171.0	6.	2	231.0	8.1	2	291.0	10.2
53	53.	1.8	3	112.0	3.9	3	172.0	6.	3	232.0	8.1	3	292.0	10.2
54	54.	1.9	4	113.0	4.	4	173.0	6.1	4	233.0	8.2	4	293.0	10.3
55	55.	1.9	5	114.0	4.	5	174.0	6.1	5	234.0	8.2	5	294.0	10.3
56	56.	2.	6	115.0	4.	6	175.0	6.1	6	235.0	8.2	6	295.0	10.3
57	57.	2.	7	116.0	4.1	7	176.0	6.2	7	236.0	8.3	7	296.0	10.4
58	58.	2.	8	117.0	4.1	8	177.0	6.2	8	237.0	8.3	8	297.0	10.4
59	59.	2.1	9	118.0	4.2	9	178.0	6.2	9	238.0	8.3	9	298.0	10.4
60	60.	2.1	190	119.9	4.2	190	179.9	6.3	240	239.9	8.4	300	299.9	10.5

Distance, Departure and Diff Latitude.

Course 90°

Course 30°.

Distance, Diff. Latitude and Departure

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	1.	0.1	61	60.9	3.9	121	120.8	6.3	181	180.8	9.5	241	240.7	12.6
2	2.	0.1	62	61.9	3.9	2	121.8	6.4	2	181.8	9.5	2	241.7	12.7
3	3.	0.9	63	62.9	3.3	3	122.8	6.4	3	182.7	9.6	3	242.7	12.7
4	4.	0.2	64	63.9	3.3	4	123.8	6.5	4	183.7	9.6	4	243.7	12.8
5	5.	0.3	65	64.9	3.4	5	124.8	6.5	5	184.7	9.7	5	244.7	12.8
6	6.	0.3	66	65.9	3.5	6	125.8	6.6	6	185.7	9.7	6	245.7	12.9
7	7.	0.4	67	66.9	3.5	7	126.8	6.6	7	186.7	9.8	7	246.7	12.9
8	8.	0.4	68	67.9	3.6	8	127.8	6.7	8	187.7	9.8	8	247.7	13.
9	9.	0.5	69	68.9	3.6	9	128.8	6.8	9	188.7	9.9	9	248.7	13.
10	10.	0.5	70	69.9	3.7	130	129.8	6.8	190	189.7	9.9	250	249.7	13.1
11	11.	0.6	71	70.9	3.7	1	130.8	6.9	1	190.7	10	1	250.7	13.1
12	12.	0.6	72	71.9	3.8	2	131.8	6.9	2	191.7	10.	2	251.7	13.2
13	13.	0.7	73	72.9	3.8	3	132.8	7.	3	192.7	10.1	3	252.7	13.2
14	14.	0.7	74	73.9	3.9	4	133.8	7.	4	193.7	10.2	4	253.7	13.3
15	15.	0.8	75	74.9	3.9	5	134.8	7.1	5	194.7	10.2	5	254.7	13.3
16	16.	0.8	76	75.9	4.	6	135.8	7.1	6	195.7	10.3	6	255.6	13.4
17	17.	0.9	77	76.9	4.	7	136.8	7.2	7	196.7	10.3	7	256.6	13.5
18	18.	0.9	78	77.9	4.1	8	137.8	7.2	8	197.7	10.4	8	257.6	13.5
19	19.	1.	79	78.9	4.1	9	138.8	7.3	9	198.7	10.4	9	258.6	13.6
20	20.	1.	80	79.9	4.2	140	139.8	7.3	200	199.7	10.5	260	259.6	13.6
21	21.	1.1	81	80.9	4.2	1	140.8	7.4	1	200.7	10.5	1	260.6	13.7
22	22.	1.2	82	81.9	4.3	2	141.8	7.4	2	201.7	10.6	2	261.6	13.7
23	23.	1.2	83	82.9	4.3	3	142.8	7.5	3	202.7	10.6	3	262.6	13.8
24	24.	1.3	84	83.9	4.4	4	143.8	7.5	4	203.7	10.7	4	263.6	13.8
25	25.	1.3	85	84.9	4.4	5	144.8	7.6	5	204.7	10.7	5	264.6	13.9
26	26.	1.4	86	85.9	4.5	6	145.8	7.6	6	205.7	10.8	6	265.6	13.9
27	27.	1.4	87	86.9	4.6	7	146.8	7.7	7	206.7	10.8	7	266	14.
28	28.	1.5	88	87.9	4.6	8	147.8	7.7	8	207.7	10.9	8	267.6	14.
29	29.	1.5	89	88.9	4.7	9	148.8	7.8	9	208.7	10.9	9	268.6	14.1
30	30.	1.6	90	89.9	4.7	150	149.8	7.9	210	209.7	11.	270	269.6	14.1
31	31.	1.6	91	90.9	4.8	1	150.8	7.9	1	210.7	11.	1	270.6	14.2
32	32.	1.7	92	91.9	4.8	2	151.8	8.	2	211.7	11.1	2	271.6	14.2
33	33.	1.7	93	92.9	4.9	3	152.8	8.	3	212.7	11.1	3	272.6	14.3
34	34.	1.8	94	93.9	4.9	4	153.8	8.1	4	213.7	11.2	4	273.6	14.3
35	35.	1.8	95	94.9	5.	5	154.8	8.1	5	214.7	11.3	5	274.6	14.4
36	36.	1.9	96	95.9	5.	6	155.8	8.2	6	215.7	11.3	6	275.6	14.4
37	36.9	1.9	97	96.9	5.1	7	156.8	8.2	7	216.7	11.4	7	276.6	14.5
38	37.9	2.	98	97.9	5.1	8	157.8	8.3	8	217.7	11.4	8	277.6	14.5
39	38.9	2.	99	98.9	5.2	9	158.8	8.3	9	218.7	11.5	9	278.6	14.6
40	39.9	2.1	100	99.9	5.2	160	159.8	8.4	220	219.7	11.5	280	279.6	14.7
41	40.9	2.1	1	100.9	5.3	1	160.8	8.4	1	220.7	11.6	1	280.6	14.7
42	41.9	2.2	2	101.9	5.3	2	161.8	8.5	2	221.7	11.6	2	281.6	14.8
43	42.9	2.3	3	102.9	5.4	3	162.8	8.5	3	222.7	11.7	3	282.6	14.8
44	43.9	2.3	4	103.9	5.4	4	163.8	8.6	4	223.7	11.7	4	283.6	14.9
45	44.9	2.4	5	104.9	5.5	5	164.8	8.6	5	224.7	11.8	5	284.6	14.9
46	45.9	2.4	6	105.9	5.5	6	165.8	8.7	6	225.7	11.9	6	285.6	15.
47	46.9	2.5	7	106.9	5.6	7	166.8	8.7	7	226.7	11.9	7	286.6	15.
48	47.9	2.5	8	107.9	5.7	8	167.8	8.8	8	227.7	12.	8	287.6	15.1
49	48.9	2.6	9	108.9	5.7	9	168.8	8.8	9	228.7	12.	9	288.6	15.1
50	49.9	2.6	110	109.8	5.8	170	169.8	8.9	230	229.7	12.	290	289.6	15.2
51	50.9	2.7	1	110.8	5.8	1	170.8	8.9	1	230.7	12.1	1	290.6	15.2
52	51.9	2.7	2	111.8	5.9	2	171.8	9.	2	231.7	12.1	2	291.6	15.3
53	52.9	2.8	3	112.8	5.9	3	172.8	9.1	3	232.7	12.2	3	292.6	15.3
54	53.9	2.8	4	113.8	6.	4	173.8	9.1	4	233.7	12.2	4	293.6	15.4
55	54.9	2.9	5	114.8	6.	5	174.8	9.2	5	234.7	12.3	5	294.6	15.4
56	55.9	2.9	6	115.8	6.1	6	175.8	9.2	6	235.7	12.4	6	295.6	15.5
57	56.9	3.	7	116.8	6.1	7	176.8	9.3	7	236.7	12.4	7	296.6	15.6
58	57.9	3.	8	117.8	6.2	8	177.8	9.3	8	237.7	12.5	8	297.6	15.6
59	58.9	3.1	9	118.8	6.2	9	178.8	9.4	9	238.7	12.5	9	298.6	15.6
60	59.9	3.1	190	119.8	6.3	180	179.8	9.4	240	239.7	12.6	300	299.6	15.7

Distance, Departure and Diff. Latitude.

Course 87°.

Course 40.

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	1.	0.1	61	60.9	4.3	131	130.7	8.4	181	180.6	12.6	241	240.4	16.8
2	2.	0.1	62	61.8	4.3	2	131.7	8.5	2	181.6	12.7	2	241.4	16.9
3	3.	0.2	63	62.8	4.4	3	132.7	8.6	3	182.6	12.8	3	242.4	17.
4	4.	0.3	64	63.8	4.5	4	133.7	8.6	4	183.6	12.8	4	243.4	17.
5	5.	0.3	65	64.8	4.5	5	134.7	8.7	5	184.5	12.9	5	244.4	17.1
6	6.	0.4	66	65.8	4.6	6	135.7	8.8	6	185.5	13.	6	245.4	17.2
7	7.	0.5	67	66.8	4.7	7	136.7	8.9	7	186.5	13.	7	246.4	17.2
8	8.	0.6	68	67.8	4.7	8	137.7	8.9	8	187.5	13.1	8	247.4	17.3
9	9.	0.6	69	68.8	4.8	9	138.7	9.	9	188.5	13.2	9	248.4	17.4
10	10.	0.7	70	69.8	4.9	130	139.7	9.1	190	189.5	13.3	250	249.4	17.4
11	11.	0.8	71	70.8	5.	1	130.7	9.1	1	190.5	13.3	1	250.4	17.5
12	12.	0.8	72	71.8	5.	2	131.7	9.2	2	191.5	13.4	2	251.4	17.6
13	13.	0.9	73	72.8	5.1	3	132.7	9.3	3	192.5	13.5	3	252.4	17.6
14	14.	1.	74	73.8	5.2	4	133.7	9.3	4	193.5	13.5	4	253.4	17.7
15	15.	1.	75	74.8	5.2	5	134.7	9.4	5	194.5	13.6	5	254.4	17.8
16	16.	1.1	76	75.8	5.3	6	135.7	9.5	6	195.5	13.7	6	255.4	17.9
17	17.	1.2	77	76.8	5.4	7	136.7	9.6	7	196.5	13.7	7	256.4	17.9
18	18.	1.3	78	77.8	5.4	8	137.7	9.6	8	197.5	13.8	8	257.4	18.
19	19.	1.3	79	78.8	5.5	9	138.7	9.7	9	198.5	13.9	9	258.4	18.1
20	20.	1.4	80	79.8	5.6	140	139.7	9.8	200	199.5	14.	260	259.4	18.1
21	20.9	1.5	81	80.8	5.7	1	140.7	9.8	1	200.5	14.	1	260.4	18.2
22	21.0	1.5	82	81.8	5.7	2	141.7	9.9	2	201.5	14.1	2	261.4	18.3
23	20.9	1.6	83	82.8	5.8	3	142.7	10.	3	202.5	14.2	3	262.4	18.3
24	23.9	1.7	84	83.8	5.9	4	143.6	10.	4	203.5	14.2	4	263.4	18.4
25	24.9	1.7	85	84.8	5.9	5	144.6	10.1	5	204.5	14.3	5	264.4	18.5
26	25.9	1.8	86	85.8	6.	6	145.6	10.2	6	205.5	14.4	6	265.4	18.6
27	26.9	1.9	87	86.8	6.1	7	146.6	10.3	7	206.5	14.4	7	266.3	18.7
28	27.9	2.	88	87.8	6.1	8	147.6	10.3	8	207.5	14.5	8	267.3	18.7
29	28.9	2.	89	88.8	6.2	9	148.6	10.4	9	208.5	14.6	9	268.3	18.8
30	29.9	2.1	90	89.8	6.3	150	149.6	10.5	210	209.5	14.6	270	269.3	18.8
31	30.9	2.2	91	90.8	6.3	1	150.6	10.5	1	210.5	14.7	1	270.3	18.9
32	31.9	2.2	92	91.8	6.4	2	151.6	10.6	2	211.5	14.8	2	271.3	19.
33	32.9	2.3	93	92.8	6.5	3	152.6	10.7	3	212.5	14.9	3	272.3	19.
34	33.9	2.4	94	93.8	6.6	4	153.6	10.7	4	213.5	14.9	4	273.3	19.1
35	34.9	2.4	95	94.8	6.6	5	154.6	10.8	5	214.5	15.	5	274.3	19.2
36	35.9	2.5	96	95.8	6.7	6	155.6	10.9	6	215.5	15.1	6	275.3	19.3
37	36.9	2.6	97	96.8	6.8	7	156.6	11.	7	216.5	15.1	7	276.3	19.3
38	37.9	2.7	98	97.8	6.8	8	157.6	11.	8	217.5	15.2	8	277.3	19.4
39	38.9	2.7	99	98.8	6.9	9	158.6	11.1	9	218.5	15.3	9	278.3	19.5
40	39.9	2.8	100	99.8	7.	160	159.6	11.2	220	219.5	15.3	280	279.3	19.5
41	40.9	2.9	1	100.8	7.	1	160.6	11.3	1	220.5	15.4	1	280.3	19.6
42	41.9	2.9	2	101.8	7.1	2	161.6	11.3	2	221.5	15.5	2	281.3	19.7
43	42.9	3.	3	102.7	7.2	3	162.6	11.4	3	222.5	15.6	3	282.3	19.7
44	43.9	3.1	4	103.7	7.3	4	163.6	11.4	4	223.5	15.6	4	283.3	19.8
45	44.9	3.1	5	104.7	7.3	5	164.6	11.5	5	224.5	15.7	5	284.3	19.9
46	45.9	3.2	6	105.7	7.4	6	165.6	11.6	6	225.4	15.8	6	285.3	20.
47	46.9	3.3	7	106.7	7.5	7	166.6	11.6	7	226.4	15.8	7	286.3	20.
48	47.9	3.3	8	107.7	7.5	8	167.6	11.7	8	227.4	15.9	8	287.3	20.1
49	48.9	3.4	9	108.7	7.6	9	168.6	11.8	9	228.4	16.	9	288.3	20.2
50	49.9	3.5	110	109.7	7.7	170	169.6	11.9	230	229.4	16.	290	289.3	20.2
51	50.9	3.6	1	110.7	7.7	1	170.6	11.9	1	230.4	16.1	1	290.3	20.3
52	51.9	3.6	2	111.7	7.8	2	171.6	12.	2	231.4	16.2	2	291.3	20.4
53	52.9	3.7	3	112.7	7.9	3	172.6	12.1	3	232.4	16.3	3	292.3	20.4
54	53.9	3.8	4	113.7	8.	4	173.6	12.1	4	233.4	16.3	4	293.3	20.5
55	54.9	3.8	5	114.7	8.	5	174.6	12.2	5	234.4	16.4	5	294.3	20.6
56	55.9	3.9	6	115.7	8.1	6	175.6	12.3	6	235.4	16.5	6	295.3	20.6
57	56.9	4.	7	116.7	8.2	7	176.6	12.3	7	236.4	16.5	7	296.3	20.7
58	57.9	4.	8	117.7	8.3	8	177.6	12.4	8	237.4	16.6	8	297.3	20.8
59	58.9	4.1	9	118.7	8.3	9	178.6	12.5	9	238.4	16.7	9	298.3	20.9
60	59.9	4.2	190	119.7	8.4	180	179.6	12.6	240	239.4	16.7	300	299.3	20.9
dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.

Distance, Departure and Diff. Latitude

Course 60°.

Course 50.

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	1.	0.1	61	60.8	5.3	121	120.5	10.5	181	180.3	15.8	241	240.1	21.
2	2.	0.2	62	61.8	5.4	2	121.5	10.6	2	181.3	15.9	2	241.1	21.1
3	3.	0.3	63	62.8	5.5	3	122.5	10.7	3	182.3	15.9	3	242.1	21.2
4	4.	0.3	64	63.8	5.6	4	123.5	10.8	4	183.3	16.	4	243.1	21.3
5	5.	0.4	65	64.8	5.7	5	124.5	10.9	5	184.3	16.1	5	244.1	21.4
6	6.	0.5	66	65.7	5.8	6	125.5	11.	6	185.3	16.2	6	245.1	21.5
7	7.	0.6	67	66.7	5.8	7	126.5	11.1	7	186.3	16.3	7	246.1	21.5
8	8.	0.7	68	67.7	5.9	8	127.5	11.2	8	187.3	16.4	8	247.1	21.6
9	9.	0.8	69	68.7	6.	9	128.5	11.3	9	188.3	16.5	9	248.1	21.7
10	10.	0.9	70	69.7	6.1	130	129.5	11.3	190	189.3	16.6	250	249.	21.8
11	11.	1.	71	70.7	6.2	1	130.5	11.4	1	190.3	16.6	1	250.	21.9
12	12.	1.	72	71.7	6.3	2	131.5	11.5	2	191.3	16.7	2	251.	22.
13	13.	1.1	73	72.7	6.4	3	132.5	11.6	3	192.3	16.8	3	252.	22.1
14	13.9	1.2	74	73.7	6.4	4	133.5	11.7	4	193.3	16.9	4	253.	22.1
15	14.9	1.3	75	74.7	6.5	5	134.5	11.8	5	194.3	17.	5	254.	22.2
16	15.9	1.4	76	75.7	6.6	6	135.5	11.9	6	195.3	17.1	6	255.	22.3
17	16.9	1.5	77	76.7	6.7	7	136.5	11.9	7	196.3	17.2	7	256.	22.4
18	17.9	1.6	78	77.7	6.8	8	137.5	12.	8	197.3	17.3	8	257.	22.5
19	18.9	1.7	79	78.7	6.9	9	138.5	12.1	9	198.3	17.3	9	258.	22.6
20	19.9	1.7	80	79.7	7.	140	139.5	12.2	200	199.9	17.4	260	259.	22.7
21	20.9	1.8	81	80.7	7.1	1	140.5	12.3	1	200.9	17.5	1	260.	22.8
22	21.9	1.9	82	81.7	7.1	2	141.5	12.4	2	201.9	17.6	2	261.	22.8
23	22.9	2.	83	82.7	7.2	3	142.5	12.5	3	202.9	17.7	3	262.	22.9
24	23.9	2.1	84	83.7	7.3	4	143.5	12.6	4	203.9	17.8	4	263.	23.
25	24.9	2.2	85	84.7	7.4	5	144.5	12.6	5	204.9	17.9	5	264.	23.1
26	25.9	2.3	86	85.7	7.5	6	145.5	12.7	6	205.9	18.	6	265.	23.2
27	26.9	2.4	87	86.7	7.6	7	146.5	12.8	7	206.9	18.	7	266.	23.3
28	27.9	2.4	88	87.7	7.7	8	147.5	12.9	8	207.9	18.1	8	267.	23.4
29	28.9	2.5	89	88.7	7.8	9	148.5	13.	9	208.9	18.2	9	268.	23.4
30	29.9	2.6	90	89.7	7.8	150	149.5	13.1	210	209.9	18.3	270	269.	23.5
31	30.9	2.7	91	90.7	7.9	1	150.5	13.2	1	210.9	18.4	1	270.	23.6
32	31.9	2.8	92	91.6	8.	2	151.5	13.2	2	211.9	18.5	2	271.	23.7
33	32.9	2.9	93	92.6	8.1	3	152.5	13.3	3	212.9	18.6	3	272.	23.8
34	33.9	3.	94	93.6	8.2	4	153.5	13.4	4	213.9	18.7	4	273.	23.9
35	34.9	3.1	95	94.6	8.3	5	154.5	13.5	5	214.9	18.7	5	274.	24.
36	35.9	3.1	96	95.6	8.4	6	155.5	13.6	6	215.9	18.8	6	274.9	24.1
37	36.9	3.2	97	96.6	8.5	7	156.5	13.7	7	216.9	18.9	7	275.9	24.1
38	37.9	3.3	98	97.6	8.5	8	157.5	13.8	8	217.9	19.	8	276.9	24.2
39	38.9	3.4	99	98.6	8.6	9	158.5	13.9	9	218.9	19.1	9	277.9	24.3
40	39.8	3.5	100	99.6	8.7	160	159.4	13.9	220	219.2	19.2	280	278.9	24.4
41	40.8	3.6	1	100.6	8.8	1	160.4	14.	1	220.2	19.3	1	279.9	24.5
42	41.8	3.7	2	101.6	8.9	2	161.4	14.1	2	221.2	19.3	2	280.9	24.6
43	42.8	3.7	3	102.6	9.	3	162.4	14.2	3	222.2	19.4	3	281.9	24.7
44	43.8	3.8	4	103.6	9.1	4	163.4	14.3	4	223.1	19.5	4	282.9	24.8
45	44.8	3.9	5	104.6	9.2	5	164.4	14.4	5	224.1	19.6	5	283.9	24.9
46	45.8	4.	6	105.6	9.2	6	165.4	14.5	6	225.1	19.7	6	284.9	24.9
47	46.8	4.1	7	106.6	9.3	7	166.4	14.6	7	226.1	19.8	7	285.9	25.
48	47.8	4.2	8	107.6	9.4	8	167.4	14.6	8	227.1	19.9	8	286.9	25.1
49	48.8	4.3	9	108.6	9.5	9	168.4	14.7	9	228.1	20.	9	287.9	25.2
50	49.8	4.4	110	109.6	9.6	170	169.4	14.8	230	229.1	20.	290	288.9	25.3
51	50.8	4.4	1	110.6	9.7	1	170.3	14.9	1	230.1	20.1	1	289.9	25.4
52	51.8	4.5	2	111.6	9.8	2	171.3	15.	2	231.1	20.2	2	290.9	25.4
53	52.8	4.6	3	112.6	9.8	3	172.3	15.1	3	232.1	20.3	3	291.9	25.5
54	53.8	4.7	4	113.6	9.9	4	173.3	15.2	4	233.1	20.4	4	292.9	25.6
55	54.8	4.8	5	114.6	10.	5	174.3	15.3	5	234.1	20.5	5	293.9	25.7
56	55.8	4.9	6	115.6	10.1	6	175.3	15.3	6	235.1	20.6	6	294.9	25.8
57	56.8	5.	7	116.6	10.2	7	176.3	15.4	7	236.1	20.7	7	295.9	25.9
58	57.8	5.1	8	117.6	10.3	8	177.3	15.5	8	237.1	20.7	8	296.9	26.
59	58.8	5.1	9	118.6	10.4	9	178.3	15.6	9	238.1	20.8	9	297.9	26.1
60	59.8	5.2	190	119.5	10.5	180	179.3	15.7	240	239.1	20.9	300	298.9	26.1

Distance, Departure and Diff. Latitude.

Course 65°.

TABLE V.

117

Course 70.

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	1.	0.1	61	60.5	7.4	181	180.1	14.7	181	179.7	24.1	241	239.2	29.4
2	2.	0.2	62	61.5	7.6	2	181.1	14.9	2	180.6	22.3	2	240.2	29.5
3	3.	0.4	63	62.5	7.7	3	182.1	15.	3	181.6	22.3	3	241.2	29.6
4	4.	0.5	64	63.5	7.8	4	183.1	15.1	4	182.6	22.4	4	242.2	29.7
5	5.	0.6	65	64.5	7.9	5	184.1	15.2	5	183.6	22.5	5	243.2	29.9
6	6.	0.7	66	65.5	8.	6	185.1	15.4	6	184.6	22.7	6	244.2	30.
7	6.9	0.9	67	66.5	8.2	7	186.1	15.5	7	185.6	22.8	7	245.2	30.1
8	7.9	1.	68	67.5	8.3	8	187.	15.6	8	186.6	22.9	8	246.2	30.2
9	8.9	1.1	69	68.5	8.4	9	188.	15.7	9	187.6	23.	9	247.1	30.3
10	9.9	1.2	70	69.5	8.5	130	129.	15.8	190	188.6	22.9	250	248.1	30.5
11	10.9	1.3	71	70.5	8.7	1	130.	16.	1	189.6	23.3	1	249.1	30.6
12	11.9	1.5	72	71.5	8.8	2	131.	16.1	2	190.6	23.4	2	250.1	30.7
13	12.9	1.6	73	72.5	8.9	3	132.	16.2	3	191.6	23.5	3	251.1	30.8
14	13.9	1.7	74	73.4	9.	4	133.	16.3	4	192.6	23.6	4	252.1	31.
15	14.9	1.8	75	74.4	9.1	5	134.	16.5	5	193.5	23.8	5	253.1	31.1
16	15.9	1.9	76	75.4	9.3	6	135.	16.6	6	194.5	23.9	6	254.1	31.2
17	16.9	2.1	77	76.4	9.4	7	136.	16.7	7	195.5	24.	7	255.1	31.3
18	17.9	2.2	78	77.4	9.5	8	137.	16.8	8	196.5	24.1	8	256.1	31.4
19	18.9	2.3	79	78.4	9.6	9	138.	16.9	9	197.5	24.3	9	257.1	31.6
20	19.9	2.4	80	79.4	9.7	140	139.	17.1	200	198.5	24.4	260	258.1	31.7
21	20.9	2.6	81	80.4	9.9	1	139.9	17.2	1	199.5	24.5	1	259.1	31.8
22	21.8	2.7	82	81.4	10.	2	140.9	17.3	2	200.5	24.6	2	260.	31.9
23	22.8	2.8	83	82.4	10.1	3	141.9	17.4	3	201.5	24.7	3	261.	32.1
24	23.8	2.9	84	83.4	10.2	4	142.9	17.5	4	202.5	24.9	4	262.	32.2
25	24.8	3.	85	84.4	10.4	5	143.9	17.7	5	203.5	25.5	5	263.	32.3
26	25.8	3.2	86	85.4	10.5	6	144.9	17.8	6	204.5	25.1	6	264.	32.4
27	26.8	3.3	87	86.4	10.6	7	145.9	17.9	7	205.5	25.2	7	265.	32.5
28	27.8	3.4	88	87.3	10.7	8	146.9	18.	8	206.4	25.3	8	266.	32.7
29	28.8	3.5	89	88.3	10.8	9	147.9	18.2	9	207.4	25.5	9	267.	32.8
30	29.8	3.7	90	89.3	11.	150	148.9	18.3	210	208.4	25.6	270	268.	32.9
31	30.8	3.8	91	90.3	11.1	1	149.9	18.4	1	209.4	25.7	1	269.	33.
32	31.8	3.9	92	91.3	11.2	2	150.9	18.5	2	210.4	25.8	2	270.	33.1
33	32.8	4.	93	92.3	11.3	3	151.9	18.6	3	211.4	26.	3	271.	33.2
34	33.8	4.1	94	93.3	11.5	4	152.9	18.8	4	212.4	26.1	4	272.	33.3
35	34.7	4.3	95	94.3	11.6	5	153.8	18.9	5	213.4	26.2	5	273.	33.5
36	35.7	4.4	96	95.3	11.7	6	154.8	19.	6	214.4	26.3	6	273.9	33.6
37	36.7	4.5	97	96.3	11.8	7	155.8	19.1	7	215.4	26.4	7	274.9	33.8
38	37.7	4.6	98	97.3	11.9	8	156.8	19.3	8	216.4	26.6	8	275.9	33.9
39	38.7	4.8	99	98.3	12.1	9	157.8	19.4	9	217.4	26.7	9	276.9	34.
40	39.7	4.9	100	99.3	12.2	160	158.8	19.5	220	218.4	26.8	280	277.9	34.1
41	40.7	5.	1	100.2	12.3	1	158.8	19.6	1	219.4	26.9	1	278.9	34.2
42	41.7	5.1	2	101.2	12.4	2	160.8	19.7	2	220.3	27.1	2	279.9	34.3
43	42.7	5.2	3	102.2	12.6	3	161.8	19.9	3	221.3	27.2	3	280.9	34.5
44	43.7	5.4	4	103.2	12.7	4	162.8	20.	4	222.3	27.3	4	281.9	34.6
45	44.7	5.5	5	104.2	12.8	5	163.8	20.1	5	223.3	27.4	5	282.9	34.7
46	45.7	5.6	6	105.2	12.9	6	164.8	20.2	6	224.3	27.5	6	283.9	34.9
47	46.7	5.7	7	106.2	13.	7	165.8	20.4	7	225.3	27.7	7	284.9	35.
48	47.6	5.8	8	107.2	13.2	8	166.7	20.5	8	226.3	27.8	8	285.9	35.1
49	48.6	6.	9	108.2	13.3	9	167.7	20.6	9	227.3	27.9	9	286.8	35.2
50	49.6	6.1	116	109.2	13.4	170	168.7	20.7	230	228.3	28.	290	287.8	35.3
51	50.6	6.2	1	110.2	13.5	1	169.7	20.8	1	229.3	28.2	1	288.8	35.5
52	51.6	6.3	2	111.2	13.6	2	170.7	21.	2	230.3	28.3	2	289.8	35.6
53	52.6	6.5	3	112.2	13.8	3	171.7	21.1	3	231.3	28.4	3	290.8	35.7
54	53.6	6.6	4	113.2	13.9	4	172.7	21.2	4	232.3	28.5	4	291.8	35.8
55	54.6	6.7	5	114.1	14.	5	173.7	21.3	5	233.3	28.6	5	292.8	36.
56	55.6	6.8	6	115.1	14.1	6	174.7	21.4	6	234.2	28.8	6	293.8	36.1
57	56.6	6.9	7	116.1	14.3	7	175.7	21.6	7	235.2	28.9	7	294.8	36.2
58	57.6	7.1	8	117.1	14.4	8	176.7	21.7	8	236.2	29.	8	295.8	36.3
59	58.6	7.2	9	118.1	14.5	9	177.7	21.8	9	237.2	29.1	9	296.8	36.4
60	59.6	7.3	120	119.1	14.6	180	178.7	21.9	240	238.2	29.2	300	297.8	36.6

Distance, Departure and Diff. Latitude.

Course 80.

Course 80°.

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	1.	0.1	61	60.4	8.5	121	119.8	16.8	181	179.2	25.2	241	238.7	33.5
2	2.	0.3	62	61.4	8.6	2	120.8	17.	2	180.2	25.3	2	239.6	33.7
3	3.	0.4	63	62.4	8.8	3	121.8	17.1	3	181.2	25.5	3	240.6	33.8
4	4.	0.6	64	63.4	8.9	4	122.8	17.3	4	182.2	25.6	4	241.6	34.
5	5.	0.7	65	64.4	9.	5	123.8	17.4	5	183.2	25.7	5	242.6	34.1
6	5.9	0.8	66	65.4	9.2	6	124.8	17.5	6	184.2	25.9	6	243.6	34.2
7	6.9	1.	67	66.3	9.3	7	125.8	17.7	7	185.2	26.	7	244.6	34.4
8	7.9	1.1	68	67.3	9.5	8	126.8	17.8	8	186.2	26.2	8	245.6	34.5
9	8.9	1.3	69	68.3	9.6	9	127.7	18.	9	187.2	26.3	9	246.6	34.7
10	9.9	1.4	70	69.3	9.7	130	128.7	18.1	100	188.2	26.4	250	247.6	34.8
11	10.9	1.5	71	70.3	9.9	1	129.7	18.2	1	189.1	26.6	1	248.6	34.9
12	11.9	1.7	72	71.3	10.	2	130.7	18.4	2	190.1	26.7	2	249.5	35.1
13	12.9	1.8	73	72.3	10.3	3	131.7	18.5	3	191.1	26.9	3	250.5	35.2
14	13.9	1.9	74	73.3	10.3	4	132.7	18.6	4	192.1	27.	4	251.5	35.3
15	14.9	2.1	75	74.3	10.4	5	133.7	18.8	5	193.1	27.1	5	252.5	35.5
16	15.8	2.2	76	75.3	10.6	6	134.7	18.9	6	194.1	27.3	6	253.5	35.6
17	16.8	2.4	77	76.3	10.7	7	135.7	19.1	7	195.1	27.4	7	254.5	35.8
18	17.8	2.5	78	77.3	10.9	8	136.7	19.2	8	196.1	27.6	8	255.5	35.9
19	18.8	2.6	79	78.3	11.	9	137.7	19.3	9	197.1	27.7	9	256.5	36.
20	19.8	2.8	80	79.3	11.1	140	138.6	19.5	200	198.1	27.8	300	257.5	36.2
21	20.8	2.9	81	80.2	11.3	1	139.6	19.6	1	199.	28.	1	258.5	36.3
22	21.8	3.1	82	81.2	11.4	2	140.6	19.8	2	200.	28.1	2	259.5	36.5
23	22.8	3.2	83	82.2	11.6	3	141.6	19.9	3	201.	28.3	3	260.4	36.6
24	23.8	3.3	84	83.2	11.7	4	142.6	20.	4	202.	28.4	4	261.4	36.7
25	24.7	3.5	85	84.2	11.8	5	143.6	20.2	5	203.	28.5	5	262.4	36.9
26	25.7	3.6	86	85.2	12.	6	144.6	20.3	6	204.	28.7	6	263.4	37.
27	26.7	3.8	87	86.2	12.1	7	145.6	20.5	7	205.	28.8	7	264.4	37.2
28	27.7	3.9	88	87.1	12.2	8	146.6	20.6	8	206.	28.9	8	265.4	37.3
29	28.7	4.	89	88.1	12.4	9	147.5	20.7	9	207.	29.1	9	266.4	37.4
30	29.7	4.3	90	89.1	12.5	150	148.5	20.9	310	208.	29.2	410	267.4	37.6
31	30.7	4.3	91	90.1	12.7	1	149.5	21.	1	208.9	29.4	1	268.4	37.7
32	31.7	4.5	92	91.1	12.8	2	150.5	21.2	2	209.9	29.5	2	269.4	37.9
33	32.7	4.6	93	92.1	12.9	3	151.5	21.3	3	210.9	29.6	3	270.3	38.
34	33.7	4.7	94	93.1	13.1	4	152.5	21.4	4	211.9	29.8	4	271.3	38.1
35	34.6	4.9	95	94.1	13.2	5	153.5	21.6	5	212.9	29.9	5	272.3	38.3
36	35.6	5.	96	95.1	13.4	6	154.5	21.7	6	213.9	30.1	6	273.3	38.4
37	36.6	5.1	97	96.1	13.5	7	155.5	21.9	7	214.9	30.2	7	274.3	38.6
38	37.6	5.3	98	97.	13.6	8	156.5	22.	8	215.9	30.3	8	275.3	38.7
39	38.6	5.4	99	98.	13.8	9	157.5	22.1	9	216.8	30.4	9	276.3	38.8
40	39.6	5.6	100	99.	13.9	160	158.4	22.2	320	217.9	30.6	420	277.3	39.
41	40.6	5.7	1	100.	14.1	1	159.4	22.4	1	218.8	30.8	1	278.3	39.1
42	41.6	5.8	2	101.	14.2	2	160.4	22.5	2	219.8	30.9	2	279.3	39.2
43	42.6	6.	3	102.	14.3	3	161.4	22.7	3	220.8	31.	3	280.2	39.4
44	43.6	6.1	4	103.	14.5	4	162.4	22.8	4	221.8	31.2	4	281.2	39.5
45	44.6	6.3	5	104.	14.6	5	163.4	23.	5	222.8	31.3	5	282.2	39.6
46	45.5	6.4	6	105.	14.8	6	164.4	23.1	6	223.8	31.5	6	283.2	39.8
47	46.5	6.5	7	106.	14.9	7	165.4	23.2	7	224.8	31.6	7	284.2	39.9
48	47.5	6.7	8	106.9	15.	8	166.4	23.4	8	225.8	31.7	8	285.2	40.1
49	48.5	6.8	9	107.9	15.2	9	167.4	23.5	9	226.8	31.9	9	286.2	40.2
50	49.5	7.	110	108.9	15.3	170	168.3	23.7	330	227.8	32.	430	287.2	40.4
51	50.5	7.1	1	109.9	15.4	1	169.3	23.8	1	228.8	32.1	1	288.2	40.5
52	51.5	7.2	2	110.9	15.6	2	170.3	23.9	2	229.7	32.3	2	289.2	40.6
53	52.5	7.4	3	111.9	15.7	3	171.3	24.1	3	230.7	32.4	3	290.1	40.8
54	53.5	7.5	4	112.9	15.9	4	172.3	24.2	4	231.7	32.6	4	291.1	40.9
55	54.5	7.7	5	113.9	16.	5	173.3	24.4	5	232.7	32.7	5	292.1	41.1
56	55.5	7.8	6	114.9	16.1	6	174.3	24.5	6	233.7	32.8	6	293.1	41.2
57	56.4	7.9	7	115.9	16.3	7	175.3	24.6	7	234.7	33.	7	294.1	41.3
58	57.4	8.1	8	116.9	16.4	8	176.3	24.8	8	235.7	33.1	8	295.1	41.5
59	58.4	8.2	9	117.8	16.6	9	177.3	24.9	9	236.7	33.3	9	296.1	41.6
60	59.4	8.4	190	118.8	16.7	180	178.9	25.1	360	237.7	33.4	300	297.1	41.8
dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.

Distance, Departure and Diff. Latitude.

Course 80°.

Course 90°.

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	1.	0.2	61	60.2	9.5	129	119.5	18.9	181	178.8	28.3	241	238.	37.7
2	2.	0.3	62	61.2	9.7	2	120.5	19.1	2	179.8	28.5	2	239.	37.9
3	3.	0.5	63	62.2	9.9	3	121.5	19.2	3	180.7	28.6	3	240.	38.
4	4.	0.6	64	63.2	10.	4	122.5	19.4	4	181.7	28.8	4	241.	38.2
5	4.9	0.8	65	64.2	10.2	5	123.5	19.6	5	182.7	28.9	5	242.	38.3
6	5.9	0.9	66	65.2	10.3	6	124.4	19.7	6	183.7	29.1	6	243.	38.5
7	6.9	1.1	67	66.3	10.5	7	125.4	19.9	7	184.7	29.3	7	244.	38.6
8	7.9	1.3	68	67.3	10.6	8	126.4	20.	8	185.7	29.4	8	244.9	38.8
9	8.9	1.4	69	68.3	10.8	9	127.4	20.2	9	186.7	29.6	9	245.9	39.
10	9.9	1.6	70	69.1	11.	130	128.4	20.3	190	187.7	29.7	250	246.9	39.1
11	10.9	1.7	71	70.1	11.1	1	129.4	20.5	1	188.6	29.9	1	247.9	39.3
12	11.9	1.9	72	71.1	11.3	2	130.4	20.6	2	189.6	30.	2	248.9	39.4
13	12.8	2.	73	72.1	11.4	3	131.4	20.8	3	190.6	30.2	3	249.9	39.6
14	13.8	2.2	74	73.1	11.6	4	132.4	21.	4	191.6	30.3	4	250.9	39.7
15	14.8	2.3	75	74.1	11.7	5	133.3	21.1	5	192.6	30.5	5	251.9	39.9
16	15.8	2.5	76	75.1	11.9	6	134.3	21.3	6	193.6	30.7	6	252.8	40.
17	16.8	2.7	77	76.1	12.	7	135.3	21.4	7	194.6	30.8	7	253.8	40.2
18	17.8	2.8	78	77.	12.2	8	136.3	21.6	8	195.6	31.	8	254.8	40.4
19	18.8	3.	79	78.	12.4	9	137.3	21.7	9	196.5	31.1	9	255.8	40.5
20	19.7	3.1	80	79.	12.5	140	138.3	21.9	200	197.5	31.3	260	256.8	40.7
21	20.7	3.3	81	80.	12.7	1	139.3	22.1	1	198.5	31.4	1	257.8	40.8
22	21.7	3.4	82	81.	12.8	2	140.3	22.2	2	199.5	31.6	2	258.8	41.
23	22.7	3.6	83	82.	13.	3	141.3	22.4	3	200.5	31.8	3	259.8	41.1
24	23.7	3.8	84	83.	13.1	4	142.2	22.5	4	201.5	31.9	4	260.7	41.3
25	24.7	3.9	85	84.	13.3	5	143.2	22.7	5	202.5	32.1	5	261.7	41.5
26	25.7	4.1	86	84.9	13.5	6	144.2	22.8	6	203.5	32.2	6	262.7	41.6
27	26.7	4.2	87	85.9	13.6	7	145.2	23.	7	204.5	32.4	7	263.7	41.8
28	27.6	4.4	88	86.9	13.8	8	146.2	23.2	8	205.4	32.5	8	264.7	41.9
29	28.6	4.5	89	87.9	13.9	9	147.2	23.3	9	206.4	32.7	9	265.7	42.1
30	29.6	4.7	90	88.9	14.1	150	148.2	23.5	210	207.4	32.9	270	266.7	42.2
31	30.6	4.8	91	89.9	14.2	1	149.1	23.6	1	208.4	33.	1	267.7	42.4
32	31.6	5.	92	90.9	14.4	2	150.1	23.8	2	209.4	33.2	2	268.7	42.6
33	32.6	5.2	93	91.9	14.5	3	151.1	23.9	3	210.4	33.3	3	269.6	42.7
34	33.6	5.3	94	92.8	14.7	4	152.1	24.1	4	211.4	33.5	4	270.6	42.8
35	34.6	5.5	95	93.8	14.9	5	153.1	24.2	5	212.4	33.6	5	271.6	43.
36	35.5	5.6	96	94.8	15.	6	154.1	24.4	6	213.3	33.8	6	272.6	43.2
37	36.5	5.8	97	95.8	15.2	7	155.1	24.6	7	214.3	33.9	7	273.6	43.3
38	37.5	5.9	98	96.8	15.3	8	156.1	24.7	8	215.3	34.1	8	274.6	43.5
39	38.5	6.1	99	97.8	15.5	9	157.	24.9	9	216.3	34.3	9	275.6	43.6
40	39.5	6.3	100	98.8	15.6	160	158.	25.	220	217.3	34.4	280	276.6	43.8
41	40.5	6.4	1	99.8	15.8	1	159.	25.2	1	218.3	34.6	1	277.5	44.
42	41.5	6.6	2	100.7	16.	2	160.	25.3	2	219.3	34.7	2	278.5	44.1
43	42.5	6.7	3	101.7	16.1	3	161.	25.5	3	220.3	34.9	3	279.5	44.3
44	43.5	6.9	4	102.7	16.3	4	162.	25.7	4	221.2	35.	4	280.5	44.4
45	44.4	7.	5	103.7	16.4	5	163.	25.8	5	222.2	35.2	5	281.5	44.6
46	45.4	7.2	6	104.7	16.6	6	164.	26.	6	223.2	35.4	6	282.5	44.7
47	46.4	7.4	7	105.7	16.7	7	164.9	26.1	7	224.2	35.5	7	283.5	44.9
48	47.4	7.5	8	106.7	16.9	8	165.9	26.3	8	225.2	35.7	8	284.5	45.1
49	48.4	7.7	9	107.7	17.1	9	166.9	26.4	9	226.2	35.8	9	285.4	45.2
50	49.4	7.8	110	108.6	17.2	170	167.9	26.6	230	227.2	36.	290	286.4	45.4
51	50.4	8.	1	109.6	17.4	1	168.9	26.8	1	228.2	36.1	1	287.4	45.5
52	51.4	8.1	2	110.6	17.5	2	169.9	26.9	2	229.1	36.3	2	288.4	45.7
53	52.3	8.3	3	111.6	17.7	3	170.9	27.1	3	230.1	36.4	3	289.4	45.8
54	53.3	8.4	4	112.6	17.8	4	171.9	27.2	4	231.1	36.6	4	290.4	46.
55	54.3	8.6	5	113.6	18.	5	172.8	27.4	5	232.1	36.8	5	291.4	46.1
56	55.3	8.8	6	114.6	18.1	6	173.8	27.5	6	233.1	36.9	6	292.4	46.3
57	56.3	8.9	7	115.6	18.3	7	174.8	27.7	7	234.1	37.1	7	293.3	46.5
58	57.3	9.1	8	116.5	18.5	8	175.8	27.8	8	235.1	37.2	8	294.3	46.6
59	58.3	9.2	9	117.5	18.6	9	176.8	28.	9	236.1	37.4	9	295.3	46.8
60	59.3	9.4	120	118.5	18.8	180	177.8	28.2	240	237.	37.5	300	296.3	46.9

Distance Departure and Diff. Latitude.

Course 81°.

Course 100°.

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	1.	0.2	61	60.1	10.6	121	119.2	21.	181	178.3	31.4	241	237.3	41.8
2	2.	0.3	62	61.1	10.8	2	120.1	21.2	2	179.2	31.6	2	238.3	42.
3	3.	0.5	63	62.	10.9	3	121.1	21.4	3	180.9	31.8	3	239.3	42.9
4	3.9	0.7	64	63.	11.1	4	122.1	21.5	4	181.2	32.	4	240.3	42.4
5	4.9	0.9	65	64.	11.3	5	123.1	21.7	5	182.2	32.1	5	241.3	42.5
6	5.9	1.	66	65.	11.5	6	124.1	21.9	6	183.2	32.3	6	242.3	42.7
7	6.9	1.2	67	66.	11.6	7	125.1	22.1	7	184.2	32.5	7	243.2	42.9
8	7.9	1.4	68	67.	11.8	8	126.1	22.2	8	185.1	32.6	8	244.2	43.1
9	8.9	1.6	69	68.	11.9	9	127.	22.4	9	186.1	32.8	9	245.2	43.2
10	9.8	1.7	70	68.9	12.2	130	128.	22.6	190	187.1	33.	250	246.2	43.4
11	10.8	1.9	71	69.9	12.3	1	129.	22.7	1	188.1	33.2	1	247.2	43.6
12	11.8	2.1	72	70.9	12.5	2	130.	22.9	2	189.1	33.3	2	248.2	43.8
13	12.8	2.3	73	71.9	12.7	3	131.	23.1	3	190.1	33.5	3	249.2	43.9
14	13.8	2.4	74	72.9	12.8	4	132.	23.3	4	191.1	33.7	4	250.1	44.1
15	14.8	2.6	75	73.9	13.	5	133.9	23.4	5	192.	33.9	5	251.1	44.3
16	15.7	2.8	76	74.8	13.2	6	133.9	23.6	6	193.	34.	6	252.1	44.5
17	16.7	3.	77	75.8	13.4	7	134.9	23.8	7	194.	34.2	7	253.1	44.6
18	17.7	3.1	78	76.8	13.5	8	135.9	24.	8	195.	34.4	8	254.1	44.8
19	18.7	3.3	79	77.8	13.7	9	136.9	24.1	9	196.	34.6	9	255.1	45.
20	19.7	3.5	80	78.8	13.9	140	137.9	24.3	200	197.	34.7	260	256.1	45.1
21	20.7	3.6	81	79.8	14.1	1	138.9	24.5	1	197.9	34.9	1	257.	45.3
22	21.7	3.8	82	80.8	14.2	2	139.8	24.7	2	198.9	35.1	2	258.	45.5
23	22.6	4.	83	81.7	14.4	3	140.8	24.8	3	199.9	35.3	3	259.	45.7
24	23.6	4.2	84	82.7	14.6	4	141.8	25.	4	200.9	35.4	4	260.	45.8
25	24.6	4.3	85	83.7	14.8	5	142.8	25.2	5	201.9	35.6	5	261.	46.
26	25.6	4.5	86	84.7	14.9	6	143.8	25.4	6	202.9	35.8	6	262.	46.2
27	26.6	4.7	87	85.7	15.1	7	144.8	25.5	7	203.9	35.9	7	262.9	46.4
28	27.6	4.9	88	86.7	15.3	8	145.8	25.7	8	204.8	36.1	8	263.9	46.5
29	28.5	5.	89	87.6	15.5	9	146.7	25.9	9	205.8	36.3	9	264.9	46.7
30	29.5	5.2	90	88.6	15.6	150	147.7	26.	210	206.8	36.5	270	265.9	46.9
31	30.5	5.4	91	89.6	15.8	1	148.7	26.2	1	207.8	36.6	1	266.9	47.1
32	31.5	5.6	92	90.6	16.	2	149.7	26.4	2	208.8	36.8	2	267.9	47.2
33	32.5	5.7	93	91.6	16.1	3	150.7	26.6	3	209.8	37.	3	268.9	47.4
34	33.5	5.9	94	92.6	16.3	4	151.7	26.7	4	210.7	37.2	4	269.8	47.6
35	34.5	6.1	95	93.6	16.5	5	152.6	26.9	5	211.7	37.3	5	270.8	47.8
36	35.4	6.3	96	94.5	16.7	6	153.6	27.1	6	212.7	37.5	6	271.8	47.9
37	36.4	6.4	97	95.5	16.8	7	154.6	27.3	7	213.7	37.7	7	272.8	48.1
38	37.4	6.6	98	96.5	17.	8	155.6	27.4	8	214.7	37.9	8	273.8	48.3
39	38.4	6.8	99	97.5	17.2	9	156.6	27.6	9	215.7	38.	9	274.8	48.4
40	39.4	6.9	100	98.5	17.4	160	157.6	27.8	220	216.7	38.2	280	275.7	48.6
41	40.4	7.1	1	99.5	17.5	1	158.6	28.	1	217.6	38.4	1	276.7	48.8
42	41.4	7.3	2	100.5	17.7	2	159.5	28.1	2	218.6	38.5	2	277.7	49.
43	42.3	7.5	3	101.4	17.9	3	160.5	28.3	3	219.6	38.7	3	278.7	49.1
44	43.3	7.6	4	102.4	18.1	4	161.5	28.5	4	220.6	38.9	4	279.7	49.3
45	44.3	7.8	5	103.4	18.2	5	162.5	28.7	5	221.6	39.1	5	280.7	49.5
46	45.3	8.	6	104.4	18.4	6	163.5	28.8	6	222.6	39.2	6	281.7	49.7
47	46.3	8.2	7	105.4	18.6	7	164.5	29.	7	223.6	39.4	7	282.6	49.8
48	47.3	8.3	8	106.4	18.8	8	165.4	29.2	8	224.5	39.6	8	283.6	50.
49	48.3	8.5	9	107.3	18.9	9	166.4	29.3	9	225.5	39.8	9	284.6	50.2
50	49.2	8.7	110	108.3	19.1	170	167.4	29.5	230	226.5	39.9	290	285.6	50.4
51	50.2	8.9	1	109.3	19.3	1	168.4	29.7	1	227.5	40.1	1	286.6	50.5
52	51.2	9.	2	110.3	19.4	2	169.4	29.9	2	228.5	40.3	2	287.6	50.7
53	52.2	9.2	3	111.3	19.6	3	170.4	30.	3	229.5	40.5	3	288.5	50.9
54	53.2	9.4	4	112.3	19.8	4	171.4	30.2	4	230.4	40.6	4	289.5	51.1
55	54.2	9.6	5	113.3	20.	5	172.3	30.4	5	231.4	40.8	5	290.5	51.2
56	55.1	9.7	6	114.2	20.1	6	173.3	30.6	6	232.4	41.	6	291.5	51.4
57	56.1	9.9	7	115.2	20.3	7	174.3	30.7	7	233.4	41.2	7	292.5	51.6
58	57.1	10.1	8	116.2	20.5	8	175.3	30.9	8	234.4	41.3	8	293.5	51.7
59	58.1	10.2	9	117.2	20.7	9	176.3	31.1	9	235.4	41.5	9	294.5	51.9
60	59.1	10.4	120	118.2	20.8	180	177.3	31.3	240	236.4	41.7	300	295.4	52.1

Distance, Departure and Diff. Latitude.

Course 80°.

TABLE V.

Course 110.

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	2.	0.2	61	59.9	11.6	121	118.6	23.1	181	177.7	34.5	241	236.6	46.
2	3.	0.4	62	60.9	11.8	2	119.6	23.3	3	178.7	34.7	4	237.6	46.2
3	2.9	0.6	63	61.8	12.	3	120.7	23.5	4	179.6	34.9	5	238.5	46.4
4	3.9	0.8	64	62.8	12.2	4	121.7	23.7	5	180.6	35.1	6	239.5	46.6
5	4.9	1.	65	63.8	12.4	5	122.7	23.9	6	181.6	35.3	7	240.5	46.7
6	5.9	1.1	66	64.8	12.6	6	123.7	24.1	7	182.6	35.5	8	241.5	46.9
7	6.9	1.3	67	65.8	12.8	7	124.7	24.3	8	183.6	35.7	9	242.5	47.1
8	7.9	1.5	68	66.8	13.	8	125.6	24.4	9	184.5	35.9	8	243.4	47.3
9	8.8	1.7	69	67.7	13.2	9	126.6	24.6	9	185.5	36.1	9	244.4	47.5
10	9.8	1.9	70	68.7	13.4	130	127.6	24.8	190	186.5	36.3	280	245.4	47.7
11	10.8	2.1	71	69.7	13.5	1	128.6	25.	1	187.5	36.4	1	246.4	47.9
12	11.8	2.3	72	70.7	13.7	2	129.6	25.2	2	188.5	36.6	2	247.4	48.1
13	12.8	2.5	73	71.7	13.9	3	130.6	25.4	3	189.5	36.8	3	248.4	48.3
14	13.7	2.7	74	72.6	14.1	4	131.5	25.6	4	190.4	37.	4	249.3	48.5
15	14.7	2.9	75	73.6	14.3	5	132.5	25.8	5	191.4	37.2	5	250.3	48.7
16	15.7	3.1	76	74.6	14.5	6	133.5	26.	6	192.4	37.4	6	251.3	48.9
17	16.7	3.3	77	75.6	14.7	7	134.5	26.1	7	193.4	37.6	7	252.3	49.
18	17.7	3.4	78	76.6	14.9	8	135.5	26.3	8	194.4	37.8	8	253.3	49.3
19	18.7	3.6	79	77.5	15.1	9	136.4	26.5	9	195.3	38.	9	254.3	49.4
20	19.6	3.8	80	78.5	15.3	140	137.4	26.7	290	196.3	38.2	280	255.2	49.6
21	20.6	4.	81	79.5	15.5	1	138.4	26.9	1	197.3	38.4	1	256.2	49.8
22	21.6	4.2	82	80.5	15.8	2	139.4	27.1	2	198.3	38.5	2	257.2	50.
23	22.6	4.4	83	81.5	15.8	3	140.4	27.3	3	199.3	38.7	3	258.2	50.2
24	23.6	4.6	84	82.5	16.	4	141.4	27.5	4	200.3	38.9	4	259.1	50.4
25	24.5	4.8	85	83.4	16.2	5	142.3	27.7	5	201.2	39.1	5	260.1	50.6
26	25.5	5.	86	84.4	16.4	6	143.3	27.9	6	202.2	39.3	6	261.1	50.8
27	26.5	5.2	87	85.4	16.6	7	144.3	28.	7	203.2	39.5	7	262.1	50.9
28	27.5	5.3	88	86.4	16.8	8	145.3	28.2	8	204.2	39.7	8	263.1	51.1
29	28.5	5.5	89	87.4	17.	9	146.3	28.4	9	205.2	39.9	9	264.1	51.3
30	29.4	5.7	90	88.3	17.2	150	147.2	28.6	310	206.1	40.1	270	265.	51.5
31	30.4	5.9	91	89.3	17.4	1	148.2	28.8	1	207.1	40.3	1	266.	51.7
32	31.4	6.1	92	90.3	17.6	2	149.2	29.	2	208.1	40.5	2	267.	51.9
33	32.4	6.3	93	91.3	17.7	3	150.2	29.2	3	209.1	40.6	3	268.	52.1
34	33.4	6.5	94	92.3	17.9	4	151.2	29.4	4	210.1	40.8	4	269.	52.3
35	34.4	6.7	95	93.3	18.1	5	152.2	29.6	5	211.	41.	5	270.	52.5
36	35.3	6.9	96	94.2	18.3	6	153.1	29.8	6	212.	41.2	6	271.	52.7
37	36.3	7.1	97	95.2	18.5	7	154.1	30.	7	213.	41.4	7	271.9	52.9
38	37.3	7.3	98	96.2	18.7	8	155.1	30.1	8	214.	41.6	8	272.9	53.
39	38.2	7.4	99	97.2	18.9	9	156.1	30.3	9	215.	41.8	9	273.9	53.2
40	39.2	7.6	100	98.2	19.1	160	157.1	30.5	290	216.	42.	280	274.9	53.4
41	40.2	7.8	1	99.1	19.3	1	158.	30.7	1	216.9	42.2	1	275.8	53.6
42	41.2	8.	2	100.1	19.5	2	159.	30.9	2	217.9	42.4	2	276.8	53.8
43	42.2	8.2	3	101.1	19.7	3	160.	31.1	3	218.9	42.6	3	277.8	54.
44	43.2	8.4	4	102.1	19.8	4	161.	31.3	4	219.9	42.7	4	278.8	54.2
45	44.2	8.6	5	103.1	20.	5	162.	31.5	5	220.9	42.9	5	279.8	54.4
46	45.2	8.8	6	104.1	20.2	6	163.	31.7	6	221.8	43.1	6	280.7	54.6
47	46.1	9.	7	105.	20.4	7	163.9	31.9	7	222.8	43.3	7	281.7	54.8
48	47.1	9.2	8	106.	20.6	8	164.9	32.1	8	223.8	43.5	8	282.7	55.
49	48.1	9.3	9	107.	20.8	9	165.9	32.3	9	224.8	43.7	9	283.7	55.1
50	49.1	9.5	110	108.	21.	170	166.9	32.4	290	225.8	43.9	280	284.7	55.3
51	50.1	9.7	1	109.	21.2	1	167.9	32.6	1	226.8	44.1	1	285.7	55.5
52	51.	9.9	2	109.9	21.4	2	168.8	32.8	2	227.7	44.3	2	286.6	55.7
53	52.	10.1	3	110.9	21.6	3	169.8	33.	3	228.7	44.5	3	287.6	55.9
54	53.	10.3	4	111.9	21.8	4	170.8	33.2	4	229.7	44.6	4	288.6	56.1
55	54.	10.5	5	112.9	21.9	5	171.8	33.4	5	230.7	44.8	5	289.6	56.3
56	55.	10.7	6	113.9	22.1	6	172.8	33.6	6	231.7	45.	6	290.6	56.5
57	56.	10.9	7	114.9	22.3	7	173.7	33.8	7	232.6	45.2	7	291.5	56.7
58	56.9	11.1	8	115.8	22.5	8	174.7	34.	8	233.6	45.4	8	292.5	56.9
59	57.9	11.3	9	116.8	22.7	9	175.7	34.2	9	234.6	45.6	9	293.5	57.1
60	58.9	11.4	190	117.8	22.9	180	176.7	34.3	240	235.6	45.8	300	294.5	57.2

Distance, Departure and Diff. Latitude.

Course 70°.

Course 150.
Distance, DiE Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	1.	0.3	61	58.9	15.8	130	116.9	31.3	181	174.8	46.8	241	232.8	62.4
2	1.9	0.5	62	59.9	16.	2	117.8	31.6	2	175.8	47.1	3	233.8	62.6
3	2.9	0.8	63	60.9	16.3	3	118.8	31.8	3	176.8	47.4	3	234.7	62.9
4	3.9	1.	64	61.8	16.6	4	119.8	32.1	4	177.7	47.6	4	235.7	63.2
5	4.8	1.3	65	62.8	16.8	5	120.7	32.4	5	178.7	47.9	5	236.7	63.4
6	5.8	1.6	66	63.8	17.1	6	121.7	32.6	6	179.7	48.1	6	237.6	63.7
7	6.8	1.8	67	64.7	17.3	7	122.7	32.9	7	180.6	48.4	7	238.6	63.9
8	7.7	2.1	68	65.7	17.6	8	123.6	33.1	8	181.6	48.7	8	239.5	64.2
9	8.7	2.3	69	66.6	17.9	9	124.6	33.4	9	182.6	48.9	9	240.5	64.4
10	9.7	2.6	70	67.6	18.1	130	125.6	33.6	190	183.5	49.3	250	241.5	64.7
11	10.6	2.8	71	68.6	18.4	1	126.5	33.9	1	184.5	49.4	1	242.4	65.
12	11.6	3.1	72	69.5	18.6	2	127.5	34.2	2	185.5	49.7	2	243.4	65.2
13	12.6	3.4	73	70.5	18.9	3	128.5	34.4	3	186.4	50.	3	244.4	65.5
14	13.5	3.6	74	71.5	19.2	4	129.4	34.7	4	187.4	50.2	4	245.3	65.7
15	14.5	3.9	75	72.4	19.4	5	130.4	34.9	5	188.4	50.5	5	246.3	66.
16	15.5	4.1	76	73.4	19.7	6	131.4	35.2	6	189.3	50.7	6	247.3	66.3
17	16.4	4.4	77	74.4	19.9	7	132.3	35.5	7	190.3	51.	7	248.2	66.5
18	17.4	4.7	78	75.3	20.2	8	133.3	35.7	8	191.3	51.2	8	249.2	66.8
19	18.4	4.9	79	76.3	20.4	9	134.3	36.	9	192.3	51.5	9	250.2	67.
20	19.3	5.2	80	77.3	20.7	140	135.3	36.2	200	193.2	51.8	300	251.1	67.3
21	20.3	5.4	81	78.2	21.	1	136.2	36.5	1	194.2	52.	1	252.1	67.6
22	21.3	5.7	82	79.2	21.2	2	137.2	36.8	2	195.1	52.3	2	253.1	67.8
23	22.2	6.	83	80.2	21.5	3	138.1	37.	3	196.1	52.5	3	254.	68.1
24	23.2	6.2	84	81.1	21.7	4	139.1	37.3	4	197.	52.8	4	255.	68.3
25	24.1	6.5	85	82.1	22.	5	140.1	37.5	5	198.	53.1	5	256.	68.6
26	25.1	6.7	86	83.1	22.3	6	141.	37.8	6	199.	53.3	6	257.9	68.8
27	26.1	7.	87	84.	22.5	7	142.	38.	7	199.9	53.6	7	259.9	69.1
28	27.	7.2	88	85.	22.8	8	143.	38.3	8	200.9	53.8	8	259.9	69.4
29	28.	7.5	89	86.	23.	9	143.9	38.6	9	201.9	54.1	9	259.9	69.6
30	29.	7.8	90	86.9	23.3	150	144.9	38.8	310	202.8	54.4	370	260.8	69.9
31	29.9	8.	91	87.9	23.6	1	145.9	39.1	1	203.8	54.6	1	261.8	70.1
32	30.9	8.2	92	88.9	23.9	2	146.8	39.3	2	204.8	54.9	2	262.7	70.4
33	31.9	8.5	93	89.8	24.1	3	147.8	39.6	3	205.7	55.1	3	263.7	70.7
34	32.8	8.8	94	90.8	24.3	4	148.8	39.9	4	206.7	55.4	4	264.7	70.9
35	33.8	9.1	95	91.8	24.6	5	149.7	40.1	5	207.7	55.6	5	265.6	71.2
36	34.8	9.3	96	92.7	24.8	6	150.7	40.4	6	208.6	55.9	6	266.6	71.4
37	35.7	9.6	97	93.7	25.1	7	151.7	40.6	7	209.6	56.2	7	267.6	71.7
38	36.7	9.8	98	94.7	25.4	8	152.6	40.9	8	210.6	56.4	8	268.5	72.
39	37.7	10.1	99	95.6	25.6	9	153.6	41.3	9	211.5	56.7	9	269.5	72.2
40	38.6	10.4	100	96.6	25.9	160	154.5	41.4	320	212.5	56.9	380	270.5	72.5
41	39.6	10.6	1	97.6	26.1	1	155.5	41.7	1	213.5	57.2	1	271.4	72.7
42	40.6	10.9	2	98.5	26.4	2	156.5	41.9	2	214.4	57.5	2	272.4	73.
43	41.5	11.1	3	99.5	26.7	3	157.4	42.2	3	215.4	57.7	3	273.4	73.2
44	42.5	11.4	4	100.5	26.9	4	158.4	42.4	4	216.4	58.	4	274.3	73.5
45	43.5	11.6	5	101.4	27.2	5	159.4	42.7	5	217.3	58.2	5	275.3	73.8
46	44.4	11.9	6	102.4	27.4	6	160.3	43.	6	218.3	58.5	6	276.3	74.
47	45.4	12.2	7	103.4	27.7	7	161.3	43.2	7	219.3	58.8	7	277.2	74.3
48	46.4	12.4	8	104.3	28.	8	162.3	43.5	8	220.3	59.	8	278.2	74.5
49	47.3	12.7	9	105.3	28.3	9	163.3	43.7	9	221.3	59.3	9	279.2	74.8
50	48.3	12.9	110	106.3	28.5	170	164.3	44.	330	222.2	59.5	390	280.1	75.1
51	49.3	13.2	1	107.2	28.7	1	165.2	44.3	1	223.1	59.8	1	281.1	75.3
52	50.3	13.5	2	108.2	29.	2	166.1	44.5	2	224.1	60.	2	282.1	75.6
53	51.2	13.7	3	109.1	29.2	3	167.1	44.8	3	225.1	60.3	3	283.	75.8
54	52.2	14.	4	110.1	29.5	4	168.1	45.	4	226.	60.6	4	284.	76.1
55	53.1	14.3	15	111.1	29.8	5	169.	45.3	5	227.	60.8	5	284.9	76.4
56	54.1	14.5	16	112.	30.	6	170.	45.6	6	228.	61.1	6	285.9	76.6
57	55.1	14.8	17	113.	30.3	7	171.	45.8	7	228.9	61.3	7	286.9	76.9
58	56.	15.	18	114.	30.5	8	171.9	46.1	8	229.9	61.6	8	287.8	77.1
59	57.	15.3	19	114.9	30.8	9	172.9	46.3	9	230.9	61.9	9	288.8	77.4
60	58.	15.5	190	115.9	31.1	180	173.9	46.6	240	231.8	62.1	300	289.8	77.6
dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.

Distance, Departure and DiE Latitude

Course 170.
 Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	1.	0.3	61	58.3	17.8	121	115.7	35.4	181	173.1	52.9	241	230.5	70.5
2	1.9	0.6	62	59.3	18.1	2	116.7	35.7	2	174.	53.2	2	231.4	70.8
3	2.9	0.9	63	60.2	18.4	3	117.6	36.	3	175.	53.5	3	232.4	71.
4	3.8	1.2	64	61.2	18.7	4	118.6	36.3	4	176.	53.8	4	233.3	71.3
5	4.8	1.5	65	62.2	19.	5	119.5	36.5	5	176.9	54.1	5	234.3	71.6
6	5.7	1.8	66	63.1	19.3	6	120.5	36.8	6	177.9	54.4	6	235.3	71.9
7	6.7	2.	67	64.1	19.6	7	121.5	37.1	7	178.8	54.7	7	236.2	72.2
8	7.7	2.3	68	65.	19.9	8	122.4	37.4	8	179.8	55.	8	237.2	72.5
9	8.6	2.6	69	66.	20.2	9	123.4	37.7	9	180.7	55.3	9	238.1	72.8
10	9.6	2.9	70	66.9	20.5	130	124.3	38.	190	181.7	55.6	250	239.1	73.1
11	10.5	3.2	71	67.9	20.8	1	125.3	38.3	1	182.7	55.8	1	240.	73.4
12	11.5	3.5	72	68.9	21.1	2	126.2	38.6	2	183.6	56.1	2	241.	73.7
13	12.4	3.8	73	69.8	21.3	3	127.2	38.9	3	184.6	56.4	3	241.9	74.
14	13.4	4.1	74	70.8	21.6	4	128.1	39.2	4	185.5	56.7	4	242.9	74.3
15	14.3	4.4	75	71.7	21.9	5	129.1	39.5	5	186.5	57.	5	243.9	74.6
16	15.3	4.7	76	72.7	22.2	6	130.1	39.8	6	187.4	57.3	6	244.8	74.8
17	16.3	5.	77	73.6	22.5	7	131.	40.1	7	188.4	57.6	7	245.8	75.1
18	17.2	5.3	78	74.6	22.8	8	132.	40.3	8	189.3	57.9	8	246.7	75.4
19	18.2	5.6	79	75.5	23.1	9	133.9	40.6	9	190.3	58.2	9	247.7	75.7
20	19.1	5.8	80	76.5	23.4	140	133.9	40.9	200	191.3	58.5	260	248.6	76.
21	20.1	6.1	81	77.5	23.7	1	134.8	41.2	1	192.2	58.8	1	249.6	76.3
22	21.	6.4	82	78.4	24.	2	135.8	41.5	2	193.2	59.1	2	250.6	76.6
23	22.	6.7	83	79.4	24.3	3	136.8	41.8	3	194.1	59.4	3	251.5	76.9
24	23.	7.	84	80.3	24.6	4	137.7	42.1	4	195.1	59.6	4	252.5	77.2
25	23.9	7.3	85	81.3	24.9	5	138.7	42.4	5	196.	59.9	5	253.4	77.5
26	24.9	7.6	86	82.2	25.1	6	139.6	42.7	6	197.	60.2	6	254.4	77.8
27	25.8	7.9	87	83.2	25.4	7	140.6	43.	7	198.	60.5	7	255.3	78.1
28	26.8	8.2	88	84.2	25.7	8	141.5	43.3	8	198.9	60.8	8	256.3	78.4
29	27.7	8.5	89	85.1	26.	9	142.5	43.6	9	199.9	61.1	9	257.2	78.6
30	28.7	8.8	90	86.1	26.3	150	143.4	43.9	210	200.8	61.4	270	258.2	78.9
31	29.6	9.1	91	87.	26.6	1	144.4	44.1	1	201.8	61.7	1	259.2	79.2
32	30.6	9.4	92	88.	26.9	2	145.4	44.4	2	202.7	62.	2	260.1	79.5
33	31.6	9.6	93	88.9	27.2	3	146.3	44.7	3	203.7	62.3	3	261.1	79.8
34	32.5	9.9	94	89.9	27.5	4	147.3	45.	4	204.6	62.6	4	262.	80.1
35	33.5	10.2	95	90.8	27.8	5	148.2	45.3	5	205.6	62.9	5	263.	80.4
36	34.4	10.5	96	91.8	28.1	6	149.2	45.6	6	206.6	63.2	6	263.9	80.7
37	35.4	10.8	97	92.8	28.4	7	150.1	45.9	7	207.5	63.4	7	264.9	81.
38	36.3	11.1	98	93.7	28.7	8	151.1	46.2	8	208.5	63.7	8	265.9	81.3
39	37.3	11.4	99	94.7	29.	9	152.1	46.5	9	209.4	64.	9	266.8	81.6
40	38.3	11.7	100	95.6	29.2	160	153.	46.8	220	210.4	64.3	280	267.8	81.9
41	39.2	12.	1	96.6	29.5	1	154.	47.1	1	211.3	64.6	1	268.7	82.2
42	40.2	12.3	2	97.5	29.8	2	154.9	47.4	2	212.3	64.9	2	269.7	82.4
43	41.1	12.6	3	98.5	30.1	3	155.9	47.7	3	213.3	65.2	3	270.6	82.7
44	42.1	12.9	4	99.5	30.4	4	156.8	47.9	4	214.2	65.5	4	271.6	83.
45	43.	13.2	5	100.4	30.7	5	157.8	48.2	5	215.2	65.8	5	272.5	83.3
46	44.	13.4	6	101.4	31.	6	158.7	48.5	6	216.1	66.1	6	273.5	83.6
47	44.9	13.7	7	102.3	31.3	7	159.7	48.8	7	217.1	66.4	7	274.5	83.9
48	45.9	14.	8	103.3	31.6	8	160.7	49.1	8	218.	66.7	8	275.4	84.2
49	46.9	14.3	9	104.2	31.9	9	161.6	49.4	9	219.	67.	9	276.4	84.5
50	47.8	14.6	110	105.2	32.2	170	162.6	49.7	230	220.	67.2	290	277.3	84.8
51	48.8	14.9	1	106.1	32.5	1	163.5	50.	1	220.9	67.5	1	278.3	85.1
52	49.7	15.2	2	107.1	32.7	2	164.5	50.3	2	221.9	67.8	2	279.2	85.4
53	50.7	15.5	3	108.1	33.	3	165.4	50.6	3	222.8	68.1	3	280.2	85.7
54	51.6	15.8	4	109.	33.3	4	166.4	50.9	4	223.8	68.4	4	281.2	86.
55	52.6	16.1	5	110.	33.6	5	167.4	51.2	5	224.7	68.7	5	282.1	86.2
56	53.6	16.4	6	110.9	33.9	6	168.3	51.5	6	225.7	69.	6	283.1	86.5
57	54.5	16.7	7	111.9	34.2	7	169.3	51.7	7	226.6	69.3	7	284.	86.8
58	55.5	17.	8	112.8	34.5	8	170.2	52.	8	227.6	69.6	8	285.	87.1
59	56.4	17.2	9	113.8	34.8	9	171.2	52.3	9	228.6	69.9	9	285.9	87.4
60	57.4	17.5	190	114.8	35.1	180	172.1	52.6	240	229.5	70.2	300	286.9	87.7

Distance, Departure and Diff. Latitude.

Course 73°.

Course 180.
Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	1.	0.3	61	58.	18.9	121	115.1	37.4	181	172.1	55.9	241	299.3	74.5
2	1.9	0.6	62	59.	19.2	9	116.	37.7	2	173.1	56.9	2	230.9	74.8
3	2.9	0.9	63	59.9	19.5	3	117.	38.	3	174.	56.6	3	231.1	75.1
4	3.8	1.2	64	60.9	19.8	4	117.9	38.3	4	175.	56.9	4	231.1	75.4
5	4.8	1.5	65	61.8	20.1	5	118.9	38.6	5	175.9	57.2	5	232.	75.7
6	5.7	1.9	66	62.8	20.4	6	119.8	38.9	6	176.9	57.5	6	234.	76.
7	6.7	2.2	67	63.7	20.7	7	120.8	39.2	7	177.8	57.8	7	234.9	76.3
8	7.6	2.5	68	64.7	21.	8	121.7	39.5	8	178.8	58.1	8	235.9	76.6
9	8.6	2.8	69	65.6	21.3	9	122.7	39.9	9	179.7	58.4	9	236.8	76.9
10	9.5	3.1	70	66.6	21.6	130	123.6	40.2	190	180.7	58.7	250	237.8	77.3
11	10.5	3.4	71	67.5	21.9	1	124.6	40.5	1	181.7	59.	1	238.7	77.6
12	11.4	3.7	72	68.5	22.2	2	125.5	40.8	2	182.6	59.3	2	239.7	77.9
13	12.4	4.	73	69.4	22.5	3	126.5	41.1	3	183.6	59.6	3	240.6	78.2
14	13.3	4.3	74	70.4	22.9	4	127.4	41.4	4	184.5	59.9	4	241.6	78.5
15	14.3	4.6	75	71.3	23.2	5	128.4	41.7	5	185.5	60.3	5	242.5	78.8
16	15.3	4.9	76	72.3	23.5	6	129.3	42.	6	186.4	60.6	6	243.5	79.1
17	16.3	5.3	77	73.2	23.8	7	130.3	42.3	7	187.4	60.9	7	244.4	79.4
18	17.1	5.6	78	74.2	24.1	8	131.2	42.6	8	188.3	61.2	8	245.4	79.7
19	18.1	5.9	79	75.1	24.4	9	132.2	43.	9	189.3	61.5	9	246.3	80.
20	19.	6.2	80	76.1	24.7	140	133.1	43.3	200	190.2	61.8	260	247.3	80.3
21	20.	6.5	81	77.	25.	1	134.1	43.6	1	191.2	62.1	1	248.2	80.7
22	20.9	6.8	82	78.	25.3	2	135.1	43.9	2	192.1	62.4	2	249.2	81.
23	21.9	7.1	83	78.9	25.6	3	136.	44.2	3	193.1	62.7	3	250.1	81.3
24	22.8	7.4	84	79.9	26.	4	137.	44.5	4	194.	63.	4	251.1	81.6
25	23.8	7.7	85	80.8	26.3	5	137.9	44.8	5	195.	63.3	5	252.1	81.9
26	24.7	8.	86	81.8	26.6	6	138.9	45.1	6	195.9	63.7	6	253.	82.2
27	25.7	8.3	87	82.7	26.9	7	139.8	45.4	7	196.9	64.	7	253.9	82.5
28	26.6	8.7	88	83.7	27.2	8	140.8	45.7	8	197.8	64.3	8	254.9	82.8
29	27.6	9.	89	84.6	27.5	9	141.7	46.	9	198.8	64.6	9	255.8	83.1
30	28.5	9.3	90	85.6	27.8	150	142.7	46.4	210	199.7	64.9	270	256.8	83.4
31	29.5	9.6	91	86.5	28.1	1	143.6	46.7	1	200.7	65.2	1	257.7	83.7
32	30.4	9.9	92	87.5	28.4	2	144.6	47.	2	201.6	65.5	2	258.7	84.1
33	31.4	10.2	93	88.4	28.7	3	145.5	47.3	3	202.6	65.8	3	259.6	84.4
34	32.3	10.5	94	89.4	29.	4	146.5	47.6	4	203.5	66.1	4	260.6	84.7
35	33.3	10.8	95	90.4	29.4	5	147.4	47.9	5	204.5	66.4	5	261.5	85.
36	34.3	11.1	96	91.3	29.7	6	148.4	48.2	6	205.4	66.7	6	262.5	85.3
37	35.3	11.4	97	92.3	30.	7	149.3	48.5	7	206.4	67.1	7	263.4	85.6
38	36.1	11.7	98	93.2	30.3	8	150.3	48.9	8	207.3	67.4	8	264.4	85.9
39	37.1	12.1	99	94.2	30.6	9	151.2	49.1	9	208.3	67.7	9	265.3	86.2
40	38.	12.4	100	95.1	30.9	160	152.2	49.4	220	209.2	68.	280	266.3	86.5
41	39.	12.7	1	96.1	31.2	1	153.1	49.8	1	210.2	68.3	1	267.3	86.8
42	39.9	13.	2	97.	31.5	2	154.1	50.1	2	211.1	68.6	2	268.2	87.1
43	40.9	13.3	3	98.	31.8	3	155.	50.4	3	212.1	68.9	3	269.1	87.5
44	41.8	13.6	4	98.9	32.1	4	156.	50.7	4	213.	69.2	4	270.1	87.8
45	42.8	13.9	5	99.9	32.4	5	156.9	51.	5	214.	69.5	5	271.1	88.1
46	43.7	14.2	6	100.8	32.8	6	157.9	51.3	6	214.9	69.8	6	272.	88.4
47	44.7	14.5	7	101.8	33.1	7	158.8	51.6	7	215.9	70.1	7	273.	88.7
48	45.7	14.8	8	102.7	33.4	8	159.8	51.9	8	216.8	70.5	8	273.9	89.
49	46.6	15.1	9	103.7	33.7	9	160.7	52.2	9	217.8	70.8	9	274.9	89.3
50	47.6	15.5	110	104.6	34.	170	161.7	52.5	230	218.7	71.1	290	275.8	89.6
51	48.5	15.8	1	105.6	34.3	1	162.6	52.8	1	219.7	71.4	1	276.8	89.9
52	49.5	16.1	2	106.5	34.6	2	163.6	53.2	2	220.6	71.7	2	277.7	90.2
53	50.4	16.4	3	107.5	34.9	3	164.5	53.5	3	221.6	72.	3	278.7	90.5
54	51.4	16.7	4	108.4	35.2	4	165.5	53.8	4	222.5	72.3	4	279.6	90.9
55	52.3	17.	5	109.4	35.5	5	166.4	54.1	5	223.5	72.6	5	280.6	91.2
56	53.3	17.3	6	110.3	35.8	6	167.4	54.4	6	224.4	72.9	6	281.5	91.5
57	54.3	17.6	7	111.3	36.2	7	168.3	54.7	7	225.4	73.2	7	282.5	91.8
58	55.3	17.9	8	112.2	36.5	8	169.3	55.	8	226.4	73.5	8	283.4	92.1
59	56.1	18.2	9	113.2	36.8	9	170.2	55.3	9	227.3	73.9	9	284.4	92.4
60	57.1	18.5	190	114.1	37.1	180	171.2	55.6	240	228.3	74.2	300	285.3	92.7

Distance, Departure and Diff. Latitude.

Course 79°.

Course 85°.

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	0.9	0.4	61	55.3	25.8	121	100.7	51.1	181	164.	76.5	241	218.4	101.9
2	1.8	0.8	62	55.3	26.3	2	110.6	51.6	2	164.9	76.9	3	219.3	102.3
3	2.7	1.3	63	57.1	26.6	3	111.5	52.	3	165.9	77.3	3	220.2	102.7
4	3.6	1.7	64	58.	27.	4	112.4	52.4	4	166.8	77.8	4	221.1	103.1
5	4.5	2.1	65	58.9	27.5	5	113.3	52.8	5	167.7	78.2	5	222.	103.5
6	5.4	2.5	66	59.8	27.9	6	114.9	53.2	6	168.6	78.6	6	223.	104.
7	6.3	3.	67	60.7	28.3	7	115.1	53.7	7	169.5	79.	7	223.9	104.4
8	7.3	3.4	68	61.6	28.7	8	116.	54.1	8	170.4	79.5	8	224.8	104.8
9	8.3	3.8	69	62.5	29.2	9	116.9	54.5	9	171.3	79.9	9	225.7	105.2
10	9.1	4.2	70	63.4	29.6	130	117.8	54.9	190	172.2	80.3	250	226.6	105.7
11	10.	4.6	71	64.3	30.	1	118.7	55.4	1	173.1	80.7	1	227.5	106.1
12	10.9	5.1	72	65.3	30.4	2	119.6	55.8	2	174.	81.1	2	228.4	106.5
13	11.8	5.5	73	66.2	30.9	3	120.5	56.2	3	174.9	81.6	3	229.3	106.9
14	12.7	5.9	74	67.1	31.3	4	121.4	56.6	4	175.8	82.	4	230.2	107.3
15	13.6	6.3	75	68.	31.7	5	122.4	57.1	5	176.7	82.4	5	231.1	107.8
16	14.5	6.8	76	68.9	32.1	6	123.3	57.5	6	177.6	82.8	6	232.	108.2
17	15.4	7.2	77	69.8	32.5	7	124.2	57.9	7	178.5	83.3	7	232.9	108.6
18	16.3	7.6	78	70.7	33.	8	125.1	58.3	8	179.4	83.7	8	233.8	109.
19	17.2	8.	79	71.6	33.4	9	126.	58.7	9	180.4	84.1	9	234.7	109.5
20	18.1	8.5	80	72.5	33.8	140	126.9	59.2	200	181.3	84.5	260	235.6	109.9
21	19.	8.9	81	73.4	34.2	1	127.8	59.6	1	182.2	84.9	1	236.5	110.3
22	19.9	9.3	82	74.3	34.7	2	128.7	60.	2	183.1	85.4	2	237.5	110.7
23	20.8	9.7	83	75.2	35.1	3	129.6	60.4	3	184.	85.8	3	238.4	111.1
24	21.8	10.1	84	76.1	35.5	4	130.5	60.9	4	184.9	86.2	4	239.3	111.6
25	22.7	10.6	85	77.	35.9	5	131.4	61.3	5	185.8	86.6	5	240.2	112.
26	23.6	11.	86	77.9	36.3	6	132.3	61.7	6	186.7	87.1	6	241.1	112.4
27	24.5	11.4	87	78.8	36.8	7	133.2	62.1	7	187.6	87.5	7	242.	112.8
28	25.4	11.8	88	79.7	37.2	8	134.1	62.5	8	188.5	87.9	8	242.9	113.3
29	26.3	12.3	89	80.6	37.6	9	135.	63.	9	189.4	88.3	9	243.8	113.7
30	27.2	12.7	90	81.6	38.	150	135.9	63.4	210	190.3	88.7	270	244.7	114.1
31	28.1	13.1	91	82.5	38.5	1	136.9	63.8	1	191.2	89.2	1	245.6	114.5
32	29.	13.5	92	83.4	38.9	2	137.8	64.2	2	192.1	89.6	2	246.5	115.
33	29.9	13.9	93	84.3	39.3	3	138.7	64.7	3	193.	90.	3	247.4	115.4
34	30.8	14.4	94	85.2	39.7	4	139.6	65.1	4	193.9	90.4	4	248.3	115.8
35	31.7	14.8	95	86.1	40.1	5	140.5	65.5	5	194.9	90.9	5	249.2	116.2
36	32.6	15.2	96	87.	40.6	6	141.4	65.9	6	195.8	91.3	6	250.1	116.6
37	33.5	15.6	97	87.9	41.	7	142.3	66.4	7	196.7	91.7	7	251.	117.1
38	34.4	16.1	98	88.8	41.4	8	143.2	66.8	8	197.6	92.1	8	252.	117.5
39	35.3	16.5	99	89.7	41.8	9	144.1	67.2	9	198.5	92.6	9	252.9	117.9
40	36.3	16.9	100	90.6	42.3	160	145.	67.6	220	199.4	93.	280	253.8	118.3
41	37.2	17.3	1	91.5	42.7	1	145.9	68.	1	200.3	93.4	1	254.7	118.8
42	38.1	17.7	2	92.4	43.1	2	146.8	68.5	2	201.2	93.8	2	255.6	119.2
43	39.	18.2	3	93.3	43.5	3	147.7	68.9	3	202.1	94.2	3	256.5	119.6
44	39.9	18.6	4	94.3	44.	4	148.6	69.3	4	203.	94.7	4	257.4	120.
45	40.8	19.	5	95.2	44.4	5	149.5	69.7	5	203.9	95.1	5	258.3	120.4
46	41.7	19.4	6	96.1	44.8	6	150.4	70.2	6	204.8	95.6	6	259.2	120.9
47	42.6	19.9	7	97.	45.2	7	151.4	70.6	7	205.7	95.9	7	260.1	121.3
48	43.5	20.3	8	97.9	45.6	8	152.3	71.	8	206.6	96.4	8	261.	121.7
49	44.4	20.7	9	98.8	46.1	9	153.2	71.4	9	207.5	96.8	9	261.9	122.1
50	45.3	21.1	110	99.7	46.5	170	154.1	71.8	230	208.5	97.2	290	262.8	122.6
51	46.2	21.6	1	100.6	46.9	1	155.	72.3	1	209.4	97.6	1	263.7	123.
52	47.1	22.	2	101.5	47.3	2	155.9	72.7	2	210.3	98.	2	264.6	123.4
53	48.	22.4	3	102.4	47.8	3	156.8	73.1	3	211.2	98.5	3	265.5	123.8
54	48.9	22.8	4	103.3	48.2	4	157.7	73.5	4	212.1	98.9	4	266.4	124.2
55	49.8	23.2	5	104.2	48.6	5	158.6	74.	5	213.	99.3	5	267.4	124.7
56	50.8	23.7	6	105.1	49.	6	159.5	74.4	6	213.9	99.7	6	268.3	125.1
57	51.7	24.1	7	106.	49.4	7	160.4	74.8	7	214.8	100.2	7	269.2	125.5
58	52.6	24.5	8	106.9	49.9	8	161.3	75.2	8	215.7	100.6	8	270.1	125.9
59	53.5	24.9	9	107.9	50.3	9	162.2	75.6	9	216.6	101.	9	271.	126.4
60	54.4	25.4	190	108.8	50.7	180	163.1	76.1	240	217.5	101.4	300	271.9	126.8
dist.	dep.	d.lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.

Distance, Departure and Diff. Latitude.

Course 65°.

Course 360.

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	0.9	0.4	61	54.8	26.7	131	106.8	53.	181	162.7	79.3	241	216.6	105.6
2	1.8	0.9	62	55.7	27.2	2	109.7	53.5	2	163.6	79.8	242	217.5	106.1
3	2.7	1.3	63	56.6	27.6	3	110.6	53.9	3	164.5	80.2	243	218.4	106.5
4	3.6	1.8	64	57.5	28.1	4	111.5	54.4	4	165.4	80.7	244	219.3	107.
5	4.5	2.2	65	58.4	28.5	5	112.3	54.8	5	166.3	81.1	5	220.3	107.4
6	5.4	2.6	66	59.3	28.9	6	113.2	55.2	6	167.2	81.5	6	221.1	107.8
7	6.3	3.1	67	60.2	29.4	7	114.1	55.7	7	168.1	82.	7	222	108.3
8	7.2	3.5	68	61.1	29.8	8	115.	56.1	8	169.	82.4	8	222.9	108.7
9	8.1	3.9	69	62.	30.2	9	115.9	56.5	9	169.9	82.9	9	223.8	109.2
10	9.	4.4	70	62.9	30.7	130	116.8	57.	190	170.8	83.3	250	224.7	109.6
11	9.9	4.8	71	63.9	31.1	1	117.7	57.4	1	171.7	83.7	1	225.6	110.
12	10.8	5.3	72	64.7	31.6	2	118.6	57.9	2	172.6	84.2	2	226.5	110.5
13	11.7	5.7	73	65.6	32.	3	119.5	58.3	3	173.5	84.6	3	227.4	110.9
14	12.6	6.1	74	66.5	32.4	4	120.4	58.7	4	174.4	85.	4	228.3	111.3
15	13.5	6.6	75	67.4	32.9	5	121.3	59.2	5	175.3	85.5	5	229.2	111.8
16	14.4	7.	76	68.3	33.3	6	122.2	59.6	6	176.2	85.9	6	230.1	112.2
17	15.3	7.5	77	69.2	33.8	7	123.1	60.1	7	177.1	86.4	7	231.	112.7
18	16.2	7.9	78	70.1	34.2	8	124.	60.5	8	178.	86.8	8	231.9	113.1
19	17.1	8.3	79	71.	34.6	9	124.9	60.9	9	178.9	87.2	9	232.8	113.5
20	18.	8.8	80	71.9	35.1	140	125.8	61.4	200	179.8	87.7	260	233.7	114.
21	18.9	9.2	81	72.8	35.5	1	126.7	61.8	1	180.7	88.1	1	234.6	114.4
22	19.8	9.6	82	73.7	35.9	2	127.6	62.2	2	181.6	88.6	2	235.5	114.9
23	20.7	10.1	83	74.6	36.4	3	128.5	62.7	3	182.5	89.	3	236.4	115.3
24	21.6	10.5	84	75.5	36.8	4	129.4	63.1	4	183.4	89.4	4	237.3	115.7
25	22.5	11.	85	76.4	37.3	5	130.3	63.6	5	184.3	89.9	5	238.2	116.2
26	23.4	11.4	86	77.3	37.7	6	131.2	64.	6	185.2	90.3	6	239.1	116.6
27	24.3	11.8	87	78.2	38.1	7	132.1	64.4	7	186.1	90.7	7	240.	117.
28	25.2	12.2	88	79.1	38.6	8	133.	64.9	8	187.	91.2	8	240.9	117.5
29	26.1	12.7	89	80.	39.	9	133.9	65.3	9	187.8	91.6	9	241.8	117.9
30	27.	13.2	90	80.9	39.5	150	134.8	65.8	210	188.7	92.1	270	242.7	118.4
31	27.9	13.6	91	81.8	39.9	1	135.7	66.2	1	189.6	92.5	1	243.6	118.8
32	28.8	14.	92	82.7	40.3	2	136.6	66.6	2	190.5	92.9	2	244.5	119.2
33	29.7	14.5	93	83.6	40.8	3	137.5	67.1	3	191.4	93.4	3	245.4	119.7
34	30.6	14.9	94	84.5	41.2	4	138.4	67.5	4	192.3	93.8	4	246.3	120.1
35	31.5	15.3	95	85.4	41.6	5	139.3	67.9	5	193.2	94.2	5	247.2	120.6
36	32.4	15.8	96	86.3	42.1	6	140.2	68.4	6	194.1	94.7	6	248.1	121.
37	33.3	16.2	97	87.2	42.5	7	141.1	68.8	7	195.	95.1	7	249.	121.4
38	34.2	16.7	98	88.1	43.	8	142.	69.3	8	195.9	95.6	8	249.9	121.9
39	35.1	17.1	99	89.	43.4	9	142.9	69.7	9	196.8	96.	9	250.8	122.3
40	36.	17.5	100	89.9	43.8	160	143.8	70.1	220	197.7	96.4	280	251.7	122.7
41	36.9	18.	1	90.8	44.3	1	144.7	70.6	1	198.6	96.9	1	252.6	123.2
42	37.7	18.4	2	91.7	44.7	2	145.6	71.	2	199.5	97.3	2	253.5	123.6
43	38.6	18.8	3	92.6	45.2	3	146.5	71.5	3	200.4	97.8	3	254.4	124.1
44	39.5	19.3	4	93.5	45.6	4	147.4	71.9	4	201.3	98.2	4	255.3	124.5
45	40.4	19.7	5	94.4	46.	5	148.3	72.3	5	202.2	98.6	5	256.2	124.9
46	41.3	20.2	6	95.3	46.5	6	149.2	72.8	6	203.1	99.1	6	257.1	125.4
47	42.2	20.6	7	96.2	46.9	7	150.1	73.2	7	204.	99.5	7	258.	125.8
48	43.1	21.	8	97.1	47.3	8	151.	73.6	8	204.9	99.9	8	258.9	126.3
49	44.	21.5	9	98.	47.8	9	151.9	74.1	9	205.8	100.4	9	259.8	126.7
50	44.9	21.9	110	98.9	48.2	170	152.8	74.5	230	206.7	100.8	290	260.7	127.1
51	45.8	22.4	1	99.8	48.7	1	153.7	75.	1	207.6	101.3	1	261.6	127.6
52	46.7	22.8	2	100.7	49.1	2	154.6	75.4	2	208.5	101.7	2	262.5	128.
53	47.6	23.2	3	101.6	49.5	3	155.5	75.8	3	209.4	102.1	3	263.4	128.4
54	48.5	23.7	4	102.5	50.	4	156.4	76.3	4	210.3	102.6	4	264.3	128.9
55	49.4	24.1	5	103.4	50.4	5	157.3	76.7	5	211.2	103.	5	265.1	129.3
56	50.3	24.5	6	104.3	50.9	6	158.2	77.2	6	212.1	103.5	6	266.	129.8
57	51.2	25.	7	105.2	51.3	7	159.1	77.6	7	213.	103.9	7	266.9	130.2
58	52.1	25.4	8	106.1	51.7	8	160.	78.	8	213.9	104.3	8	267.8	130.6
59	53.	25.9	9	107.	52.2	9	160.9	78.5	9	214.8	104.8	9	268.7	131.1
60	53.9	26.3	190	107.9	52.6	180	161.8	78.9	240	215.7	105.2	300	269.6	131.5
dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.

Distance, Departure and Diff. Latitude.

Course 640.

TABLE V.

Course 37°.

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	0.9	0.5	61	54.4	37.7	121	107.8	54.9	181	161.3	82.2	241	314.7	109.4
2	1.8	0.9	62	53.2	38.1	2	108.7	55.4	2	162.2	82.6	2	315.6	109.9
3	2.7	1.4	63	56.1	38.6	3	109.6	55.8	3	163.1	83.1	3	316.5	110.3
4	3.6	1.8	64	57.	39.1	4	110.5	56.3	4	163.9	83.5	4	317.4	110.8
5	4.5	2.3	65	57.9	39.5	5	111.4	56.7	5	164.8	84.	5	318.3	111.9
6	5.3	2.7	66	58.8	30.	6	112.3	57.2	6	165.7	84.4	6	319.2	111.7
7	6.2	3.2	67	59.7	30.4	7	113.2	57.7	7	166.6	84.9	7	320.1	112.1
8	7.1	3.6	68	60.6	30.9	8	114.	58.1	8	167.5	85.4	8	321.	112.6
9	8.	4.1	69	61.5	31.3	9	114.9	58.6	9	168.4	85.8	9	321.9	113.
10	8.9	4.5	70	62.4	31.8	130	115.8	59.	190	169.3	86.3	250	322.8	113.5
11	9.8	5.	71	63.3	32.2	1	116.7	59.5	1	170.2	86.7	1	323.6	114.
12	10.7	5.4	72	64.2	32.7	2	117.6	59.9	2	171.1	87.2	2	324.5	114.4
13	11.6	5.9	73	65.	33.1	3	118.5	60.4	3	172.	87.6	3	325.4	114.9
14	12.5	6.4	74	65.9	33.6	4	119.4	60.8	4	172.9	88.1	4	326.3	115.3
15	13.4	6.8	75	66.8	34.	5	120.3	61.3	5	173.7	88.5	5	327.2	115.8
16	14.3	7.3	76	67.7	34.5	6	121.2	61.7	6	174.6	89.	6	328.1	116.9
17	15.1	7.7	77	68.6	35.	7	122.1	62.2	7	175.5	89.4	7	329.	117.7
18	16.	8.2	78	69.5	35.4	8	123.	62.7	8	176.4	89.8	8	329.9	118.1
19	16.9	8.6	79	70.4	35.9	9	123.8	63.1	9	177.3	90.2	9	330.8	117.6
20	17.8	9.1	80	71.3	36.3	140	124.7	63.6	200	178.2	90.8	260	331.7	118.
21	18.7	9.5	81	72.2	36.8	1	125.6	64.	1	179.1	91.2	1	332.6	118.5
22	19.6	10.	82	73.1	37.2	2	126.5	64.5	2	180.	91.7	2	333.4	118.9
23	20.5	10.4	83	74.	37.7	3	127.4	64.9	3	180.9	92.2	3	334.3	119.4
24	21.4	10.9	84	74.8	38.1	4	128.3	65.4	4	181.8	92.6	4	335.2	119.9
25	22.3	11.3	85	75.7	38.6	5	129.2	65.8	5	182.7	93.1	5	336.1	120.3
26	23.2	11.8	86	76.6	39.	6	130.1	66.3	6	183.5	93.5	6	337.	120.8
27	24.1	12.3	87	77.5	39.5	7	131.	66.7	7	184.4	94.	7	337.9	121.2
28	24.9	12.7	88	78.4	40.	8	131.9	67.2	8	185.3	94.4	8	338.8	121.7
29	25.8	13.2	89	79.3	40.4	9	132.8	67.6	9	186.2	94.9	9	339.7	122.1
30	26.7	13.6	90	80.2	40.9	150	133.7	68.1	210	187.1	95.3	270	340.6	122.6
31	27.6	14.1	91	81.1	41.3	1	134.5	68.6	1	188.	95.8	1	341.5	123.
32	28.5	14.5	92	82.	41.8	2	135.4	69.	2	188.9	96.2	2	342.4	123.5
33	29.4	15.	93	82.9	42.2	3	136.3	69.5	3	189.8	96.7	3	343.3	123.9
34	30.3	15.4	94	83.8	42.7	4	137.2	69.9	4	190.7	97.2	4	344.1	124.4
35	31.2	15.9	95	84.6	43.1	5	138.1	70.4	5	191.6	97.6	5	345.	124.8
36	32.1	16.3	96	85.5	43.6	6	139.	70.8	6	192.5	98.1	6	345.9	125.3
37	33.	16.8	97	86.4	44.	7	139.9	71.3	7	193.3	98.5	7	346.8	125.8
38	33.9	17.3	98	87.3	44.5	8	140.8	71.7	8	194.2	99.	8	347.7	126.2
39	34.7	17.7	99	88.2	44.9	9	141.7	72.2	9	195.1	99.4	9	348.6	126.9
40	35.6	18.2	100	89.1	45.4	160	142.6	72.6	220	196.	99.9	280	349.5	127.1
41	36.5	18.6	1	90.	45.9	1	143.5	73.1	1	196.9	100.3	1	350.4	127.6
42	37.4	19.1	2	90.9	46.3	2	144.3	73.5	2	197.8	100.8	2	351.3	128.
43	38.3	19.5	3	91.8	46.8	3	145.2	74.	3	198.7	101.2	3	352.2	128.5
44	39.2	20.	4	92.7	47.2	4	146.1	74.5	4	199.6	101.7	4	353.	128.9
45	40.1	20.4	5	93.6	47.7	5	147.	74.9	5	200.5	102.1	5	353.9	129.4
46	41.	20.9	6	94.4	48.1	6	147.9	75.4	6	201.4	102.6	6	354.8	129.8
47	41.9	21.3	7	95.3	48.6	7	148.8	75.8	7	202.3	103.1	7	355.7	130.3
48	42.8	21.8	8	96.2	49.	8	149.7	76.3	8	203.1	103.5	8	356.6	130.7
49	43.7	22.2	9	97.1	49.5	9	150.6	76.7	9	204.	104.	9	357.5	131.2
50	44.6	22.7	110	98.	49.9	170	151.5	77.2	230	204.9	104.4	290	358.4	131.7
51	45.4	23.2	1	98.9	50.4	1	152.4	77.6	1	205.8	104.9	1	359.3	132.1
52	46.3	23.6	2	99.8	50.8	2	153.3	78.1	2	206.7	105.3	2	360.2	132.6
53	47.2	24.1	3	100.7	51.3	3	154.1	78.5	3	207.6	105.8	3	361.1	133.
54	48.1	24.5	4	101.6	51.8	4	155.	79.	4	208.5	106.2	4	362.	133.5
55	49.	25.	5	102.5	52.2	5	155.9	79.4	5	209.4	106.7	5	362.8	133.9
56	49.9	25.4	6	103.4	52.7	6	156.8	79.9	6	210.3	107.1	6	363.7	134.4
57	50.8	25.9	7	104.3	53.1	7	157.7	80.4	7	211.2	107.6	7	364.6	134.8
58	51.7	26.3	8	105.1	53.6	8	158.6	80.8	8	212.1	108.	8	365.5	135.3
59	52.6	26.8	9	106.	54.	9	159.5	81.3	9	213.	108.5	9	366.4	135.7
60	53.5	27.2	120	106.9	54.5	180	160.4	81.7	240	213.8	109.	300	367.3	136.3
dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.

Distance, Departure and Diff. Latitude.

Course 63°.

Course 90°.

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	0.0	0.5	61	52.9	23.6	121	106.8	56.8	181	159.8	85.	241	212.8	112.1
2	1.8	0.9	62	54.7	25.1	2	107.7	57.3	2	160.7	85.4	2	213.7	112.6
3	2.6	1.4	63	55.8	25.6	3	108.6	57.7	3	161.8	85.9	3	214.6	113.1
4	3.5	1.9	64	56.5	26.	4	109.5	58.2	4	162.5	86.4	4	215.4	113.6
5	4.4	2.3	65	57.4	26.5	5	110.4	58.7	5	163.3	86.9	5	216.3	114.
6	5.3	2.8	66	58.3	27.	6	111.3	59.2	6	164.2	87.3	6	217.2	114.5
7	6.2	3.3	67	59.2	27.5	7	112.1	59.6	7	165.1	87.8	7	218.1	115.
8	7.1	3.8	68	60.	27.9	8	113.	60.1	8	166.	88.3	8	219.	115.4
9	7.9	4.3	69	60.9	28.4	9	113.9	60.6	9	166.9	88.7	9	219.9	115.9
10	8.8	4.7	70	61.8	28.9	120	114.8	61.	190	167.8	89.2	250	220.7	117.0
11	9.7	5.2	71	62.7	29.3	1	115.7	61.5	1	168.6	89.7	1	221.6	117.5
12	10.6	5.6	72	63.6	29.8	2	116.5	62.	2	169.5	90.1	2	222.5	118.0
13	11.5	6.1	73	64.5	30.3	3	117.4	62.4	3	170.4	90.6	3	223.4	118.5
14	12.4	6.6	74	65.3	30.7	4	118.3	62.9	4	171.3	91.1	4	224.3	119.0
15	13.3	7.	75	66.2	31.2	5	119.2	63.4	5	172.2	91.5	5	225.2	119.5
16	14.1	7.5	76	67.1	31.7	6	120.1	63.8	6	173.1	92.	6	226.	120.
17	15.	8.	77	68.	32.1	7	121.	64.3	7	174.2	92.5	7	226.9	120.7
18	15.9	8.5	78	68.9	32.6	8	121.8	64.8	8	174.8	93.	8	227.8	121.1
19	16.8	8.9	79	69.8	33.1	9	122.7	65.3	9	175.7	93.4	9	228.7	121.6
20	17.7	9.4	80	70.6	33.6	140	123.6	65.7	200	176.6	93.9	260	229.6	122.1
21	18.5	9.9	81	71.5	34.	1	124.5	66.2	1	177.5	94.4	1	230.4	122.6
22	19.4	10.3	82	72.4	34.5	2	125.4	66.7	2	178.4	94.8	2	231.3	123.
23	20.3	10.8	83	73.3	35.	3	126.3	67.1	3	179.2	95.3	3	232.2	123.5
24	21.2	11.3	84	74.2	35.4	4	127.1	67.6	4	180.1	95.8	4	233.1	124.
25	22.1	11.7	85	75.1	35.9	5	128.	68.1	5	181.	96.2	5	234.	124.4
26	23.	12.2	86	75.9	36.4	6	128.9	68.5	6	181.9	96.7	6	234.9	124.9
27	23.8	12.7	87	76.8	36.9	7	129.8	69.	7	182.8	97.2	7	235.7	125.3
28	24.7	13.1	88	77.7	37.4	8	130.7	69.5	8	183.7	97.7	8	236.6	125.8
29	25.6	13.6	89	78.6	37.8	9	131.6	70.	9	184.5	98.1	9	237.5	126.3
30	26.5	14.1	90	79.5	38.3	150	132.4	70.4	210	185.4	98.6	270	238.4	126.8
31	27.4	14.6	91	80.3	38.7	1	133.3	70.9	1	186.3	99.1	1	239.3	127.3
32	28.3	15.	92	81.2	39.2	2	134.2	71.4	2	187.2	99.5	2	240.2	127.8
33	29.1	15.5	93	82.1	39.7	3	135.1	71.8	3	188.1	100.	3	241.	128.3
34	30.	16.	94	83.	40.1	4	136.	72.3	4	189.	100.5	4	241.9	128.8
35	30.9	16.4	95	83.9	40.6	5	136.9	72.8	5	189.8	100.9	5	242.8	129.1
36	31.8	16.9	96	84.8	41.1	6	137.7	73.2	6	190.7	101.4	6	243.7	129.6
37	32.7	17.4	97	85.6	41.5	7	138.6	73.7	7	191.6	101.9	7	244.6	130.
38	33.6	17.8	98	86.5	42.	8	139.5	74.2	8	192.5	102.3	8	245.5	130.5
39	34.4	18.3	99	87.4	42.5	9	140.4	74.6	9	193.4	102.8	9	246.3	131.
40	35.3	18.8	100	88.3	42.9	160	141.3	75.1	220	194.3	103.3	280	247.2	131.5
41	36.2	19.3	1	89.2	47.4	1	142.2	75.6	1	195.1	103.8	1	248.1	132.
42	37.1	19.7	2	90.1	47.9	2	143.	76.1	2	196.	104.3	2	249.	132.4
43	38.	20.3	3	90.9	48.4	3	143.9	76.5	3	196.9	104.7	3	249.9	132.9
44	38.8	20.7	4	91.8	48.8	4	144.8	77.	4	197.8	105.2	4	250.8	133.3
45	39.7	21.1	5	92.7	49.3	5	145.7	77.5	5	198.7	105.6	5	251.6	133.8
46	40.6	21.6	6	93.6	49.8	6	146.6	77.9	6	199.5	106.1	6	252.5	134.2
47	41.5	22.1	7	94.5	50.2	7	147.5	78.4	7	200.4	106.6	7	253.4	134.7
48	42.4	22.5	8	95.4	50.7	8	148.3	78.9	8	201.3	107.	8	254.3	135.2
49	43.3	23.	9	96.3	51.3	9	149.2	79.3	9	202.2	107.5	9	255.2	135.7
50	44.1	23.5	110	97.1	51.6	170	150.1	79.8	230	203.1	108.	290	256.1	136.1
51	45.	24.1	1	98.	52.1	1	151.	80.3	1	204.	108.4	1	256.9	136.6
52	45.9	24.4	2	98.9	52.6	2	151.9	80.7	2	204.8	108.9	2	257.8	137.1
53	46.8	24.9	3	99.8	53.1	3	152.7	81.2	3	205.7	109.3	3	258.7	137.6
54	47.7	25.4	4	100.7	53.5	4	153.6	81.7	4	206.6	109.8	4	259.6	138.
55	48.6	25.8	5	101.5	54.	5	154.5	82.2	5	207.5	110.3	5	260.5	138.5
56	49.4	26.3	6	102.4	54.5	6	155.4	82.6	6	208.4	110.8	6	261.4	139.
57	50.3	26.8	7	103.3	54.9	7	156.3	83.1	7	209.3	111.3	7	262.3	139.4
58	51.2	27.3	8	104.2	55.4	8	157.2	83.6	8	210.1	111.7	8	263.1	139.9
59	52.1	27.7	9	105.1	55.9	9	158.	84.	9	211.	112.2	9	264.	140.4
60	53.	28.3	120	106.	56.3	180	158.9	84.5	240	211.9	112.7	300	264.9	140.8

Distance, Departure and Diff. Latitude.

Course 90°.

Course 30°.

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	0.9	0.5	61	33.8	30.5	121	104.8	60.5	181	156.8	90.5	241	208.7	120.5
2	1.7	1.	62	33.7	31.	2	105.7	61.	2	157.6	91.	2	209.6	121.
3	2.6	1.5	63	34.6	31.5	3	106.5	61.5	3	158.5	91.5	3	210.4	121.5
4	3.5	2.	64	35.4	32.	4	107.4	62.	4	159.3	92.	4	211.3	122.
5	4.3	2.5	65	36.3	32.5	5	108.3	62.5	5	160.3	92.5	5	212.2	122.5
6	5.2	3.	66	37.2	33.	6	109.1	63.	6	161.1	93.	6	213.	123.
7	6.1	3.5	67	38.	33.5	7	110.	63.5	7	161.9	93.5	7	213.9	123.5
8	6.9	4.	68	38.9	34.	8	110.9	64.	8	162.8	94.	8	214.8	124.
9	7.8	4.5	69	39.8	34.5	9	111.7	64.5	9	163.7	94.5	9	215.6	124.5
10	8.7	5.	70	40.6	35.	130	112.6	65.	190	164.5	95.	250	216.5	125.
11	9.5	5.5	71	41.5	35.5	1	113.4	65.5	1	165.4	95.5	1	217.4	125.5
12	10.4	6.	72	42.4	36.	2	114.3	66.	2	166.3	96.	2	218.2	126.
13	11.3	6.5	73	43.3	36.5	3	115.2	66.5	3	167.1	96.5	3	219.1	126.5
14	12.1	7.	74	44.1	37.	4	116.	67.	4	168.	97.	4	220.	127.
15	13.	7.5	75	45.	37.5	5	116.9	67.5	5	168.9	97.5	5	220.8	127.5
16	13.9	8.	76	45.8	38.	6	117.8	68.	6	169.7	98.	6	221.7	128.
17	14.7	8.5	77	46.7	38.5	7	118.6	68.5	7	170.6	98.5	7	222.6	128.5
18	15.6	9.	78	47.5	39.	8	119.5	69.	8	171.5	99.	8	223.4	129.
19	16.5	9.5	79	48.4	39.5	9	120.4	69.5	9	172.3	99.5	9	224.3	129.5
20	17.3	10.	80	49.3	40.	140	121.2	70.	200	173.2	100.	260	225.2	130.
21	18.2	10.5	81	50.1	40.5	1	122.1	70.5	1	174.1	100.5	1	226.	130.5
22	19.1	11.	82	51.	41.	2	123.	71.	2	174.9	101.	2	226.9	131.
23	19.9	11.5	83	51.9	41.5	3	123.8	71.5	3	175.8	101.5	3	227.8	131.5
24	20.8	12.	84	52.7	42.	4	124.7	72.	4	176.7	102.	4	228.6	132.
25	21.7	12.5	85	53.6	42.5	5	125.6	72.5	5	177.5	102.5	5	229.5	132.5
26	22.5	13.	86	54.5	43.	6	126.4	73.	6	178.4	103.	6	230.4	133.
27	23.4	13.5	87	55.3	43.5	7	127.3	73.5	7	179.3	103.5	7	231.2	133.5
28	24.2	14.	88	56.2	44.	8	128.2	74.	8	180.1	104.	8	232.1	134.
29	25.1	14.5	89	57.1	44.5	9	129.	74.5	9	181.	104.5	9	233.	134.5
30	26.	15.	90	57.9	45.	150	129.9	75.	210	181.9	105.	270	233.8	135.
31	26.8	15.5	91	58.8	45.5	1	130.8	75.5	1	182.7	105.5	1	234.7	135.5
32	27.7	16.	92	59.7	46.	2	131.6	76.	2	183.6	106.	2	235.6	136.
33	28.6	16.5	93	60.5	46.5	3	132.5	76.5	3	184.5	106.5	3	236.4	136.5
34	29.4	17.	94	61.4	47.	4	133.4	77.	4	185.3	107.	4	237.3	137.
35	30.3	17.5	95	62.3	47.5	5	134.2	77.5	5	186.2	107.5	5	238.2	137.5
36	31.2	18.	96	63.1	48.	6	135.1	78.	6	187.1	108.	6	239.	138.
37	32.	18.5	97	64.	48.5	7	136	78.5	7	187.9	108.5	7	239.9	138.5
38	32.9	19.	98	64.9	49.	8	136.8	79.	8	188.8	109.	8	240.8	139.
39	33.8	19.5	99	65.7	49.5	9	137.7	79.5	9	189.7	109.5	9	241.6	139.5
40	34.6	20.	100	66.6	50.	160	138.6	80.	220	190.5	110.	280	242.5	140.
41	35.5	20.5	1	67.5	50.5	1	139.4	80.5	1	191.4	110.5	1	243.4	140.5
42	36.4	21.	2	68.3	51.	2	140.3	81.	2	192.3	111.	2	244.2	141.
43	37.3	21.5	3	69.2	51.5	3	141.2	81.5	3	193.1	111.5	3	245.1	141.5
44	38.1	22.	4	70.1	52.	4	142.	82.	4	194.	112.	4	246.	142.
45	39.	22.5	5	70.9	52.5	5	142.9	82.5	5	194.9	112.5	5	246.8	142.5
46	39.8	23.	6	71.8	53.	6	143.8	83.	6	195.7	113.	6	247.7	143.
47	40.7	23.5	7	72.7	53.5	7	144.6	83.5	7	196.6	113.5	7	248.5	143.5
48	41.6	24.	8	73.5	54.	8	145.5	84.	8	197.5	114.	8	249.4	144.
49	42.4	24.5	9	74.4	54.5	9	146.4	84.5	9	198.3	114.5	9	250.3	144.5
50	43.3	25.	110	75.3	55.	170	147.2	85.	230	199.2	115.	290	251.1	145.
51	44.2	25.5	1	76.1	55.5	1	148.1	85.5	1	200.1	115.5	1	252.	145.5
52	45.	26.	2	77.	56.	2	149.	86.	2	200.9	116.	2	252.9	146.
53	45.9	26.5	3	77.9	56.5	3	149.8	86.5	3	201.8	116.5	3	253.7	146.5
54	46.8	27.	4	78.7	57.	4	150.7	87.	4	202.6	117.	4	254.6	147.
55	47.6	27.5	5	79.6	57.5	5	151.6	87.5	5	203.5	117.5	5	255.5	147.5
56	48.5	28.	6	80.5	58.	6	152.4	88.	6	204.4	118.	6	256.3	148.
57	49.4	28.5	7	81.3	58.5	7	153.3	88.5	7	205.3	118.5	7	257.2	148.5
58	50.3	29.	8	82.2	59.	8	154.2	89.	8	206.1	119.	8	258.1	149.
59	51.1	29.5	9	83.1	59.5	9	155.	89.5	9	207.	119.5	9	259.0	149.5
60	52.	30.	120	84.0	60.	180	155.9	90.	240	207.8	120.	300	259.8	150.
dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.

Distance, Departure and Diff. Latitude.

Course 60°.

Course 31°.

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	0.9	0.5	61	52.3	31.4	121	103.7	62.3	181	153.1	93.3	241	206.6	124.1
2	1.7	1.	62	53.1	31.9	2	104.6	62.8	2	156.	93.7	2	207.4	124.6
3	2.6	1.5	63	54.	32.4	3	105.4	63.3	3	156.9	94.3	3	208.3	125.2
4	3.4	2.1	64	54.9	33.	4	106.3	63.9	4	157.7	94.8	4	209.1	125.7
5	4.3	2.6	65	55.7	33.5	5	107.1	64.4	5	158.6	95.3	5	210.	126.2
6	5.1	3.1	66	56.6	34.	6	108.	64.9	6	159.4	95.8	6	210.9	126.7
7	6.	3.6	67	57.4	34.5	7	108.9	65.4	7	160.3	96.3	7	211.7	127.2
8	6.9	4.1	68	58.3	35.	8	109.7	65.9	8	161.1	96.8	8	212.6	127.7
9	7.7	4.6	69	59.1	35.5	9	110.6	66.4	9	162.	97.3	9	213.4	128.2
10	8.6	5.3	70	60.	36.1	130	111.4	67.	190	162.9	97.9	250	214.3	128.8
11	9.4	5.7	71	60.9	36.6	1	112.3	67.5	1	163.7	98.4	1	215.1	129.3
12	10.3	6.2	72	61.7	37.1	2	113.1	68.	2	164.6	98.9	2	216.	129.8
13	11.1	6.7	73	62.6	37.6	3	114.	68.5	3	165.4	99.4	3	216.9	130.3
14	12.	7.2	74	63.4	38.1	4	114.9	69.	4	166.3	99.9	4	217.7	130.8
15	12.9	7.7	75	64.3	38.6	5	115.7	69.5	5	167.1	100.4	5	218.6	131.3
16	13.7	8.2	76	65.1	39.1	6	116.6	70.	6	168.	100.9	6	219.4	131.8
17	14.6	8.8	77	66.	39.7	7	117.4	70.6	7	168.9	101.5	7	220.3	132.4
18	15.4	9.3	78	66.9	40.2	8	118.3	71.1	8	169.7	102.	8	221.1	132.9
19	16.3	9.8	79	67.7	40.7	9	119.1	71.6	9	170.6	102.5	9	222.	133.4
20	17.1	10.3	80	68.6	41.2	140	120.	72.1	200	171.4	103.	260	222.9	133.9
21	18.	10.8	81	69.4	41.7	1	120.9	72.6	1	172.3	103.5	1	223.7	134.4
22	18.9	11.3	82	70.3	42.2	2	121.7	73.1	2	173.1	104.	2	224.6	134.9
23	19.7	11.8	83	71.1	42.7	3	122.6	73.7	3	174.	104.6	3	225.4	135.5
24	20.6	12.4	84	72.	43.3	4	123.4	74.2	4	174.9	105.1	4	226.3	136.
25	21.4	12.9	85	72.9	43.8	5	124.3	74.7	5	175.7	105.6	5	227.1	136.5
26	22.3	13.4	86	73.7	44.3	6	125.1	75.2	6	176.6	106.1	6	228.	137.
27	23.1	13.9	87	74.6	44.8	7	126.	75.7	7	177.4	106.6	7	228.9	137.5
28	24.	14.4	88	75.4	45.3	8	126.9	76.2	8	178.3	107.1	8	229.7	138.
29	24.9	14.9	89	76.3	45.8	9	127.7	76.7	9	179.1	107.6	9	230.6	138.5
30	25.7	15.5	90	77.1	46.4	150	128.6	77.3	210	180.	108.2	270	231.4	139.1
31	26.6	16.	91	78.	46.9	1	129.4	77.8	1	180.9	108.7	1	232.3	139.6
32	27.4	16.5	92	78.9	47.4	2	130.3	78.3	2	181.7	109.2	2	233.1	140.1
33	28.3	17.	93	79.7	47.9	3	131.1	78.8	3	182.6	109.7	3	234.	140.6
34	29.1	17.5	94	80.6	48.4	4	132.	79.3	4	183.4	110.2	4	234.9	141.1
35	30.	18.	95	81.4	48.9	5	132.9	79.8	5	184.3	110.7	5	235.7	141.6
36	30.9	18.5	96	82.3	49.4	6	133.7	80.3	6	185.1	111.2	6	236.6	142.2
37	31.7	19.1	97	83.1	50.	7	134.6	80.9	7	186.	111.8	7	237.4	142.7
38	32.6	19.6	98	84.	50.5	8	135.4	81.4	8	186.9	112.3	8	238.3	143.2
39	33.4	20.1	99	84.9	51.	9	136.3	81.9	9	187.7	112.8	9	239.1	143.7
40	34.3	20.6	100	85.7	51.5	160	137.1	82.4	220	188.6	113.3	280	240.	144.2
41	35.1	21.1	1	86.6	52.	1	138.	82.9	1	189.4	113.8	1	240.9	144.7
42	36.	21.6	2	87.4	52.5	2	138.9	83.4	2	190.3	114.3	2	241.7	145.2
43	36.9	22.1	3	88.3	53.	3	139.7	84.	3	191.1	114.9	3	242.6	145.8
44	37.7	22.7	4	89.1	53.6	4	140.6	84.5	4	192.	115.4	4	243.4	146.3
45	38.6	23.2	5	90.	54.1	5	141.4	85.	5	192.9	115.9	5	244.3	146.8
46	39.4	23.7	6	90.9	54.6	6	142.3	85.5	6	193.7	116.4	6	245.1	147.3
47	40.3	24.2	7	91.7	55.1	7	143.1	86.	7	194.6	116.9	7	246.	147.8
48	41.1	24.7	8	92.6	55.6	8	144.	86.5	8	195.4	117.4	8	246.9	148.3
49	42.	25.2	9	93.4	56.1	9	144.9	87.	9	196.3	117.9	9	247.7	148.8
50	42.9	25.8	110	94.3	56.7	170	145.7	87.6	230	197.1	118.5	290	248.6	149.4
51	43.7	26.3	1	95.1	57.2	1	146.6	88.1	1	198.	119.	1	249.4	149.9
52	44.6	26.8	2	96.	57.7	2	147.4	88.6	2	198.9	119.5	2	250.3	150.4
53	45.4	27.3	3	96.9	58.2	3	148.3	89.1	3	199.7	120.	3	251.2	150.9
54	46.3	27.8	4	97.7	58.7	4	149.1	89.6	4	200.6	120.5	4	252.	151.4
55	47.1	28.3	5	98.6	59.2	5	150.	90.1	5	201.4	121.	5	252.9	151.9
56	48.	28.8	6	99.4	59.7	6	150.9	90.6	6	202.3	121.5	6	253.7	152.5
57	48.9	29.4	7	100.3	60.3	7	151.7	91.2	7	203.1	122.1	7	254.6	153.
58	49.7	29.9	8	101.1	60.8	8	152.6	91.7	8	204.	122.6	8	255.4	153.5
59	50.6	30.4	9	102.	61.3	9	153.4	92.2	9	204.9	123.1	9	256.3	154.
60	51.4	30.9	120	102.9	61.8	180	154.3	92.7	240	205.7	123.6	300	257.1	154.5
dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.

Distance, Departure and Diff. Latitude.

Course 59°.

Course 33°.

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	0.8	0.5	61	51.9	33.9	121	101.5	65.9	191	151.8	98.8	241	202.1	131.3
2	1.7	1.1	62	52.	33.8	2	102.3	66.4	2	152.6	99.1	2	203.	131.8
3	2.5	1.6	63	52.8	34.3	3	103.2	67.	3	153.5	99.7	3	203.8	132.3
4	3.4	2.2	64	53.7	34.9	4	104.	67.5	4	154.3	100.2	4	204.6	132.9
5	4.2	2.7	65	54.5	35.4	5	104.8	68.1	5	155.2	100.8	5	205.5	133.4
6	5.	3.3	66	55.4	35.9	6	105.7	68.6	6	156.	101.3	6	206.3	134.
7	5.9	3.8	67	56.2	36.5	7	106.5	69.2	7	156.8	101.8	7	207.4	134.5
8	6.7	4.4	68	57.	37.	8	107.3	69.7	8	157.7	102.4	8	208.	135.1
9	7.5	4.9	69	57.9	37.6	9	108.2	70.3	9	158.5	102.9	9	208.8	135.6
10	8.4	5.4	70	58.7	38.1	130	109.	70.8	190	159.3	103.5	250	209.7	136.2
11	9.2	6.	71	59.5	38.7	1	109.9	71.3	1	160.2	104.	1	210.5	136.7
12	10.1	6.5	72	60.4	39.2	2	110.7	71.9	2	161.	104.6	2	211.3	137.2
13	10.9	7.1	73	61.3	39.8	3	111.5	72.4	3	161.9	105.1	3	212.2	137.8
14	11.7	7.6	74	62.1	40.3	4	112.4	73.	4	162.7	105.7	4	213.	138.3
15	12.6	8.2	75	62.9	40.8	5	113.2	73.5	5	163.5	106.2	5	213.9	138.9
16	13.4	8.7	76	63.7	41.4	6	114.1	74.1	6	164.4	106.7	6	214.7	139.4
17	14.3	9.3	77	64.6	41.9	7	114.9	74.6	7	165.3	107.3	7	215.5	140.
18	15.1	9.8	78	65.4	42.5	8	115.7	75.2	8	166.1	107.8	8	216.4	140.5
19	15.9	10.3	79	66.3	43.	9	116.6	75.7	9	166.9	108.4	9	217.2	141.1
20	16.8	10.9	80	67.1	43.6	140	117.4	76.2	200	167.7	108.9	260	218.1	141.6
21	17.6	11.4	81	67.9	44.1	1	118.3	76.8	1	168.6	109.5	1	218.9	142.2
22	18.5	12.	82	68.8	44.7	2	119.1	77.3	2	169.4	110.	2	219.7	142.7
23	19.3	12.5	83	69.6	45.2	3	119.9	77.9	3	170.3	110.6	3	220.6	143.2
24	20.1	13.1	84	70.4	45.7	4	120.8	78.4	4	171.1	111.1	4	221.4	143.8
25	21.	13.6	85	71.3	46.3	5	121.6	78.	5	171.9	111.7	5	222.2	144.3
26	21.8	14.2	86	72.1	46.8	6	122.4	79.5	6	172.8	112.2	6	223.1	144.9
27	22.6	14.7	87	73.	47.4	7	123.3	80.1	7	173.6	112.7	7	223.9	145.4
28	23.5	15.2	88	73.8	47.9	8	124.1	80.6	8	174.4	113.3	8	224.8	146.
29	24.3	15.8	89	74.6	48.5	9	125.	81.2	9	175.3	113.8	9	225.6	146.5
30	25.2	16.3	90	75.5	49.	150	125.8	81.7	210	176.1	114.4	270	226.4	147.1
31	26.	16.9	91	76.3	49.6	1	126.6	82.2	1	177.	114.9	1	227.3	147.6
32	26.8	17.4	92	77.2	50.1	2	127.5	82.8	2	177.8	115.5	2	228.1	148.1
33	27.7	18.	93	78.	50.7	3	128.3	83.3	3	178.6	116.	3	229.	148.7
34	28.5	18.5	94	78.8	51.2	4	129.2	83.9	4	179.5	116.6	4	229.8	149.2
35	29.4	19.1	95	79.7	51.7	5	130.	84.4	5	180.3	117.1	5	230.6	149.8
36	30.2	19.6	96	80.5	52.3	6	130.8	85.	6	181.3	117.6	6	231.5	150.3
37	31.	20.2	97	81.4	52.8	7	131.7	85.5	7	182.	118.2	7	232.3	150.9
38	31.9	20.7	98	82.2	53.4	8	132.5	86.1	8	182.8	118.7	8	233.2	151.4
39	32.7	21.2	99	83.	53.9	9	133.3	86.6	9	183.7	119.3	9	234.	152.
40	33.5	21.8	100	83.9	54.5	160	134.2	87.1	220	184.5	119.8	280	234.8	152.5
41	34.4	22.3	1	84.7	55.	1	135.	87.7	1	185.3	120.4	1	235.7	153.
42	35.2	22.9	2	85.5	55.6	2	135.9	88.2	2	186.2	120.9	2	236.5	153.6
43	36.1	23.4	3	86.4	56.1	3	136.7	88.8	3	187.	121.5	3	237.3	154.1
44	36.9	24.	4	87.2	56.6	4	137.5	89.3	4	187.9	122.	4	238.2	154.7
45	37.7	24.5	5	88.1	57.2	5	138.4	89.9	5	188.7	122.5	5	239.	155.2
46	38.6	25.1	6	88.9	57.7	6	139.3	90.4	6	189.5	123.1	6	239.9	155.8
47	39.4	25.6	7	89.7	58.3	7	140.1	91.	7	190.4	123.6	7	240.7	156.3
48	40.3	26.1	8	90.6	58.8	8	140.9	91.5	8	191.2	124.2	8	241.5	156.9
49	41.1	26.7	9	91.4	59.4	9	141.7	92.	9	192.1	124.7	9	242.4	157.4
50	41.9	27.2	110	92.3	59.9	170	142.6	92.6	230	192.9	125.2	290	243.2	157.9
51	42.8	27.8	1	93.1	60.5	1	143.4	93.1	1	193.7	125.8	1	244.1	158.5
52	43.6	28.3	2	93.9	61.	2	144.3	93.7	2	194.6	126.4	2	244.9	159.
53	44.4	28.9	3	94.8	61.5	3	145.1	94.2	3	195.4	126.9	3	245.7	159.6
54	45.3	29.4	4	95.6	62.1	4	145.9	94.8	4	196.2	127.4	4	246.6	160.1
55	46.1	30.	5	96.4	62.6	5	146.8	95.3	5	197.1	128.	5	247.4	160.7
56	47.	30.5	6	97.3	63.2	6	147.6	95.9	6	197.9	128.5	6	248.2	161.2
57	47.8	31.	7	98.1	63.7	7	148.4	96.4	7	198.8	129.1	7	249.1	161.8
58	48.6	31.6	8	98.	64.3	8	149.3	96.9	8	199.6	129.6	8	249.9	162.3
59	49.5	32.1	9	99.8	64.8	9	150.1	97.5	9	200.4	130.2	9	250.8	162.8
60	50.3	32.7	190	100.6	65.4	180	151.	98.	240	201.3	130.7	300	251.6	163.4
dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.

Distance, Departure and Diff. Latitude.

Course 57°.

Course 34°.

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	0.8	0.6	61	50.6	34.1	181	100.3	67.7	181	150.1	101.9	241	199.8	134.6
2	1.7	1.1	62	51.4	34.7	2	101.1	68.2	2	150.9	101.8	2	200.6	135.3
3	2.5	1.7	63	52.2	35.2	3	102	68.8	3	151.7	102.3	3	201.5	135.9
4	3.3	2.2	64	53.1	35.8	4	102.8	69.3	4	152.5	102.9	4	202.3	136.4
5	4.1	2.8	65	53.9	36.3	5	103.6	69.9	5	153.4	103.5	5	203.1	137.
6	5.	3.4	66	54.7	36.9	6	104.5	70.5	6	154.3	104.	6	203.9	137.6
7	5.8	3.9	67	55.5	37.5	7	105.3	71.	7	155.	104.6	7	204.8	138.1
8	6.6	4.5	68	56.4	38.	8	106.1	71.6	8	155.9	105.1	8	205.6	138.7
9	7.5	5.	69	57.3	38.6	9	106.9	72.1	9	156.7	105.7	9	206.4	139.3
10	8.3	5.6	70	58.	39.1	130	107.8	72.7	190	157.5	106.2	250	207.3	139.8
11	9.1	6.2	71	58.9	39.7	1	108.6	73.3	1	158.3	106.8	1	208.1	140.4
12	9.9	6.7	72	59.7	40.3	2	109.4	73.8	2	159.2	107.4	2	208.9	140.9
13	10.8	7.3	73	60.5	40.8	3	110.3	74.4	3	160.	107.9	3	209.7	141.5
14	11.6	7.8	74	61.3	41.4	4	111.1	74.9	4	160.8	108.5	4	210.6	142.
15	12.4	8.4	75	62.2	41.9	5	111.9	75.5	5	161.7	109.	5	211.4	142.6
16	13.3	8.9	76	63.	42.5	6	112.7	76.1	6	162.5	109.6	6	212.2	143.2
17	14.1	9.5	77	63.8	43.1	7	113.6	76.6	7	163.3	110.2	7	213.1	143.7
18	14.9	10.1	78	64.7	43.6	8	114.4	77.3	8	164.1	110.7	8	213.9	144.3
19	15.8	10.6	79	65.5	44.2	9	115.2	77.7	9	165.	111.3	9	214.7	144.8
20	16.6	11.2	80	66.3	44.7	140	116.1	78.3	200	165.8	111.8	260	215.5	145.4
21	17.4	11.7	81	67.2	45.3	1	116.9	78.8	1	166.6	112.4	1	216.4	145.9
22	18.2	12.3	82	68.	45.9	2	117.7	79.4	2	167.5	113.	2	217.2	146.5
23	19.1	12.9	83	68.8	46.4	3	118.6	80.	3	168.3	113.5	3	218.	147.1
24	19.9	13.4	84	69.6	47.	4	119.4	80.5	4	169.1	114.1	4	218.9	147.6
25	20.7	14.	85	70.5	47.5	5	120.2	81.1	5	170.	114.6	5	219.7	148.2
26	21.6	14.5	86	71.3	48.1	6	121.	81.6	6	170.8	115.2	6	220.5	148.7
27	22.4	15.1	87	72.1	48.6	7	121.9	82.2	7	171.6	115.8	7	221.4	149.3
28	23.2	15.7	88	73.	49.2	8	122.7	82.8	8	172.4	116.3	8	222.2	149.9
29	24.	16.3	89	73.8	49.8	9	123.5	83.3	9	173.3	116.9	9	223.	150.4
30	24.9	16.8	90	74.6	50.3	150	124.4	83.9	210	174.1	117.4	270	223.8	151.
31	25.7	17.3	91	75.4	50.9	1	125.2	84.4	1	174.9	118.	1	224.7	151.5
32	26.5	17.9	92	76.3	51.4	2	126.	85.	2	175.8	118.5	2	225.5	152.1
33	27.4	18.5	93	77.1	52.	3	126.8	85.6	3	176.6	119.1	3	226.3	152.7
34	28.2	19.	94	77.9	52.6	4	127.7	86.1	4	177.4	119.7	4	227.2	153.2
35	29.	19.6	95	78.8	53.1	5	128.5	86.7	5	178.2	120.2	5	228.	153.8
36	29.8	20.1	96	79.6	53.7	6	129.3	87.2	6	179.	120.8	6	228.8	154.3
37	30.7	20.7	97	80.4	54.2	7	130.2	87.8	7	179.9	121.3	7	229.6	154.9
38	31.5	21.2	98	81.2	54.8	8	131.	88.4	8	180.7	121.9	8	230.5	155.5
39	32.3	21.8	99	82.1	55.4	9	131.8	88.9	9	181.6	122.5	9	231.3	156.
40	33.2	22.4	100	82.9	55.9	160	132.6	89.5	220	182.4	123.	280	232.1	156.6
41	34.	22.9	1	83.7	56.5	1	133.5	90.	1	183.2	123.6	1	233.	157.1
42	34.8	23.5	2	84.6	57.	2	134.3	90.6	2	184.	124.1	2	233.8	157.7
43	35.6	24.	3	85.4	57.6	3	135.1	91.1	3	184.9	124.7	3	234.6	158.3
44	36.5	24.6	4	86.3	58.2	4	136.	91.7	4	185.7	125.3	4	235.4	158.8
45	37.3	25.2	5	87.	58.7	5	136.8	92.3	5	186.5	125.8	5	236.3	159.4
46	38.1	25.7	6	87.9	59.3	6	137.6	92.8	6	187.4	126.4	6	237.1	159.9
47	39.	26.3	7	88.7	59.8	7	138.4	93.4	7	188.2	126.9	7	237.9	160.5
48	39.8	26.8	8	89.5	60.4	8	139.3	93.9	8	189.	127.5	8	238.6	161.
49	40.6	27.4	9	90.4	61.	9	140.1	94.5	9	189.8	128.1	9	239.6	161.6
50	41.5	28.	110	91.2	61.5	170	140.9	95.1	230	190.7	128.6	290	240.4	162.2
51	42.3	28.5	1	92.	62.1	1	141.8	95.6	1	191.5	129.2	1	241.2	162.7
52	43.1	29.1	2	92.9	62.6	2	142.6	96.2	2	192.3	129.7	2	242.1	163.3
53	43.9	29.6	3	93.7	63.2	3	143.4	96.7	3	193.2	130.3	3	242.9	163.8
54	44.8	30.2	4	94.5	63.7	4	144.3	97.3	4	194.	130.9	4	243.7	164.4
55	45.6	30.8	5	95.3	64.3	5	145.1	97.9	5	194.8	131.4	5	244.6	165.
56	46.4	31.3	6	96.2	64.9	6	145.9	98.4	6	195.7	132.	6	245.4	165.5
57	47.3	31.9	7	97.	65.4	7	146.7	99.	7	196.5	132.5	7	246.2	166.1
58	48.1	32.4	8	97.8	66.	8	147.6	99.5	8	197.3	133.1	8	247.1	166.6
59	48.9	33.	9	98.7	66.5	9	148.4	100.1	9	198.1	133.6	9	247.9	167.2
60	49.7	33.6	130	99.5	67.1	180	149.2	100.7	240	199.	134.2	300	248.7	167.8
dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.

Distance, Departure and Diff. Latitude.

Course 56°.

Course 35°.

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	0.8	0.6	61	50.	35.	121	99.1	69.4	181	148.3	103.8	241	197.4	138.9
2	1.6	1.1	62	50.8	35.6	2	99.9	70.	2	149.1	104.4	2	198.2	139.8
3	2.5	1.7	63	51.6	36.1	3	100.8	70.5	3	149.9	105.	3	199.1	140.4
4	3.3	2.3	64	52.4	36.7	4	101.6	71.1	4	150.7	105.5	4	199.9	140.
5	4.1	2.9	65	53.2	37.3	5	102.4	71.7	5	151.5	106.1	5	200.7	140.5
6	4.9	3.4	66	54.1	37.9	6	103.2	72.3	6	152.4	106.7	6	201.5	141.1
7	5.7	4.	67	54.9	38.4	7	104.	72.8	7	153.2	107.3	7	202.3	141.7
8	6.6	4.6	68	55.7	39.	8	104.9	73.4	8	154.	107.8	8	203.1	142.2
9	7.4	5.2	69	56.5	39.6	9	105.7	74.	9	154.8	108.4	9	204.	142.8
10	8.2	5.7	70	57.3	40.2	130	106.5	74.6	190	155.6	109.	250	204.8	143.4
11	9.	6.3	71	58.2	40.7	1	107.3	75.1	1	156.5	109.6	1	205.6	144.
12	9.8	6.9	72	59.	41.3	2	108.1	75.7	2	157.3	110.1	2	206.4	144.5
13	10.6	7.5	73	59.8	41.9	3	108.9	76.3	3	158.1	110.7	3	207.2	145.1
14	11.5	8.	74	60.6	42.4	4	109.8	76.9	4	158.9	111.3	4	208.1	145.7
15	12.3	8.6	75	61.4	43.	5	110.6	77.4	5	159.7	111.8	5	208.9	146.3
16	13.1	9.2	76	62.3	43.6	6	111.4	78.	6	160.6	112.4	6	209.7	146.8
17	13.9	9.8	77	63.1	44.2	7	112.2	78.6	7	161.4	113.	7	210.5	147.4
18	14.7	10.3	78	63.9	44.8	8	113.	79.2	8	162.2	113.6	8	211.3	148.
19	15.6	10.9	79	64.7	45.3	9	113.9	79.7	9	163.	114.1	9	212.2	148.6
20	16.4	11.5	80	65.5	45.9	140	114.7	80.3	200	163.8	114.7	260	213.	149.1
21	17.2	12.	81	66.4	46.5	1	115.5	80.9	1	164.6	115.3	1	213.8	149.7
22	18.	12.6	82	67.2	47.	2	116.3	81.4	2	165.5	115.9	2	214.6	150.3
23	18.8	13.2	83	68.	47.6	3	117.1	82.	3	166.3	116.4	3	215.4	150.9
24	19.7	13.8	84	68.8	48.2	4	118.	82.6	4	167.1	117.	4	216.3	151.4
25	20.5	14.3	85	69.6	48.8	5	118.8	83.2	5	167.9	117.6	5	217.1	152.
26	21.3	14.9	86	70.4	49.3	6	119.6	83.7	6	168.7	118.2	6	217.9	152.6
27	22.1	15.5	87	71.3	49.9	7	120.4	84.3	7	169.6	118.7	7	218.7	153.1
28	22.9	16.1	88	72.1	50.5	8	121.2	84.9	8	170.4	119.3	8	219.5	153.7
29	23.8	16.6	89	72.9	51.	9	122.1	85.5	9	171.2	119.9	9	220.4	154.3
30	24.6	17.2	90	73.7	51.6	150	122.9	86.	210	172.	120.5	270	221.2	154.9
31	25.4	17.8	91	74.5	52.2	1	123.7	86.6	1	172.8	121.	1	222.	155.4
32	26.2	18.4	92	75.4	52.8	2	124.5	87.2	2	173.7	121.6	2	222.8	156.
33	27.	18.9	93	76.2	53.3	3	125.3	87.8	3	174.5	122.2	3	223.6	156.6
34	27.9	19.5	94	77.	53.9	4	126.1	88.3	4	175.3	122.7	4	224.4	157.2
35	28.7	20.1	95	77.8	54.5	5	127.	88.9	5	176.1	123.3	5	225.3	157.7
36	29.5	20.6	96	78.6	55.1	6	127.8	89.5	6	176.9	123.9	6	226.1	158.3
37	30.3	21.2	97	79.5	55.6	7	128.6	90.1	7	177.8	124.5	7	226.9	158.9
38	31.1	21.8	98	80.3	56.2	8	129.4	90.6	8	178.6	125.	8	227.7	159.5
39	31.9	22.4	99	81.1	56.8	9	130.2	91.2	9	179.4	125.6	9	228.5	160.
40	32.8	22.9	100	81.9	57.4	160	131.1	91.8	220	180.2	126.2	280	229.4	160.6
41	33.6	23.5	1	82.7	57.9	1	131.9	92.3	1	181.	126.8	1	230.2	161.2
42	34.4	24.1	2	83.6	58.5	2	132.7	92.9	2	181.9	127.3	2	231.	161.7
43	35.2	24.7	3	84.4	59.1	3	133.5	93.5	3	182.7	127.9	3	231.8	162.3
44	36.	25.3	4	85.2	59.7	4	134.3	94.1	4	183.5	128.5	4	232.6	162.9
45	36.9	25.8	5	86.	60.2	5	135.2	94.6	5	184.3	129.1	5	233.5	163.5
46	37.7	26.4	6	86.8	60.8	6	136.	95.2	6	185.1	129.6	6	234.3	164.
47	38.5	27.	7	87.6	61.4	7	136.8	95.8	7	185.9	130.2	7	235.1	164.6
48	39.3	27.5	8	88.5	61.9	8	137.6	96.4	8	186.8	130.8	8	235.9	165.2
49	40.1	28.1	9	89.3	62.5	9	138.4	96.9	9	187.6	131.3	9	236.7	165.8
50	41.	28.7	110	90.1	63.1	170	139.3	97.5	230	188.4	131.9	290	237.6	166.3
51	41.8	29.3	1	90.9	63.7	1	140.1	98.1	1	189.2	132.5	1	238.4	166.9
52	42.6	29.8	2	91.7	64.2	2	140.9	98.7	2	190.	133.1	2	239.3	167.5
53	43.4	30.4	3	92.6	64.8	3	141.7	99.2	3	190.9	133.6	3	240.	168.1
54	44.2	31.	4	93.4	65.4	4	142.5	99.8	4	191.7	134.2	4	240.8	168.6
55	45.1	31.5	5	94.2	66.	5	143.4	100.4	5	192.5	134.8	5	241.6	169.2
56	45.9	32.1	6	95.	66.5	6	144.2	100.9	6	193.3	135.4	6	242.5	169.8
57	46.7	32.7	7	95.8	67.1	7	145.	101.5	7	194.1	135.9	7	243.3	170.4
58	47.5	33.3	8	96.7	67.7	8	145.8	102.1	8	195.	136.5	8	244.1	170.9
59	48.3	33.8	9	97.5	68.3	9	146.6	102.7	9	195.8	137.1	9	244.9	171.5
60	49.1	34.4	120	98.3	68.8	180	147.4	103.2	240	196.6	137.7	300	245.7	172.1
dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.

Distance, Departure and Diff. Latitude.

Course 55°.

T

Course 36°.

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	0.8	0.6	61	49.4	35.9	121	97.9	71.1	181	146.4	106.4	241	195.	141.7
2	1.6	1.2	62	50.2	36.4	9	98.7	71.7	2	147.2	107.	2	195.8	142.2
3	2.4	1.8	63	51.	37.	3	99.5	72.3	3	148.1	107.6	3	196.6	142.8
4	3.2	2.4	64	51.8	37.6	4	100.3	72.9	4	148.9	108.2	4	197.4	143.4
5	4.	2.9	65	52.6	38.2	5	101.1	73.5	5	149.7	108.7	5	198.2	144.
6	4.9	3.5	66	53.4	38.8	6	101.9	74.1	6	150.5	109.3	6	199.	144.6
7	5.7	4.1	67	54.2	39.4	7	102.7	74.6	7	151.3	109.9	7	199.8	145.2
8	6.5	4.7	68	55.	40.	8	103.6	75.2	8	152.1	110.5	8	200.6	145.8
9	7.3	5.3	69	55.8	40.6	9	104.4	75.8	9	152.9	111.1	9	201.4	146.4
10	8.1	5.9	70	56.6	41.1	130	105.2	76.4	190	153.7	111.7	250	202.2	146.9
11	8.9	6.5	71	57.4	41.7	1	106.	77.	1	154.5	112.3	1	203.1	147.5
12	9.7	7.1	72	58.2	42.3	2	106.8	77.6	2	155.3	112.9	2	203.9	148.1
13	10.5	7.6	73	59.1	42.9	3	107.6	78.2	3	156.1	113.4	3	204.7	148.7
14	11.3	8.2	74	59.9	43.5	4	108.4	78.8	4	156.9	114.	4	205.5	149.3
15	12.1	8.8	75	60.7	44.1	5	109.2	79.4	5	157.8	114.6	5	206.3	149.9
16	12.9	9.4	76	61.5	44.7	6	110.	79.9	6	158.6	115.2	6	207.1	150.5
17	13.8	10.	77	62.3	45.3	7	110.8	80.5	7	159.4	115.8	7	207.9	151.1
18	14.6	10.6	78	63.1	45.8	8	111.6	81.1	8	160.2	116.4	8	208.7	151.6
19	15.4	11.2	79	63.9	46.4	9	112.5	81.7	9	161.	117.	9	209.5	152.2
20	16.2	11.8	80	64.7	47.	140	113.3	82.3	200	161.8	117.6	260	210.3	152.8
21	17.	12.3	81	65.5	47.6	1	114.1	82.9	1	162.6	118.1	1	211.2	153.4
22	17.8	12.9	82	66.3	48.2	2	114.9	83.5	2	163.4	118.7	2	212.	154.
23	18.6	13.5	83	67.1	48.8	3	115.7	84.1	3	164.2	119.3	3	212.8	154.6
24	19.4	14.1	84	68.	49.4	4	116.5	84.6	4	165.	119.9	4	213.6	155.2
25	20.2	14.7	85	68.8	50.	5	117.3	85.2	5	165.8	120.5	5	214.4	155.8
26	21.	15.3	86	69.6	50.5	6	118.1	85.8	6	166.7	121.1	6	215.2	156.4
27	21.8	15.9	87	70.4	51.1	7	118.9	86.4	7	167.5	121.7	7	216.	156.9
28	22.7	16.5	88	71.2	51.7	8	119.7	87.	8	168.3	122.3	8	216.8	157.5
29	23.5	17.	89	72.	52.3	9	120.5	87.6	9	169.1	122.9	9	217.6	158.1
30	24.3	17.6	90	72.8	52.9	150	121.4	88.2	210	169.9	123.4	270	218.4	158.7
31	25.1	18.2	91	73.6	53.5	1	122.2	88.8	1	170.7	124.	1	219.2	159.3
32	25.9	18.8	92	74.4	54.1	2	123.	89.3	2	171.5	124.6	2	220.1	159.9
33	26.7	19.4	93	75.2	54.7	3	123.8	89.9	3	172.3	125.2	3	220.9	160.5
34	27.5	20.	94	76.	55.3	4	124.6	90.5	4	173.1	125.8	4	221.7	161.1
35	28.3	20.6	95	76.9	55.8	5	125.4	91.1	5	173.9	126.4	5	222.5	161.6
36	29.1	21.2	96	77.7	56.4	6	126.2	91.7	6	174.7	127.	6	223.3	162.2
37	29.9	21.7	97	78.5	57.	7	127.	92.3	7	175.6	127.5	7	224.1	162.8
38	30.7	22.3	98	79.3	57.6	8	127.8	92.9	8	176.4	128.1	8	224.9	163.4
39	31.6	22.9	99	80.1	58.2	9	128.6	93.5	9	177.2	128.7	9	225.7	164.
40	32.4	23.5	100	80.9	58.8	160	129.4	94.	220	178.	129.3	280	226.5	164.6
41	33.2	24.1	1	81.7	59.4	1	130.3	94.6	1	178.8	129.9	1	227.3	165.2
42	34.	24.7	2	82.5	60.	2	131.1	95.2	2	179.6	130.5	2	228.1	165.8
43	34.8	25.3	3	83.3	60.5	3	131.9	95.8	3	180.4	131.1	3	229.	166.3
44	35.6	25.9	4	84.1	61.1	4	132.7	96.4	4	181.2	131.7	4	229.8	166.9
45	36.4	26.5	5	84.9	61.7	5	133.5	97.	5	182.	132.3	5	230.6	167.5
46	37.2	27.	6	85.8	62.3	6	134.3	97.6	6	182.8	132.8	6	231.4	168.1
47	38.	27.6	7	86.6	62.9	7	135.1	98.2	7	183.6	133.4	7	232.2	168.7
48	38.8	28.2	8	87.4	63.5	8	135.9	98.7	8	184.5	134.	8	233.	169.3
49	39.6	28.8	9	88.2	64.1	9	136.7	99.3	9	185.3	134.6	9	233.8	169.9
50	40.5	29.4	110	89.	64.7	170	137.5	99.9	230	186.1	135.2	290	234.6	170.5
51	41.3	30.	1	89.8	65.2	1	138.3	100.5	1	186.9	135.8	1	235.4	171.
52	42.1	30.6	2	90.6	65.8	2	139.2	101.1	2	187.7	136.4	2	236.2	171.6
53	42.9	31.2	3	91.4	66.4	3	140.	101.7	3	188.5	137.	3	237.	172.2
54	43.7	31.7	4	92.2	67.	4	140.8	102.3	4	189.3	137.5	4	237.9	172.8
55	44.5	32.3	5	93.	67.6	5	141.6	102.9	5	190.1	138.1	5	238.7	173.4
56	45.3	32.9	6	93.8	68.2	6	142.4	103.5	6	190.9	138.7	6	239.5	174.
57	46.1	33.5	7	94.7	68.8	7	143.2	104.	7	191.7	139.3	7	240.3	174.6
58	46.9	34.1	8	95.5	69.4	8	144.	104.6	8	192.5	139.9	8	241.1	175.2
59	47.7	34.7	9	96.3	69.9	9	144.8	105.2	9	193.4	140.5	9	241.9	175.7
60	48.5	35.3	190	97.1	70.5	180	145.6	105.8	240	194.2	141.1	300	242.7	176.3

Distance, Departure and Diff. Latitude.

Course 54°.

Course 37°.

Distance, Dist. Latitude and Departure

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	0.8	0.6	61	48.7	36.7	121	96.6	72.8	181	144.6	108.9	241	192.5	145.
2	1.6	1.2	63	49.5	37.3	2	97.4	73.4	2	145.4	109.5	2	193.3	145.6
3	2.4	1.8	63	50.3	37.9	3	98.2	74.	3	146.2	110.1	3	194.1	146.2
4	3.2	2.4	64	51.1	38.5	4	99.	74.6	4	146.9	110.7	4	194.9	146.8
5	4.	3.	65	51.9	39.1	5	99.8	75.2	5	147.7	111.3	5	195.7	147.4
6	4.8	3.6	66	52.7	39.7	6	100.6	75.8	6	148.5	111.9	6	196.5	148.
7	5.6	4.2	67	53.5	40.3	7	101.4	76.4	7	149.3	112.5	7	197.3	148.6
8	6.4	4.8	68	54.3	40.9	8	102.2	77.	8	150.1	113.1	8	198.1	149.3
9	7.2	5.4	69	55.1	41.5	9	103.	77.6	9	150.9	113.7	9	198.9	149.9
10	8.	6.	70	55.9	42.1	130	103.8	78.2	190	151.7	114.3	250	199.7	150.5
11	8.8	6.6	71	56.7	42.7	1	104.6	78.8	1	152.5	114.9	1	200.5	151.1
12	9.6	7.2	72	57.5	43.3	2	105.4	79.4	2	153.3	115.5	2	201.3	151.7
13	10.4	7.8	73	58.3	43.9	3	106.2	80.	3	154.1	116.1	3	202.1	152.3
14	11.2	8.4	74	59.1	44.5	4	107.	80.6	4	154.9	116.7	4	202.9	152.9
15	12.	9.	75	59.9	45.1	5	107.8	81.2	5	155.7	117.3	5	203.7	153.5
16	12.8	9.6	76	60.7	45.7	6	108.6	81.8	6	156.5	117.9	6	204.5	154.1
17	13.6	10.2	77	61.5	46.3	7	109.4	82.4	7	157.3	118.5	7	205.3	154.7
18	14.4	10.8	78	62.3	46.9	8	110.2	83.1	8	158.1	119.1	8	206.	155.3
19	15.2	11.4	79	63.1	47.5	9	111.	83.7	9	158.9	119.7	9	206.8	155.9
20	16.	12.	80	63.9	48.1	140	111.8	84.3	200	159.7	120.4	260	207.6	156.5
21	16.8	12.6	81	64.7	48.7	1	112.6	84.9	1	160.5	121.	1	208.4	157.1
22	17.6	13.2	82	65.5	49.3	2	113.4	85.5	2	161.3	121.6	2	209.2	157.7
23	18.4	13.8	83	66.3	49.9	3	114.2	86.1	3	162.1	122.2	3	210.	158.3
24	19.2	14.4	84	67.1	50.6	4	115.	86.7	4	162.9	122.8	4	210.8	158.9
25	20.	15.	85	67.9	51.2	5	115.8	87.3	5	163.7	123.4	5	211.6	159.5
26	20.8	15.6	86	68.7	51.8	6	116.6	87.9	6	164.5	124.	6	212.4	160.1
27	21.6	16.2	87	69.5	52.4	7	117.4	88.5	7	165.3	124.6	7	213.2	160.7
28	22.4	16.9	88	70.3	53.	8	118.2	89.1	8	166.1	125.2	8	214.	161.3
29	23.2	17.5	89	71.1	53.6	9	119.	89.7	9	166.9	125.8	9	214.8	161.9
30	24.	18.1	90	71.9	54.2	150	119.8	90.3	210	167.7	126.4	270	215.6	162.5
31	24.8	18.7	91	72.7	54.8	1	120.6	90.9	1	168.5	127.	1	216.4	163.1
32	25.6	19.3	92	73.5	55.4	2	121.4	91.5	2	169.3	127.6	2	217.2	163.7
33	26.4	19.9	93	74.3	56.	3	122.2	92.1	3	170.1	128.2	3	218.	164.3
34	27.2	20.5	94	75.1	56.6	4	123.	92.7	4	170.9	128.8	4	218.8	164.9
35	28.	21.1	95	75.9	57.2	5	123.8	93.3	5	171.7	129.4	5	219.6	165.5
36	28.8	21.7	96	76.7	57.8	6	124.6	93.9	6	172.5	130.	6	220.4	166.1
37	29.6	22.3	97	77.5	58.4	7	125.4	94.5	7	173.3	130.6	7	221.2	166.7
38	30.3	22.9	98	78.3	59.	8	126.2	95.1	8	174.1	131.2	8	222.	167.3
39	31.1	23.5	99	79.1	59.6	9	127.	95.7	9	174.9	131.8	9	222.8	167.9
40	31.9	24.1	100	79.9	60.2	160	127.8	96.3	220	175.7	132.4	280	223.6	168.5
41	32.7	24.7	1	80.7	60.8	1	128.6	96.9	1	176.5	133.	1	224.4	169.1
42	33.5	25.3	2	81.5	61.4	2	129.4	97.5	2	177.3	133.6	2	225.2	169.7
43	34.3	25.9	3	82.3	62.	3	130.2	98.1	3	178.1	134.2	3	226.	170.3
44	35.1	26.5	4	83.1	62.6	4	131.	98.7	4	178.9	134.8	4	226.8	170.9
45	35.9	27.1	5	83.9	63.2	5	131.8	99.3	5	179.7	135.4	5	227.6	171.5
46	36.7	27.7	6	84.7	63.8	6	132.6	99.9	6	180.5	136.	6	228.4	172.1
47	37.5	28.3	7	85.5	64.4	7	133.4	100.5	7	181.3	136.6	7	229.2	172.7
48	38.3	28.9	8	86.3	65.	8	134.2	101.1	8	182.1	137.2	8	230.	173.3
49	39.1	29.5	9	87.1	65.6	9	135.	101.7	9	182.9	137.8	9	230.8	173.9
50	39.9	30.1	110	87.8	66.2	170	135.8	102.3	230	183.7	138.4	290	231.6	174.5
51	40.7	30.7	1	88.6	66.8	1	136.6	102.9	1	184.5	139.	1	232.4	175.1
52	41.5	31.3	2	89.4	67.4	2	137.4	103.5	2	185.3	139.6	2	233.2	175.7
53	42.3	31.9	3	90.2	68.	3	138.2	104.1	3	186.1	140.2	3	234.	176.3
54	43.1	32.5	4	91.	68.6	4	139.	104.7	4	186.9	140.8	4	234.8	176.9
55	43.9	33.1	5	91.8	69.2	5	139.8	105.3	5	187.7	141.4	5	235.6	177.5
56	44.7	33.7	6	92.6	69.8	6	140.6	105.9	6	188.5	142.	6	236.4	178.1
57	45.5	34.3	7	93.4	70.4	7	141.4	106.5	7	189.3	142.6	7	237.2	178.7
58	46.3	34.9	8	94.2	71.	8	142.2	107.1	8	190.1	143.2	8	238.	179.3
59	47.1	35.5	9	95.	71.6	9	143.	107.7	9	190.9	143.8	9	238.8	179.9
60	47.9	36.1	190	95.8	72.2	180	143.8	108.3	240	191.7	144.4	300	239.6	180.5

Distance, Departure and Dist. Latitude.

Course 53°.

Course 30°.

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	0.8	0.6	61	48.1	37.6	121	95.3	74.5	181	142.6	111.4	241	189.9	148.4
2	1.6	1.2	62	48.9	38.2	2	96.1	75.1	2	143.4	112.1	3	190.7	149.
3	2.4	1.8	63	49.6	38.8	3	96.9	75.7	3	144.2	112.7	4	191.5	149.6
4	3.2	2.5	64	50.4	39.4	4	97.7	76.3	4	145.	113.3	5	192.3	150.2
5	3.9	3.1	65	51.3	40.	5	98.5	77.	5	145.8	113.9	6	193.1	150.8
6	4.7	3.7	66	52.	40.6	6	99.3	77.6	6	146.6	114.6	7	193.9	151.5
7	5.5	4.3	67	52.8	41.2	7	100.1	78.2	7	147.4	115.1	8	194.6	152.1
8	6.3	4.9	68	53.6	41.9	8	100.9	78.8	8	148.1	115.7	9	195.4	152.7
9	7.1	5.5	69	54.4	42.5	9	101.7	79.4	9	148.9	116.4	10	196.2	153.3
10	7.9	6.2	70	55.2	43.1	130	102.4	80.	190	149.7	117.	290	197.	153.9
11	8.7	6.8	71	55.9	43.7	1	103.2	80.7	1	150.5	117.6	1	197.8	154.5
12	9.5	7.4	72	56.7	44.3	2	104.	81.2	2	151.3	118.2	2	198.6	155.1
13	10.2	8.	73	57.5	44.9	3	104.8	81.9	3	152.1	118.8	3	199.4	155.8
14	11.	8.6	74	58.3	45.6	4	105.6	82.5	4	152.9	119.4	4	200.2	156.4
15	11.8	9.2	75	59.1	46.2	5	106.4	83.1	5	153.7	120.1	5	200.9	157.
16	12.6	9.9	76	59.9	46.8	6	107.2	83.7	6	154.5	120.7	6	201.7	157.6
17	13.4	10.5	77	60.7	47.4	7	108.	84.3	7	155.2	121.3	7	202.5	158.2
18	14.2	11.1	78	61.5	48.	8	108.7	85.	8	156.	121.9	8	203.3	158.8
19	15.	11.7	79	62.3	48.6	9	109.5	85.6	9	156.8	122.5	9	204.1	159.5
20	15.8	12.3	80	63.	49.2	140	110.3	86.3	200	157.6	123.	300	204.9	160.1
21	16.5	12.9	81	63.8	49.9	1	111.1	86.8	1	158.4	123.7	1	205.7	160.7
22	17.3	13.5	82	64.6	50.5	2	111.9	87.4	2	159.2	124.3	2	206.5	161.3
23	18.1	14.2	83	65.4	51.1	3	112.7	88.	3	160.	125.	3	207.3	161.9
24	18.9	14.8	84	66.2	51.7	4	113.5	88.7	4	160.8	125.6	4	208.1	162.5
25	19.7	15.5	85	67.	52.3	5	114.3	89.3	5	161.5	126.2	5	208.9	163.1
26	20.5	16.	86	67.8	52.9	6	115.	89.9	6	162.3	126.8	6	209.6	163.8
27	21.3	16.6	87	68.6	53.6	7	115.8	90.5	7	163.1	127.4	7	210.4	164.4
28	22.1	17.2	88	69.3	54.2	8	116.6	91.1	8	163.9	128.1	8	211.2	165.
29	22.9	17.9	89	70.1	54.8	9	117.4	91.7	9	164.7	128.7	9	212.	165.6
30	23.6	18.5	90	70.9	55.4	150	118.2	92.3	210	165.5	129.3	310	212.8	166.2
31	24.4	19.1	91	71.7	56.	1	119.	93.	1	166.3	129.9	1	213.6	166.8
32	25.2	19.7	92	72.5	56.6	2	119.8	93.6	2	167.1	130.5	2	214.3	167.5
33	26.	20.3	93	73.3	57.3	3	120.6	94.3	3	167.9	131.1	3	215.1	168.1
34	26.8	20.9	94	74.1	57.9	4	121.4	94.8	4	168.6	131.8	4	215.9	168.7
35	27.6	21.5	95	74.9	58.5	5	122.1	95.4	5	169.4	132.4	5	216.7	169.3
36	28.4	22.2	96	75.6	59.1	6	122.9	96.	6	170.2	133.	6	217.5	169.9
37	29.2	22.8	97	76.4	59.7	7	123.7	96.7	7	171.	133.6	7	218.3	170.5
38	29.9	23.4	98	77.2	60.3	8	124.5	97.3	8	171.8	134.2	8	219.1	171.1
39	30.7	24.	99	78.	61.	9	125.3	97.9	9	172.6	134.8	9	219.9	171.8
40	31.5	24.6	100	78.8	61.6	160	126.1	98.5	260	173.4	135.4	360	220.6	172.4
41	32.3	25.2	1	79.6	62.2	1	126.9	99.1	1	174.2	136.1	1	221.4	173.
42	33.1	25.9	2	80.4	62.8	2	127.7	99.7	2	174.9	136.7	2	222.2	173.6
43	33.9	26.5	3	81.2	63.4	3	128.4	100.4	3	175.7	137.3	3	223.	174.2
44	34.7	27.1	4	82.	64.	4	129.2	101.	4	176.5	137.9	4	223.8	174.8
45	35.5	27.7	5	82.7	64.6	5	130.	101.6	5	177.3	138.5	5	224.6	175.4
46	36.3	28.3	6	83.5	65.3	6	130.8	102.2	6	178.1	139.1	6	225.4	176.1
47	37.	28.9	7	84.3	65.9	7	131.6	102.8	7	178.9	139.8	7	226.2	176.7
48	37.8	29.6	8	85.1	66.5	8	132.4	103.4	8	179.7	140.4	8	227.	177.3
49	38.6	30.2	9	85.9	67.1	9	133.2	104.	9	180.5	141.	9	227.7	177.9
50	39.4	30.8	110	86.7	67.7	170	134.	104.7	230	181.3	141.8	300	228.5	178.5
51	40.2	31.4	1	87.5	68.3	1	134.7	105.3	1	182.	142.3	1	229.3	179.1
52	41.	32.	2	88.3	69.	2	135.5	105.9	2	182.8	142.9	2	230.1	179.8
53	41.8	32.6	3	89.	69.6	3	136.3	106.5	3	183.6	143.4	3	230.9	180.4
54	42.6	33.2	4	89.8	70.2	4	137.1	107.1	4	184.4	144.1	4	231.7	181.
55	43.3	33.9	5	90.6	70.8	5	137.9	107.7	5	185.2	144.7	5	232.5	181.6
56	44.1	34.5	6	91.4	71.4	6	138.7	108.4	6	186.	145.3	6	233.3	182.2
57	44.9	35.1	7	92.2	72.	7	139.5	109.	7	186.8	145.9	7	234.	182.9
58	45.7	35.7	8	93.	72.6	8	140.3	109.6	8	187.5	146.5	8	234.8	183.5
59	46.5	36.3	9	93.8	73.3	9	141.1	110.2	9	188.3	147.1	9	235.6	184.1
60	47.3	36.9	130	94.6	73.9	180	141.8	110.8	240	189.1	147.8	300	236.4	184.7
dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.

Distance, Departure and Diff. Latitude.

Course 50°.

Course 39°

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	0.6	0.6	61	47.4	38.4	131	94.	76.1	181	140.7	113.9	241	187.3	151.7
2	1.6	1.3	62	48.2	39.	132	94.8	76.8	2	141.4	114.5	242	188.1	152.3
3	2.3	1.9	63	49.	39.6	3	95.6	77.4	3	142.9	115.2	3	188.8	152.9
4	3.1	2.5	64	49.7	40.3	4	96.4	78.	4	143.	115.8	4	189.6	153.6
5	3.9	3.1	65	50.5	40.9	5	97.1	78.7	5	143.8	116.4	5	190.4	154.2
6	4.7	3.8	66	51.3	41.5	6	97.9	79.3	6	144.5	117.1	6	191.2	154.8
7	5.4	4.4	67	52.1	42.2	7	98.7	79.9	7	145.3	117.7	7	192.	155.4
8	6.2	5.	68	52.8	42.8	8	99.5	80.6	8	146.1	118.3	8	192.7	156.1
9	7.	5.7	69	53.6	43.4	9	100.3	81.2	9	146.9	118.9	9	193.5	156.7
10	7.8	6.3	70	54.4	44.1	130	101.	81.8	190	147.7	119.6	250	194.3	157.3
11	8.5	6.9	71	55.2	44.7	1	101.8	82.4	1	148.4	120.2	1	195.1	158.
12	9.3	7.6	72	56.	45.3	2	102.6	83.1	2	149.2	120.8	2	195.8	158.6
13	10.1	8.2	73	56.7	45.9	3	103.4	83.7	3	150.	121.5	3	196.6	159.2
14	10.9	8.8	74	57.5	46.6	4	104.1	84.3	4	150.8	122.1	4	197.4	159.8
15	11.7	9.4	75	58.3	47.2	5	104.9	85.	5	151.5	122.7	5	198.2	160.5
16	12.4	10.1	76	59.1	47.8	6	105.7	85.6	6	152.3	123.3	6	199.	161.1
17	13.2	10.7	77	59.8	48.5	7	106.5	86.2	7	153.1	124.	7	199.7	161.7
18	14.	11.3	78	60.6	49.1	8	107.3	86.8	8	153.9	124.6	8	200.5	162.4
19	14.8	12.	79	61.4	49.7	9	108.	87.5	9	154.7	125.2	9	201.3	163.
20	15.5	12.6	80	62.2	50.3	140	108.8	88.1	200	155.4	125.9	260	202.1	163.6
21	16.3	13.2	81	62.9	51.	1	109.6	88.7	1	156.2	126.5	1	202.8	164.3
22	17.1	13.8	82	63.7	51.6	2	110.4	89.4	2	157.	127.1	2	203.6	164.9
23	17.9	14.5	83	64.5	52.2	3	111.1	90.	3	157.8	127.8	3	204.4	165.5
24	18.7	15.1	84	65.3	52.9	4	111.9	90.6	4	158.5	128.4	4	205.2	166.1
25	19.4	15.7	85	66.1	53.5	5	112.7	91.3	5	159.3	129.	5	205.9	166.8
26	20.2	16.4	86	66.8	54.1	6	113.5	91.9	6	160.1	129.6	6	206.7	167.4
27	21.	17.	87	67.6	54.8	7	114.3	92.5	7	160.9	130.3	7	207.5	168.
28	21.8	17.6	88	68.4	55.4	8	115.	93.1	8	161.6	130.9	8	208.3	168.7
29	22.6	18.3	89	69.2	56.	9	115.8	93.8	9	162.4	131.5	9	209.1	169.3
30	23.3	18.9	90	69.9	56.6	150	116.6	94.4	210	163.2	132.2	270	209.8	169.9
31	24.1	19.5	91	70.7	57.3	1	117.3	95.	1	164.	132.8	1	210.6	170.5
32	24.9	20.1	92	71.5	57.9	2	118.1	95.7	2	164.8	133.4	2	211.4	171.1
33	25.6	20.8	93	72.3	58.5	3	118.9	96.3	3	165.5	134.	3	212.2	171.8
34	26.4	21.4	94	73.1	59.2	4	119.7	96.9	4	166.3	134.7	4	212.9	172.4
35	27.2	22.	95	73.8	59.8	5	120.5	97.5	5	167.1	135.3	5	213.7	173.1
36	28.	22.7	96	74.6	60.4	6	121.3	98.2	6	167.9	135.9	6	214.5	173.7
37	28.8	23.3	97	75.4	61.	7	122.	98.8	7	168.6	136.6	7	215.3	174.3
38	29.5	23.9	98	76.2	61.7	8	122.8	99.4	8	169.4	137.2	8	216.	175.
39	30.3	24.5	99	76.9	62.3	9	123.6	100.1	9	170.2	137.8	9	216.8	175.6
40	31.1	25.2	100	77.7	62.9	160	124.3	100.7	220	171.	138.5	280	217.6	176.2
41	31.9	25.8	1	78.5	63.6	1	125.1	101.3	1	171.7	139.1	1	218.4	176.8
42	32.6	26.4	2	79.3	64.2	2	125.9	101.9	2	172.5	139.7	2	219.2	177.5
43	33.4	27.1	3	80.	64.8	3	126.7	102.6	3	173.3	140.3	3	219.9	178.1
44	34.2	27.7	4	80.8	65.4	4	127.5	103.2	4	174.1	141.	4	220.7	178.7
45	35.	28.3	5	81.6	66.1	5	128.3	103.8	5	174.9	141.6	5	221.5	179.4
46	35.7	28.9	6	82.4	66.7	6	129.	104.5	6	175.6	142.2	6	222.3	180.
47	36.5	29.6	7	83.2	67.3	7	129.8	105.1	7	176.4	142.9	7	223.	180.6
48	37.3	30.2	8	83.9	68.	8	130.6	105.7	8	177.2	143.5	8	223.8	181.2
49	38.1	30.8	9	84.7	68.6	9	131.3	106.4	9	178.	144.1	9	224.6	181.9
50	38.9	31.5	110	85.5	69.2	170	132.1	107.	230	178.7	144.7	290	225.4	182.5
51	39.6	32.1	1	86.3	69.9	1	132.9	107.6	1	179.5	145.4	1	226.1	183.1
52	40.4	32.7	2	87.	70.5	2	133.7	108.2	2	180.3	146.	2	226.9	183.8
53	41.2	33.4	3	87.8	71.1	3	134.4	108.9	3	181.1	146.6	3	227.7	184.4
54	42.	34.	4	88.6	71.7	4	135.2	109.5	4	181.9	147.3	4	228.5	185.
55	42.7	34.6	5	89.4	72.4	5	136.	110.1	5	182.6	147.9	5	229.3	185.6
56	43.5	35.2	6	90.1	73.	6	136.8	110.8	6	183.4	148.5	6	230.	186.3
57	44.3	35.9	7	90.9	73.6	7	137.6	111.4	7	184.2	149.1	7	230.8	186.9
58	45.1	36.5	8	91.7	74.3	8	138.3	112.	8	185.	149.8	8	231.6	187.5
59	45.9	37.1	9	92.5	74.9	9	139.1	112.6	9	185.7	150.4	9	232.4	188.2
60	46.6	37.8	190	93.3	75.5	190	139.9	113.3	240	186.5	151.	300	233.1	188.8
dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.

Distance, Departure and Diff. Latitude.

Course 51°.

Course 40°.

Distance, Diff. Latitude and Departure.

dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.	dist.	d. lat.	dep.
1	0.8	0.6	61	46.7	39.9	181	92.7	77.8	181	138.7	116.3	241	184.6	154.9
2	1.5	1.3	62	47.5	39.9	9	93.5	78.4	2	139.4	117.	2	185.4	155.6
3	2.3	1.9	63	48.3	40.5	3	94.9	79.1	3	140.2	117.6	3	186.1	156.2
4	3.1	2.6	64	49.	41.1	4	95.	79.7	4	141.	118.3	4	186.9	156.8
5	3.8	3.2	65	49.8	41.8	5	95.8	80.3	5	141.7	118.9	5	187.7	157.5
6	4.6	3.9	66	50.6	42.4	6	96.5	81.	6	142.5	119.6	6	188.4	158.1
7	5.4	4.5	67	51.3	43.1	7	97.3	81.6	7	143.3	120.2	7	189.2	158.6
8	6.1	5.1	68	52.1	43.7	8	98.1	82.3	8	144.	120.8	8	190.	159.4
9	6.9	5.8	69	52.9	44.4	9	98.8	82.9	9	144.8	121.5	9	190.7	160.1
10	7.7	6.4	70	53.6	45.	130	99.6	83.6	100	145.5	122.1	250	191.5	160.7
11	8.4	7.1	71	54.4	45.6	1	100.4	84.3	1	146.3	122.8	1	192.3	161.3
12	9.2	7.7	72	55.2	46.3	2	101.1	84.8	2	147.1	123.4	2	193.	162.
13	10.	8.4	73	55.9	46.9	3	101.9	85.5	3	147.8	124.1	3	193.8	162.6
14	10.7	9.	74	56.7	47.6	4	102.6	86.1	4	148.6	124.7	4	194.6	163.3
15	11.5	9.6	75	57.5	48.2	5	103.4	86.8	5	149.4	125.3	5	195.3	163.9
16	12.3	10.3	76	58.3	48.9	6	104.2	87.4	6	150.1	126.	6	196.1	164.6
17	13.	10.9	77	59.	49.5	7	104.9	88.1	7	150.9	126.6	7	196.9	165.2
18	13.8	11.6	78	59.8	50.1	8	105.7	88.7	8	151.7	127.3	8	197.6	165.8
19	14.6	12.3	79	60.5	50.8	9	106.5	89.3	9	152.4	127.9	9	198.4	166.5
20	15.3	12.9	80	61.3	51.4	140	107.2	90.	200	153.2	128.6	260	199.2	167.1
21	16.1	13.5	81	62.	52.1	1	108.	90.6	1	154.	129.2	1	199.9	167.8
22	16.9	14.1	82	62.8	52.7	2	108.8	91.3	2	154.7	129.8	2	200.7	168.4
23	17.6	14.8	83	63.6	53.4	3	109.5	91.9	3	155.5	130.5	3	201.5	169.1
24	18.4	15.4	84	64.3	54.	4	110.3	92.6	4	156.3	131.1	4	202.3	169.7
25	19.2	16.1	85	65.1	54.6	5	111.1	93.2	5	157.	131.8	5	203.	170.3
26	19.9	16.7	86	65.9	55.3	6	111.8	93.8	6	157.8	132.4	6	203.8	171.
27	20.7	17.4	87	66.6	55.9	7	112.6	94.5	7	158.6	133.1	7	204.5	171.6
28	21.4	18.	88	67.4	56.6	8	113.4	95.1	8	159.3	133.7	8	205.3	172.3
29	22.3	18.6	89	68.2	57.3	9	114.1	95.8	9	160.1	134.3	9	206.1	172.9
30	23.	19.3	90	68.9	57.9	150	114.9	96.4	210	160.9	135.	270	206.8	173.6
31	23.7	19.9	91	69.7	58.5	1	115.7	97.1	1	161.6	135.6	1	207.6	174.2
32	24.5	20.6	92	70.5	59.1	2	116.4	97.7	2	162.4	136.3	2	208.4	174.8
33	25.3	21.2	93	71.3	59.8	3	117.2	98.3	3	163.3	136.9	3	209.1	175.5
34	26.	21.9	94	72.	60.4	4	118.	99.	4	163.9	137.6	4	209.9	176.1
35	26.8	22.5	95	72.8	61.1	5	118.7	99.6	5	164.7	138.2	5	210.7	176.8
36	27.6	23.1	96	73.5	61.7	6	119.5	100.3	6	165.5	138.8	6	211.4	177.4
37	28.3	23.8	97	74.3	62.4	7	120.3	100.9	7	166.3	139.5	7	212.2	178.1
38	29.1	24.4	98	75.1	63.	8	121.	101.6	8	167.	140.1	8	213.	178.7
39	29.9	25.1	99	75.8	63.6	9	121.8	102.2	9	167.8	140.8	9	213.7	179.3
40	30.6	25.7	100	76.6	64.3	160	122.6	102.8	220	168.5	141.4	280	214.5	180.
41	31.4	26.4	1	77.4	64.9	1	123.3	103.5	1	169.3	142.1	1	215.3	180.6
42	32.2	27.	2	78.1	65.6	2	124.1	104.1	2	170.1	142.7	2	216.	181.3
43	32.9	27.6	3	78.9	66.3	3	124.9	104.8	3	170.8	143.3	3	216.8	181.9
44	33.7	28.3	4	79.7	66.8	4	125.6	105.4	4	171.6	144.	4	217.6	182.6
45	34.5	28.9	5	80.4	67.5	5	126.4	106.1	5	172.4	144.6	5	218.3	183.2
46	35.2	29.6	6	81.2	68.1	6	127.2	106.7	6	173.1	145.3	6	219.1	183.8
47	36.	30.3	7	82.	68.8	7	127.9	107.3	7	173.9	145.9	7	219.9	184.5
48	36.8	30.9	8	82.7	69.4	8	128.7	108.	8	174.7	146.6	8	220.6	185.1
49	37.5	31.5	9	83.5	70.1	9	129.5	108.6	9	175.4	147.2	9	221.4	185.8
50	38.3	32.1	110	84.3	70.7	170	130.2	109.3	230	176.2	147.8	290	222.2	186.4
51	39.1	32.8	1	85.	71.3	1	131.	109.9	1	177.	148.5	1	223.0	187.1
52	39.8	33.4	2	85.8	72.	2	131.8	110.6	2	177.7	149.1	2	223.7	187.7
53	40.6	34.1	3	86.6	72.6	3	132.5	111.2	3	178.5	149.8	3	224.5	188.3
54	41.4	34.7	4	87.3	73.3	4	133.3	111.8	4	179.3	150.4	4	225.2	189.
55	42.1	35.4	5	88.1	73.9	5	134.1	112.5	5	180.	151.1	5	226.	189.6
56	42.9	36.	6	88.9	74.6	6	134.8	113.1	6	180.8	151.7	6	226.7	190.3
57	43.7	36.7	7	89.6	75.2	7	135.6	113.8	7	181.6	152.3	7	227.5	190.9
58	44.4	37.3	8	90.4	75.8	8	136.4	114.4	8	182.3	153.	8	228.3	191.6
59	45.2	37.9	9	91.2	76.5	9	137.1	115.1	9	183.1	153.6	9	229.	192.3
60	46.	38.6	130	91.9	77.1	180	137.9	115.7	240	183.9	154.2	300	229.8	192.9
dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.	dist.	dep.	d. lat.

Distance, Departure and Diff. Latitude.

Course 50°.

TABLE VI.

MERIDIONAL PARTS.

Meridional Parts.

	29°	30°	31°	32°	33°	34°	35°	36°	37°	38°	39°	40°	41°	42°	
0	1819	1889	1938	2008	2100	2171	2244	2318	2393	2468	2545	2623	2702	2782	0
1	21	90	59	30	01	73	46	19	94	70	46	24	03	83	1
2	28	91	60	31	02	74	47	20	95	71	48	25	04	84	2
3	23	92	62	32	03	75	48	22	96	72	49	27	06	86	3
4	24	93	63	33	04	76	49	23	98	73	50	28	07	87	4
5	25	94	64	34	06	78	50	24	99	75	51	29	08	88	5
6	26	95	65	35	07	79	52	25	2490	76	53	31	10	90	6
7	27	96	66	37	08	80	53	27	01	77	54	32	11	91	7
8	29	98	67	38	09	81	54	28	03	78	55	33	12	92	8
9	30	99	69	39	10	82	55	29	04	80	57	34	14	94	9
10	31	1000	70	40	11	84	57	30	05	81	58	36	15	95	10
11	32	01	71	41	13	85	58	32	06	82	59	37	16	97	11
12	33	02	72	43	14	86	59	33	08	84	60	38	18	98	12
13	34	03	73	44	15	87	60	34	09	85	62	40	19	99	13
14	35	05	74	45	16	88	61	35	10	86	63	41	20	201	14
15	37	06	76	46	17	90	63	37	11	87	64	42	22	02	15
16	38	07	77	47	19	91	64	38	13	89	66	44	23	03	16
17	39	08	78	48	20	92	65	39	14	90	67	45	24	05	17
18	40	09	79	50	21	93	66	40	15	91	68	46	26	06	18
19	41	10	80	51	22	94	68	42	16	92	69	48	27	07	19
20	42	11	81	52	23	96	69	43	18	94	71	49	28	09	20
21	43	13	83	53	25	97	70	44	19	95	72	50	29	10	21
22	45	14	84	54	26	98	71	45	20	96	73	51	31	11	22
23	46	15	85	56	27	2000	72	46	22	98	75	53	32	13	23
24	47	16	86	57	28	01	74	48	23	99	76	54	33	14	24
25	48	17	87	58	29	02	75	49	24	2500	77	55	35	15	25
26	49	19	88	59	31	03	76	50	25	01	78	57	36	17	26
27	50	20	90	60	32	04	77	51	27	03	80	58	37	18	27
28	52	21	91	61	33	05	79	53	28	04	81	59	39	20	28
29	53	22	92	63	34	07	80	54	29	05	82	61	40	21	29
30	54	23	93	64	35	08	81	55	30	06	84	62	42	22	30
31	55	25	94	65	37	09	82	56	32	08	85	63	43	24	31
32	56	26	95	66	38	10	83	58	33	09	86	65	44	25	32
33	57	27	97	67	39	11	85	59	34	10	88	66	46	26	33
34	58	28	98	69	40	13	86	60	35	12	89	67	47	28	34
35	60	29	99	70	41	14	87	61	37	13	90	69	48	29	35
36	61	31	2000	71	43	15	88	63	38	14	91	70	50	30	36
37	62	32	01	72	44	16	90	64	39	15	92	71	51	32	37
38	63	33	02	73	45	17	91	65	40	17	94	73	52	33	38
39	64	34	04	75	46	19	92	66	42	18	95	74	54	34	39
40	65	35	05	76	47	20	93	68	43	19	97	75	55	36	40
41	66	36	06	77	49	21	95	69	44	21	98	76	56	37	41
42	68	37	07	78	50	22	96	70	45	22	99	78	58	39	42
43	69	38	08	79	51	24	97	71	47	23	2001	79	59	40	43
44	70	39	10	80	52	25	98	73	48	24	02	80	60	41	44
45	71	41	11	82	53	26	99	74	49	26	03	82	62	43	45
46	72	42	12	83	55	27	201	75	51	27	04	83	63	44	46
47	73	43	13	84	56	28	02	76	52	28	06	84	64	45	47
48	75	44	14	85	57	30	03	78	53	30	07	86	66	47	48
49	76	45	15	86	58	31	04	79	54	31	08	87	67	48	49
50	77	46	17	88	59	32	06	80	56	32	10	88	68	49	50
51	78	48	18	89	61	33	07	81	57	33	11	90	70	51	51
52	79	49	19	90	62	35	08	83	58	35	12	91	71	52	52
53	80	50	20	91	63	36	09	84	59	36	14	92	72	54	53
54	81	51	21	92	64	37	11	85	61	37	15	94	74	55	54
55	83	52	22	94	65	38	12	86	62	38	16	95	75	56	55
56	84	53	24	95	67	39	13	88	63	40	17	96	76	58	56
57	85	55	25	96	68	41	14	89	64	41	19	98	78	59	57
58	86	56	26	97	69	42	16	90	66	42	20	99	79	60	58
59	1067	1057	2027	2096	2170	2243	2317	2391	2467	2544	2621	2700	2780	2862	59
	29°	30°	31°	32°	33°	34°	35°	36°	37°	38°	39°	40°	41°	42°	

TABLE VII.

AMPLITUDES

Amplitudes.

Declination of the Sun.													
lat.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	lat.
0°	0'	10'	20'	30'	40'	50'	00'	10'	20'	30'	40'	50'	0°
1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	1	1	1	1	1	1	1	0
5	0	0	0	0	1	1	1	1	1	1	1	1	0
6	0	0	0	1	1	1	1	1	1	1	1	1	0
7	0	0	1	1	1	1	1	1	1	1	1	1	0
8	0	1	1	1	1	1	1	1	1	1	1	1	0
9	0	1	1	1	1	1	1	1	1	1	1	1	0
10	0	1	1	1	1	1	1	1	1	1	1	1	0
11	0	1	1	1	1	1	1	1	1	1	1	1	0
12	0	1	1	1	1	1	1	1	1	1	1	1	0
13	0	1	1	1	1	1	1	1	1	1	1	1	0
14	0	1	1	1	1	1	1	1	1	1	1	1	0
15	0	1	1	1	1	1	1	1	1	1	1	1	0
16	0	1	1	1	1	1	1	1	1	1	1	1	0
17	0	1	1	1	1	1	1	1	1	1	1	1	0
18	0	1	1	1	1	1	1	1	1	1	1	1	0
19	0	1	1	1	1	1	1	1	1	1	1	1	0
20	0	1	1	1	1	1	1	1	1	1	1	1	0
21	0	1	1	1	1	1	1	1	1	1	1	1	0
22	0	1	1	1	1	1	1	1	1	1	1	1	0
23	0	1	1	1	1	1	1	1	1	1	1	1	0
24	0	1	1	1	1	1	1	1	1	1	1	1	0
25	0	1	1	1	1	1	1	1	1	1	1	1	0
26	0	1	1	1	1	1	1	1	1	1	1	1	0
27	0	1	1	1	1	1	1	1	1	1	1	1	0
28	0	1	1	1	1	1	1	1	1	1	1	1	0
29	0	1	1	1	1	1	1	1	1	1	1	1	0
30	0	1	1	1	1	1	1	1	1	1	1	1	0
31	0	1	1	1	1	1	1	1	1	1	1	1	0
32	0	1	1	1	1	1	1	1	1	1	1	1	0
33	0	1	1	1	1	1	1	1	1	1	1	1	0
34	0	1	1	1	1	1	1	1	1	1	1	1	0
35	0	1	1	1	1	1	1	1	1	1	1	1	0
36	0	1	1	1	1	1	1	1	1	1	1	1	0
37	0	1	1	1	1	1	1	1	1	1	1	1	0
38	0	1	1	1	1	1	1	1	1	1	1	1	0
39	0	1	1	1	1	1	1	1	1	1	1	1	0
40	0	1	1	1	1	1	1	1	1	1	1	1	0
41	0	1	1	1	1	1	1	1	1	1	1	1	0
42	0	1	1	1	1	1	1	1	1	1	1	1	0
43	0	1	1	1	1	1	1	1	1	1	1	1	0
44	0	1	1	1	1	1	1	1	1	1	1	1	0
45	0	1	1	1	1	1	1	1	1	1	1	1	0
46	0	1	1	1	1	1	1	1	1	1	1	1	0
47	0	1	1	1	1	1	1	1	1	1	1	1	0
48	0	1	1	1	1	1	1	1	1	1	1	1	0
49	0	1	1	1	1	1	1	1	1	1	1	1	0
50	0	1	1	1	1	1	1	1	1	1	1	1	0
51	0	1	1	1	1	1	1	1	1	1	1	1	0
52	0	1	1	1	1	1	1	1	1	1	1	1	0
53	0	1	1	1	1	1	1	1	1	1	1	1	0
54	0	1	1	1	1	1	1	1	1	1	1	1	0
55	0	1	1	1	1	1	1	1	1	1	1	1	0
56	0	1	1	1	1	1	1	1	1	1	1	1	0
57	0	1	1	1	1	1	1	1	1	1	1	1	0
58	0	1	1	1	1	1	1	1	1	1	1	1	0
59	0	1	1	1	1	1	1	1	1	1	1	1	0
60	0	1	1	1	1	1	1	1	1	1	1	1	0

Declination of the Sun.

Amplitudes.

Declination of the Sun.

lat.	120°	130°	140°	150°	160°	170°	180°	190°	200°	210°	220°	230°	lat.
0°	0	0	0	0	0	0	0	0	0	0	0	0	0°
1	0	0	0	0	0	0	0	0	0	0	0	0	1
2	0	0	0	0	0	0	0	0	0	0	0	0	2
3	1	1	1	1	1	1	1	1	1	1	1	1	3
4	2	2	2	2	2	2	2	2	2	2	2	2	4
5	3	3	3	3	3	3	3	3	3	3	3	3	5
6	4	4	4	4	4	4	4	4	4	4	4	4	6
7	5	5	5	5	5	5	5	5	5	5	5	5	7
8	6	6	6	6	6	6	6	6	6	6	6	6	8
9	7	7	7	7	7	7	7	7	7	7	7	7	9
10	8	8	8	8	8	8	8	8	8	8	8	8	10
11	9	9	9	9	9	9	9	9	9	9	9	9	11
12	10	10	10	10	10	10	10	10	10	10	10	10	12
13	11	11	11	11	11	11	11	11	11	11	11	11	13
14	12	12	12	12	12	12	12	12	12	12	12	12	14
15	13	13	13	13	13	13	13	13	13	13	13	13	15
16	14	14	14	14	14	14	14	14	14	14	14	14	16
17	15	15	15	15	15	15	15	15	15	15	15	15	17
18	16	16	16	16	16	16	16	16	16	16	16	16	18
19	17	17	17	17	17	17	17	17	17	17	17	17	19
20	18	18	18	18	18	18	18	18	18	18	18	18	20
21	19	19	19	19	19	19	19	19	19	19	19	19	21
22	20	20	20	20	20	20	20	20	20	20	20	20	22
23	21	21	21	21	21	21	21	21	21	21	21	21	23
24	22	22	22	22	22	22	22	22	22	22	22	22	24
25	23	23	23	23	23	23	23	23	23	23	23	23	25
26	24	24	24	24	24	24	24	24	24	24	24	24	26
27	25	25	25	25	25	25	25	25	25	25	25	25	27
28	26	26	26	26	26	26	26	26	26	26	26	26	28
29	27	27	27	27	27	27	27	27	27	27	27	27	29
30	28	28	28	28	28	28	28	28	28	28	28	28	30
31	29	29	29	29	29	29	29	29	29	29	29	29	31
32	30	30	30	30	30	30	30	30	30	30	30	30	32
33	31	31	31	31	31	31	31	31	31	31	31	31	33
34	32	32	32	32	32	32	32	32	32	32	32	32	34
35	33	33	33	33	33	33	33	33	33	33	33	33	35
36	34	34	34	34	34	34	34	34	34	34	34	34	36
37	35	35	35	35	35	35	35	35	35	35	35	35	37
38	36	36	36	36	36	36	36	36	36	36	36	36	38
39	37	37	37	37	37	37	37	37	37	37	37	37	39
40	38	38	38	38	38	38	38	38	38	38	38	38	40
41	39	39	39	39	39	39	39	39	39	39	39	39	41
42	40	40	40	40	40	40	40	40	40	40	40	40	42
43	41	41	41	41	41	41	41	41	41	41	41	41	43
44	42	42	42	42	42	42	42	42	42	42	42	42	44
45	43	43	43	43	43	43	43	43	43	43	43	43	45
46	44	44	44	44	44	44	44	44	44	44	44	44	46
47	45	45	45	45	45	45	45	45	45	45	45	45	47
48	46	46	46	46	46	46	46	46	46	46	46	46	48
49	47	47	47	47	47	47	47	47	47	47	47	47	49
50	48	48	48	48	48	48	48	48	48	48	48	48	50
51	49	49	49	49	49	49	49	49	49	49	49	49	51
52	50	50	50	50	50	50	50	50	50	50	50	50	52
53	51	51	51	51	51	51	51	51	51	51	51	51	53
54	52	52	52	52	52	52	52	52	52	52	52	52	54
55	53	53	53	53	53	53	53	53	53	53	53	53	55
56	54	54	54	54	54	54	54	54	54	54	54	54	56
57	55	55	55	55	55	55	55	55	55	55	55	55	57
58	56	56	56	56	56	56	56	56	56	56	56	56	58
59	57	57	57	57	57	57	57	57	57	57	57	57	59
60	58	58	58	58	58	58	58	58	58	58	58	58	60

Declination of the Sun.

TABLE VIII.

TIME OF THE SUN'S RISING AND SETTING.

Y

Time of the Sun's rising and setting.

App. time at sunrise		when the lat. and dec. are of different names. ^a												the same name.	
Declination of the Sun.															
lat.	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°	lat.	
00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
1	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
2	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
3	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
4	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
5	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
6	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
7	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
8	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
9	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
10	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
11	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
12	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
13	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
14	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
15	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
16	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
17	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
18	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
19	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
20	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
21	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
22	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
23	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
24	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
25	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
26	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
27	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
28	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
29	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
30	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
31	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
32	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
33	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
34	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
35	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
36	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
37	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
38	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
39	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
40	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
41	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
42	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
43	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
44	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
45	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
46	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
47	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
48	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
49	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
50	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
51	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
52	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
53	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
54	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
55	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
56	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
57	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
58	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
59	00	00	00	00	00	00	00	00	00	00	00	00	00	00	
60	00	00	00	00	00	00	00	00	00	00	00	00	00	00	

^a 120—App. time at sunrise, gives the time of setting.

TABLE IX.

Atmospherical Refractions. { Barom. 30 in. { Fa. Therm. 50°					
App. Alt.	Refraction.	Diff. for 10° Therm.	App. Alt.	Refraction.	Diff. for 20° Therm.
10°	5' 20''	07''	50°	1' 49''	09''
11	4 51	6	51	47	9
12	28	5	52	45	9
13	8	5	53	44	9
14	3 50	5	54	42	9
15	34	4	55	41	9
16	20	4	56	39	9
17	9	4	57	38	9
18	2 58	4	58	36	1
19	48	3	59	35	1
20	39	3	60	34	1
21	31	3	61	32	1
22	23	3	62	31	1
23	17	3	63	30	1
24	10	3	64	28	1
25	4	3	65	27	1
26	1 59	2	66	26	1
27	54	2	67	25	1
28	49	2	68	24	1
29	45	2	69	22	1
30	41	2	70	21	1
31	37	2	71	20	1
32	33	2	72	19	1
33	30	2	73	18	1
34	26	2	74	17	1
35	23	2	75	16	1
36	20	2	76	14	1
37	17	2	77	13	1
38	14	1	78	12	1
39	12	1	79	11	1
40	9	1	80	10	0
41	7	1	81	9	
42	5	1	82	8	
43	2	1	83	7	
44	0	1	84	6	
45	0 58	1	85	5	
46	56	1	86	4	
47	54	1	87	3	
48	52	1	88	2	
49	51	1	89	1	
50	49	0	90	0	

* When the Therm. is below 50° the correction in these columns is additive to the refraction.

TABLE X.

Augmentation of the moon's semi-diam't'r.	
Altitude.	Augment.
10°	3''
15	4
20	5
25	7
30	8
35	9
40	10
45	11
50	12
55	13
60	14
70	15
80	16

TABLE XI.

Sun's Alt.	P'l'x in alt.
15°	9''
35	8
45	7
55	6
65	5
70	4
75	3
80	2
85	1

MAURY'S NAVIGATION.

AN ELEMENTARY, PRACTICAL, AND THEORETICAL TREATISE ON
NAVIGATION: WITH A NEW AND EASY PLAN FOR FINDING DIFF.
LAT., DEP., COURSE, AND DISTANCE BY PROJECTION. BY M. F
MAURY, LIEUT., U. S. NAVY.

OPINIONS OF NAVIGATORS AND PROFESSORS.

Boston, April 21st, 1835.

A work of the kind you are preparing for the press, containing the demonstrations of the formulas of Nautical Astronomy, would be very useful to those who have a taste for the subject, and would like to examine the demonstrations of the rules.

Respectfully,

Your obedient servant,
N. BOWDITCH.

PHILADELPHIA, 14th April, 1835.

DEAR SIR:

I have examined, with as much care as the nature of my engagements has permitted, the book intended for the instruction of the younger officers of the navy, which you have left with me. My opinion is, that such a work will be valuable to them, and will meet with favour among them; supplying, as it does, the mathematical principles involved in the studies of their profession in a sufficiently condensed form.

Coming from one of their own profession, especially, I should anticipate that the work would be received among them, even without the injunction of the authorities, who would, however, I think, find it to the interest of the service to sanction it.

Allow me to say, that I consider the work fully to sustain the high character for scientific acquirement, which I have always heard attributed to its author.

Respectfully yours,

A. D. BACHE, Prof., &c.

The undersigned are of opinion that the work of Lieutenant M. F. Maury, on Navigation, is eminently useful as a school book for nautical students. It illustrates with clearness and simplicity, the principles on which the calculations in navigation are founded.

We felt the want of just such a book in aid of our early studies, and cheerfully recommend it to all who desire to inform themselves in this branch of education, with a view to the nautical profession.

FRANCIS H. GREGORY, Capt.
ROBERT F. STOCKTON, Capt.
FREDERICK ENGLE, Com.
G. A. MAGRUDER, Com.

MY DEAR SIR:

I have great pleasure in stating my belief that it is of the utmost importance that the midshipmen of the navy should have some established work, containing within itself all the information on mathematics and navigation, including nautical astronomy, which they are required to know in order to pass an examination for promo-

mentary, and embrace arithmetic, algebra, geometry, and plane and spherical trigonometry, so far, and so far only, as might be necessary to the construction of all the rules and formulæ requisite to solve the various problems in navigation and surveying: When I was preparing for examination, I felt an earnest desire to possess this information, and to ascend step by step to a complete understanding of the whole subject, so as to have the means of reaching all the processes of which I availed myself by my own resources, without taking any thing on trust. After my promotion I devoted my whole time and attention, for a considerable period, in attaining this object; and feeling how much my own course had been impeded by the want of books containing the required knowledge, I carefully preserved all that I had recourse to for the purpose; such as La Croix, Bezout, Légendre, Lassalle, Borda, and Callet, and determined at my earliest leisure to compile from them a treatise, narrowed down to what was indispensable, so as to spare others, ambitious of more complete information on this branch of professional knowledge than is usual, the great difficulty I had experienced in knowing where to apply for information. The appearance of your work, so exactly supplying what was needed, and, from your infinitely higher mathematical attainments, executed in so superior a manner, to what it would have been had the task been left to me, took from me all motive and desire to go on with the undertaking. After having exerted such commendable exertion, ingenuity, and judgment in accomplishing your task, I trust you may, at least, have the satisfaction of seeing your work generally used by the midshipmen, if not as a manual of practical navigation, which Bowditch's admirable work so effectually supplies, at least as an elementary treatise for the instruction of the young officers of the navy, the more acceptable and encouraging for having been supplied by one, who was, at the time, of their number.

Believe me, very truly,

And respectfully yours,

ALEX. SLIDELL MACKENZIE,
Commander, U. S. Navy.

TARRYTOWN, 14th November, 1843.

I should not hesitate to commend Maury's Navigation for the use of the midshipmen of our navy. To those of them who are advanced in the elements of the science, it supplies the *practical* information necessary to make them good navigators; others, who have commenced with

of the principles are both ample, easy, and well arranged.

The work, I know, originated in the wants of the students of navigation on board ships, and I confidently believe that it will supply those wants.

L. M. POWELL,
Commander, U. S. Navy.

U. S. NAVY YARD, GOSPORT, }
8th January, 1839. }

SIR:

I am much pleased with your "Treatise on Navigation." The mathematical investigations it affords of the principles of the science, particularly of nautical astronomy, place it upon different grounds from the treatises upon the subject in common use, and adapt it much better to the purpose of instruction. I am desirous of introducing it, as far as may be practicable, among my own classes; and recommend it to all the younger officers in our naval service, who desire to become acquainted with the theory, as well as the practice of the mathematical part of their profession. Its designation as one of the books to be used in their examination, would, I think, conduce to elevate the standard of mathematical attainments among them.

With great regard,
Your obedient servant,
JOHN H. C. COFFIN.
Prof. of Mathematics, U. S. Navy.

WASHINGTON, 20th Dec., 1842.

DEAR SIR:

I have been much interested in a hasty examination of your work upon *Navigation*, more particularly with that part of it, which relates to "Spherical Trigonometry," and "Nautical Astronomy," two branches of navigation that have been too superficially treated in the most popular works upon the subject, to answer the increasing general information of the practical navigators of the present day.

During the last forty years, but little improvement of this kind has been introduced into the standard works upon navigation, and as a general remark, it may be safely asserted that they are behind the wants of those who use them.

Ever anxious for the general diffusion of such important knowledge in our profession, I trust you will be encouraged to introduce the work into the navy as a *Text-Book*. I remain, respectfully, etc.,

JAMES GLYNN,
Commander, U. S. Navy.
Lt. M. F. MAURY, U. S. N.

U. S. SHIP DALE, 23d Oct., 1843.

I feel great pleasure in recommending Maury's *Navigation* as a work of real usefulness and importance to the young officers of our navy.

It embodies whatever can be of utility to the navigator, in a concise and perspicuous manner; and its explanations and references calculated to excite, in youthful and inquiring minds, a desire for the higher attainments. Having used

the work myself, I know its value, and, therefore, hope it will become the authorized book of study for the midshipmen of our navy.

THOS. A. DORNIN,
Commander, U. S. Navy.

FLAG SHIP PENNSYLVANIA, }
November 4th, 1843. }

MY DEAR SIR:

I have, with great pleasure and care, examined your book on *Navigation*, and do decidedly recommend and prefer it to any other in use. The mathematical principles condensed in such a form, is a sufficient recommendation to every student of navigation, and I recommend it to all mathematicians and young officers belonging to, and at schools attached to the navy, and should think that the Hon. Secretary of the Navy would so order it.

I have the pleasure to remain, with regard,
Your obedient servant,
E. P. KENNEDY,
Lt. M. F. MAURY, U. S. N. P. C., Norfolk.

U. S. SHIP PENNSYLVANIA, }
Nov. 3d, 1843. }

SIR:

I have been much gratified in the perusal of your *Treatise on Navigation*; and think it well adapted for use as a school book, and one best calculated, of any that I have seen, to induce a love for the prosecution of the study of navigation as a science, and not merely as an art.

I am, very respectfully,
Your obedient servant,
A. G. PENDLETON,
Prof. of Mathematics, U. S. Navy.
Lt. M. F. MAURY, U. S. N.

U. S. SHIP OHIO, BOSTON, }
Nov. 14th, 1843. }

SIR:

I have examined, with a good deal of attention, your *Treatise on Navigation*, and find it embraces all the elements necessary to constitute a scientific navigator. It would, I believe, be found a valuable auxiliary in our naval schools.

The work of Dr. Bowditch, although eminently useful as a *practical* one, fails almost entirely in the development of the principles from which its rules are derived.

It seems, therefore, desirable, that some work explaining more fully the *theory* of navigation should be put into the hands of our midshipmen, in order that they may become, as we all desire, *scientific*, as well as *practical* navigators.

Hoping that the pleasure of furnishing such a work for the younger officers of the service may be yours,

I remain, very respectfully,
Your obedient servant,
JOS. T. HUSTON,
Prof. of Mathematics, U. S. Navy.
Lt. M. F. MAURY, U. S. N.

U. S. SHIP BOSTON, BOSTON, }
Nov. 15th, 1843. }

DEAR SIR:

I am much pleased to learn, through the

Army and Navy Chronicle, that a new edition of your valuable Navigator is soon to be published.

I cannot doubt the success of a large second edition, and I am confident that it will add to your reputation, and secure for your book the celebrity which it deserves. Its merit is recognised in the navy, and I would recommend it in all schools of navigation, particularly on account of the manner in which many difficult and obscure points are made easy and plain.

Respectfully, etc.,

G. J. PENDERGRAST.

Commander U. S. Navy.

Lt. M. F. MAURY, U. S. Navy.

WILMINGTON, Del. Nov. 5th, 1843.

I consider Maury's Treatise on Navigation, the very work that has long been wanting in our schools, and I hope it will eventually be used in all of them (particularly the naval ones) instead of Bowditch's Practical Navigator; for I think it far superior as a book of instruction.

J. SHUBRICK,

Commander, U. S. N.

NORFOLK, Nov. 7th, 1843.

I think Maury's Navigation is admirably adapted for the instruction of the young officers of the Navy. All the problems are deduced from theorems, in such a manner as to give the young seaman a correct idea of the theory as well as the practice of navigation. The methods are simple and accurate, and the tables well and carefully constructed. The work contains all that the student of navigation can require; and were it made the authorized text book of the Navy, the standard of mathematical attainments among midshipmen, would be greatly elevated.

R. B. CUNNINGHAM.

Commander, U. S. Navy.

NOVEMBER 8th, 1843.

I possess a copy of Maury's Navigation, and consider it a valuable text-book for all nautical students, whether in the United States Naval Service, or in the commercial marine.

The author, Lieutenant M. F. Maury, U. S. Navy, is a man of science, and has produced this work under the advantage of knowing from experience what the nautical student and practical navigator require.

S. P. LEE,

Lt. U. S. Navy.

U. S. NAVAL HOSPITAL,
New York, Nov. 10th, 1843. }

SIR:

Your volume seems to me well calculated to achieve the object for which it seems designed: namely, to demonstrate the formulas of Nautical Astronomy, and explain the principles upon which the art of navigation is founded. A better book for schools of navigation, than yours, I am persuaded does not exist in our language. But

after the expression of favourable opinions of it, by such men as Bowditch, Alexander Dallas Bache, P. J. Rodriguez, Edward C. Ward, and John H. C. Coffin, all eminently qualified to judge of such a work, few can doubt its worth or set any value upon the opinion of,

Very respectfully,

Your obedient servant,

W. S. W. RUSCHENBERGER,

Surgeon, U. S. N.

Lieut. M. F. MAURY, U. S. Navy.

U. S. SHIP CUMBERLAND, }
Boston, Nov. 14th, 1843. }

DEAR SIR:

From the cursory examination I have given your treatise on navigation, I, for one, am proud that so useful and valuable a work has been furnished by one of our own corps.

With the improvements you propose to add in the new edition you are preparing, I doubt not it will possess advantages over other works of the kind, and be found a valuable auxiliary in our naval schools.

Very truly yours,

JOSEPH SMITH,

Captain U. S. Navy.

Lieut. M. F. MAURY, U. S. Navy.

WASHINGTON CITY, Nov. 23d, 1843.

DEAR MAURY,

I take this occasion to express to you the pleasure I feel, in noticing the announcement of a new edition of your treatise on navigation. Its subject matter being strictly professional, has called for the close scan of many of your brother officers, myself of the number. Its worthiness to become the text book of our young naval officers, may, with propriety, be judged of by those who are called on to exhibit a certificate of having been closely examined on, and found to possess a thorough knowledge of the subject treated of in your book alluded to.

Without an exception, all of our brother officers, and they are many whom I have heard descant freely on the merits of your work on navigation, pronounced it to be the best text book on that subject extant. In a full concurrence with that opinion,

I am with much esteem,

Respectfully yours truly,

WM. W. HUNTER,

Lieut. U. S. Navy.

Lieut. M. F. MAURY, U. S. Navy.

U. S. BRIG PERRY, }
Norfolk, Va., Nov. 24th, 1843. }

DEAR SIR:

I am pleased to learn you are preparing an improved edition of your Navigation, more especially with the view of instructing the midshipmen in the theory of navigation. To carry the student beyond the mechanical solution of nautical problems, into a comprehension of those principles of Mathematics and Astronomy, upon which these problems are based, seems to have been wanting in most treatises on Navigation.

Even Dr. Bowditch's invaluable epitome, that has laid every American, who has to trace his way on the great deep, under a lasting debt of gratitude, is wholly practical in its method. But the value of a Class Book proposing to teach the young officer the theory of one branch of his profession, which imbues his mind with some tincture of science, and raises him above the blind worker of mechanical problems, need not be enlarged upon; though it cannot be better appreciated than by those in the profession, who were deprived of the many advantages now offered to our midshipmen, and who were compelled to prepare for their examination with a Practical Navigator for their sole instructor, and a camp-stool between two guns, for their study room.

Uniformity in instruction too is a matter of great importance, and not doubting that the superior authority, who alone can prescribe the book to be taught, will fully appreciate the honorable contribution afforded by your work, to the character and benefit of the Navy, and that it will be made the standard at the examination of midshipmen,

I remain, my dear sir,
Very sincerely, &c.
S. F. DUFOUR,
Commander, U. S. N.

Lieut. M. F. MAURY, U. S. Navy.

ST. AUGUSTINE, FL. FLORIDA,
Nov. 27th, 1843. }

My opinion can contribute nothing to the established reputation of your valuable work. I can only say, that when I first looked through it, I considered it as thoroughly supplying the great desideratum of a midshipman's study of navigation, and remarked to those who were present, "How unfortunate were we, in not having such a book when we were students of navigation." My opinion has been confirmed by that of other and more competent judges, and I believe that throughout the entire service, there has not been a disparaging voice raised against it. I regard it as being to Bowditch what Bowditch was to Hamilton Moore.

Respectfully, &c.
WM. F. LYNCH,
Lieut. U. S. Navy.

Lieut. M. F. MAURY.

YALE COLLEGE, CONN. }
October 26th, 1843. }

This valuable work on Navigation, theoretical and practical, it seems to me desirable to have placed in the hands of every naval officer, who may have occasion to navigate a ship or to explain the principles of Nautical Astronomy. These are well brought out, and illustrated with examples of their application, which render the treatise clear and intelligible, and adapt it well to the purposes of a text book for learners. This book as a guide in the rational principles of navigation, and Bowditch's companion in the practical computa-

tions, the young officers of our Navy may be well prepared for the important and responsible duties of navigators. Having formerly used the work at sea, while engaged as an instructor in the naval service of the U. S., I cordially give it this recommendation.

JAMES NOONEY, Jr.

Tutor in Nat. Philosophy,
Late Prof. of Math. U. S. N.

SURVEYING SCHR. GALLATIN, }
Philadelphia, Dec. 13th, 1843. }

DEAR SIR:

I have learnt with pleasure that a second edition of your work on Navigation will shortly be published.

I have always considered this book, from its general arrangement, and the kinds of solution employed, as decidedly the best work for students extant.

Believe me, truly yours,
(Signed) GEO. S. BLAKE,
Lt. M. F. MAURY, U. S. N. Washington.

Opinion of Capt. M'INTOSH, U. S. N. }
New York, Dec. 12th, 1843. }

It affords me great pleasure to state, that I consider your work on Navigation, as one of the very best now extant, and most cheerfully recommend it as most suitable for a school book for midshipmen.

Lieut. MAURY.

Opinion of Capt. PERCIVAL, U. S. N. }
U. S. SHIP CONSTITUTION, }
Gosport, Dec. 14th, 1843. }

In compliance with your request, I have perused Maury's "New Theoretical, and Practical Treatise on Navigation," and have found it, as far as I am able to judge, a convenient reference, in illustrating the principles of Nautical Astronomy. Its explanations of the principles of Spherical Trigonometry, and its application of them to the solutions of the various astronomical problems, so essential in navigation, render it a book, in my opinion, not only useful as a supplement to our first (Bowditch's) standard work on the subject; but valuable in itself: an acquisition to the nautical student, who, if he is desirous of acquiring a correct, and practical knowledge of his profession, may be largely aided by the study thereof.

Lt. M. F. MAURY, U. S. N.

Opinion of Capt. FORREST, U. S. N. }
WASHINGTON, Dec. 18th, 1843. }

I take much pleasure in recommending your "New Theoretical and Practical Treatise on Navigation." The explanations and illustrations are rendered clear and comprehensive, and I believe it to be just such a production as we require for the instruction of the young officers of our Navy, as well as others desirous of obtaining a well grounded and accurate knowledge of the science.

Lt. M. F. MAURY, U. S. N.

*Opinion of the U. S. Naval Lyceum, }
Brooklyn, New York. }
BROOKLYN, Dec. 20th, 1843.*

The undersigned, a committee to which was referred Lieut. Maury's "Treatise on Navigation," report that they have carefully examined the same, and are of opinion that it is a work well adapted for the instruction of the young officers of the Navy, as all the problems and formulæ that are necessary in their profession, are there brought together in a condensed form, and so clearly, and concisely demonstrated, that the student may easily inform himself of the theory and principles on which his practice is founded; and the accompanying tables are so constructed as to facilitate his calculations; all of which are systematically arranged, with a simplicity that has heretofore been generally wanting in works on Navigation. They would therefore recommend the same to be adopted for the use of the Naval schools.

Signed,
J. H. STRINGHAM,
WM. D. NEWMAN,
ALEX. C. GIBSON.

The following opinions have already appeared in print, but as they have in all probability escaped the notice of many to whom these pages will be presented, they are again inserted here.

U. S. N. S., New York, January 19, 1836.

"Dear Sir,—I have had much pleasure in the perusal of your "New Theoretical and Practical Treatise on Navigation;" the plan and arrangements of which are original; it contains little or nothing superfluous, and every part of it appears to be as clear and intelligible as the nature of the subject will admit. Such a work has long been wanted in our Naval Schools, and on board our vessels of war. I intend to make use of it in the Naval School on this station; and I recommend it to be used by all the professors of Mathematics and Nautical Science, in the Navy of the United States.

"Yours Respectfully,

"EDW. C. WARD,

"Prof. Math. U. S. Navy."

"Passed Midshipman M. F. Maury,

"U. S. Navy."

U. S. Navy Yard, Gosport, March 7, 1836.

"I have examined a Treatise on Navigation written by M. F. Maury of the U. S. Navy; and have no hesitation in recommending it to the students of that science. The explanations are clear, the rules are illustrated by many examples, and the new arrangement of some of the tables simplify the calculations of the navigator. Mr. Maury is deserving of great credit for the work, and I wish him every success.

P. J. RODRIGUEZ.